In this paper, a rigid structure is introduced into the acoustic metasurface of layered medium with acoustic modulation function, which is based on the theory of the equivalent medium and different media are replaced by curly labyrinth. An acoustic metasurface is designed to achieve simultaneous acoustic modulation and attenuation in the air, the adjustment range is from 30 to 90 degrees, and sound attenuation is over 40%. According to the theory of layered medium modulation the direction of sound waves, a metasurface which can be applied to modulate direction of underwater acoustic wave is designed and the relationship between structure size and working frequency is analyzed by simulation. At last, the theory of layered medium and equivalent medium are combined to design an arbitrary underwater acoustic metasurface structure which is based on a curly labyrinth, theoretical calculation and simulation analysis are done to verify the effect of this structure. The study of this paper can be used to design a rigid underwater acoustic metasurface with low frequency, adjustable direction and sound attenuation.

Keywords: Layered medium, Equivalent medium, Modulation direction of acoustic wave, Curly labyrinth

1. Introduction

Traditional acoustic structures are basically made of homogeneous materials, their acoustic characteristics generally depend on their own physical characteristics. In early studies of electromagnetic metamaterials, negative permeability and negative permittivity were found\(^1\), negative refractive index was found in the study of photonic crystals\(^2\). Analogous to electromagnetic metamaterials and photonic crystals, in the field of acoustics, periodically distributed artificial microstructures can also demonstrate extraordinary performances that traditional acoustic materials do not possess. For example, high transmission characteristics can achieve acoustic lossless focusing\(^3\), low reflection and high absorption properties can be used for acoustic absorption\(^4\), negative refraction can realize acoustic manipulation and high resolution acoustic holography\(^5\). In addition, acoustical metasurfaces can also achieve acoustics stealth, acoustical super-lens, acoustics tunnel and other functions\(^6,7\).

Liu et al. designed a spiral-labyrinthine acoustic metamaterial that can realize acoustic isolation, acoustic stealth and acoustic supertunneling\(^8\); Bai et al. proposed a metasurface to achieve high efficient underwater sound insulation through sealed air\(^9\); Song et al. utilized an acoustic metamaterial cavity of a labyrinth structure to increase the incident energy by more than 10 dB at the resonant frequency\(^10\); A O Krushynska et al. designed a labyrinth metasurface structure that can realize low-frequency acoustic manipulation through an analog spider web\(^11\); Lan et al. proposed a gradient metasurface that can modulate the wavefront which is based on the Helmholtz resonator\(^12\); Tian et al. proposed a high-resolution acoustic holographic imaging metasurface that is based on the theory of curly labyrinth and micro perforated plate\(^13\); Mei et al. based on impedance matching and proposed a metasurface that controls acoustic transmission and reflection\(^14\).
At present, most of the researches are aimed at the metasurfaces that absorb, isolate and manipulate acoustic waves in the air at the subwavelength, this paper is based on impedance matching and equivalent medium theory, introduced a curly labyrinth structure into layered medium metasurface and designed an acoustic meta surface that can achieve underwater acoustic waves manipulation, and the relationship between structure size and working frequency is verified. Finally, based on impedance matching and equivalent medium model, an arbitrary underwater acoustical metasurface structure with low frequency, high attenuation, adjustable direction and strong stability is designed.

2. Model introduction

2.1 A layered medium model based on impedance matching

It is assumed that metasurface is formed by slits periodically embedded into rigid thin plates and slits are filled with three kinds of medium. There are \( m \) penetrating slits between thin plates, the height of plate \( h=0.5d \), width \( q=0.05d \), slit width \( w=0.2d \), then unit cell width of metasurface \( d=m(w+p) \). When acoustic waves pass through the metasurface, the transmitted wavefront must be maintained continuously and smoothly, then the phase gradient change must be satisfied \( d\Phi/dx = 2\pi/d \), adjacent slit phase gradient change is \( \Delta \Phi = 2\pi/m \). Assume that first slit is filled with background media, so phase change is \( \Delta \Phi = -k_1h \), the i-th phase change satisfies \( \Phi_i = \Phi_1 + (i-1)\Delta \Phi = \Phi_1 + 2\pi (i-1)/m = \omega h/c_i, c_i = c_0/n_i, c_i \) is the speed of acoustic waves passing through multi-layer medium, \( n_i \) is relative refractive index of a multi-layer medium, \( c_0 \) is sound speed of the background medium. When the sound wave is vertically incident from the medium 1 to the medium 2, reflection and transmission coefficients are \( r_p = (Z_2-Z_1)/(Z_2+Z_1), t_p = 2Z_2/(Z_2+Z_1) \), respectively. \( Z_1, Z_2 \) are the characteristic impedance of medium 1 and 2, and \( Z_i = \rho_i c_i \) \( (i=1,2) \). When the impedance is matched, \( Z_1 \rightarrow Z_2, r_p \rightarrow 0, t_p \rightarrow 1 \). Thus, the target of enhancing transmission and restraining reflection is achieved.

The phase change can cause the oblique transmission wave, according to the generalized Snell law, it must be satisfied \( k_1 \sin \theta - k_2 \sin \theta' = d\Phi/dx \), \( \theta \) is incident angle, \( \theta' \) is transmission angle.

In order to verify this theoretical model, background medium is air, the other two impedance matching media are argon and xenon. Each metasurface unit cell has 4 slits, according to the phase change formula, phase change in the i-th slit satisfies \( \Phi_i = k_{iA}h_{iA} + k_{iX}h_{iX} = \Phi_i + (i-1)\pi/2 \) \( (i=2,3,4) \). \( k_{A} = 2\pi f_0/c_0 \) and \( k_{X} = 2\pi f_0/c_X \) are argon and xenon wave numbers \( (f_0 \) is working frequency), \( h_{iA} \) and \( h_{iX} \) are the height of argon and xenon in the i-th slit, respectively.

In the sub-wavelength size, selecting the width of the metasurface unite cell is \( d=100 \) mm, the working wavelength \( \lambda_0 = 0.6 \) m. When the plane wave is incident vertically, incident angle \( \theta_0 = 0^\circ \). According to the generalized Snell law, the corresponding working frequency \( f_0 = 5716 \) Hz and deflection angle 36.9° can be calculated. The height of medium in different slit is calculated by layered medium model. A unit cell of the acoustic metasurface is drawn from the calculated geometric parameters by software Solidworks2013, the unit cell is arrayed four times along the x direction to obtain a layered media acoustical metasurface. This metasurface is imported into the comsol5.3a acoustic-solid coupling module for finite element simulation, the result is shown in Fig. 1(a).

The result in Fig. 1(a) shows that when working wavelength \( \lambda_0 = 0.6d \), working frequency and deflection angle calculated by the layered media model are in good agreement with the results of the finite element simulation. It is proved that this model can be used for sub-wavelength acoustical metasurface that manipulation the direction of sound waves design.
2.2 A curly labyrinth model based on equivalent medium

The theory of equivalent medium refers to the design of a metasurface by different ways of folding or curling space, so that the propagation path of acoustic waves through a metasurface is different, thus sound speed and impedance in metasurface can be controlled. Equivalent to metasurface as a homogeneous medium with controllable sound speed and impedance, a representative curly labyrinth structure is shown in Fig. 1(b).

When sound wave passes through the curly labyrinth structure in Fig. 1(b), according to the continuous equation of sound pressure and volume velocity at the mutation interface, it can be deduced that equivalent impedance and equivalent sound speed of the curly labyrinth structure are:

$$
Z = \frac{h \rho_0 c_0}{w + A} \\
c = \frac{n c_0}{(l + d + w)n + d + B}
$$

In the form, $h, t$ are width and thickness of the labyrinth structure, and $t=(w+d)n+d$, $\rho_0, c_0$ are the density and sound speed of the background medium, respectively. $l, d, w$ correspond to the structural parameters marked in Fig. 1(b), $n$ is curled times of the labyrinth structure. $A$ and $B$ are correction factors for achieving a transmission factor of 1, their values are far less than the minimum structure parameters, so they can be ignored in this paper.

3. Simulation calculation and discussion

3.1 Aeroacoustic metasurface of curly labyrinth structure

Based on the equivalent medium theory, a spiral labyrinth structure model is introduced into layered medium, and an acoustic metasurface which can achieve deflection direction in air is designed. The acoustic metasurface working frequency is $f_0=5300$Hz, deflection angle $\theta=40^\circ$. The sound speed and impedance of air, argon and xenon are substituted in the formula of phase change respectively. The height distribution of medium can be obtained as shown in Table 1 as follows.

<table>
<thead>
<tr>
<th>Slit (i)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argo height(mm)</td>
<td>—</td>
<td>36.40</td>
<td>19.61</td>
<td>2.81</td>
</tr>
<tr>
<td>Xenon height(mm)</td>
<td>—</td>
<td>13.60</td>
<td>30.39</td>
<td>47.19</td>
</tr>
</tbody>
</table>

Air as background medium is filled in the first slit of the metasurface. For the next 3 slits filled with argon and xenon, the density and impedance of different media and medium height in table 1...
are substituted into curly labyrinth model, six equivalent labyrinth microstructures can be obtained. Taking the filling medium xenon in the slit 2 as an example, the geometric parameters of the equivalent labyrinth microstructure are as follows:

When \( i=2 \), the geometric parameters of the equivalent labyrinth structure of xenon are: \( t_2=t_2\text{Xen}=13.60\text{mm}, \) \( l_2=7\text{mm}, \) \( d_2=1\text{mm}, \) \( w_2=5.3\text{mm}, \) \( n=2. \)

According to above geometric parameters, the geometric diagrams of labyrinth microstructures which is equivalent to xenon in the second slit drawn by the Solidworks 2013 are shown in Fig. 2. The metasurface unit cell consisting of six labyrinth microstructure and background medium is shown in Fig. 3(a). The unit cell was arrayed four times along the \( x \) direction in the figure to obtain an acoustical metasurface which is composed of five cells, as shown in Fig. 3(c).

![Figure 2: Schematic diagram of labyrinth microstructures (xenon equivalent in the second slit)](image)

![Figure 3: (a) Curly labyrinth equivalent acoustic metasurface cell; (b) Acoustic metasurface effect diagram; (c) Acoustic metasurface composed of five cells.](image)

The background medium is air which is the light blue part of Fig. 3(a). Rigid structure material is ABS plastic which is the yellow part in Fig. 3(a), its density is 1190kg/m\(^3\), sound speed is 1360m/s. The plane sound waves with 1Pa amplitude of sound pressure are vertically incident below the metasurface, the effect is shown in Fig. 3(b).

As can be seen from Fig. 3(b), when the working frequency is 5300Hz, deflection angle is 40°, this result is in good agreement with the prediction of layered media theory. Thus it can be seen that introducing a curly labyrinth into a layered media model, the design of the labyrinth acoustic metasurface can also manipulate sound waves. The introduction of the labyrinth structure not only increases stability of metasurface, but also equates to a medium that does not exist in nature, this greatly broadens the applications of acoustical metasurface.

### 3.2 Underwater metasurface of curly labyrinth structure
Next, for the demands of underwater acoustic control, an acoustic metasurface for underwater is designed. Based on the impedance matching theory, two kinds of media that are close to water impedance are selected, namely glycerol and cryogen c318. The physical parameters of the three media are shown in Table 2.

Table 2: Three media physical parameters of underwater acoustic metasurface

<table>
<thead>
<tr>
<th>Media</th>
<th>Parameter</th>
<th>Sound speed (m/s)</th>
<th>Density (kg/m³)</th>
<th>Impedance (Pa · s/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td></td>
<td>1,500</td>
<td>1,000</td>
<td>1.5×10⁶</td>
</tr>
<tr>
<td>glycerin</td>
<td></td>
<td>1,904</td>
<td>1,260</td>
<td>2.4×10⁷</td>
</tr>
<tr>
<td>cryogen c318</td>
<td></td>
<td>574</td>
<td>1,620</td>
<td>9.3×10⁶</td>
</tr>
</tbody>
</table>

Analogous to air layered acoustic metasurface, unit cell width \( d = 100 \text{mm} \), selected working wavelengths are \( 0.6d \), the working frequency \( f_0 = 25 \text{kHz} \) and the deflection angle \( \theta = 36.9^\circ \) are calculated by layered media theory. According to the phase change formula, the height of the filling medium in different slits is calculated.

According to the geometric parameters of the three media, corresponding underwater acoustical metasurface unit cell structure is shown in Fig. 4(a). The gray part of the figure is filled with background media water, the unit cell is arrayed four times along the \( x \) direction in Fig. 4, and the resulting metasurface composed of five unit cells is shown in Fig. 4(c). Figure. 4(b) represents the effect of underwater acoustic metasurface obtained by finite element simulation.

From the simulation results in Fig. 4 (b), the deflection angle \( \theta = 37^\circ \) at frequency \( f_0 = 25 \text{kHz} \), this is in good agreement with the results predicted by the theoretical model, it is proved that layered medium model based on impedance matching is also suitable for underwater acoustic metasurface design.

The impedance of water is about 3390 times that of the air impedance, so when the underwater and air acoustic metasurface structure is the same size, the working frequency of underwater metasurface up to 25kHz. For purpose of low frequency applications, the underwater metasurface with low frequency is designed in this paper. The width of unit cell is increased by 10 times, that means \( d=1 \text{m} \). According to structural parameters which is calculated by layered medium model, finite element simulation results of low frequency underwater acoustic metasurface are shown in Fig. 5.

It can be seen from Fig. 5, when metasurface structure size increased 10 times, the working frequency decreased 10 times, which achieves the low frequency target. Further study found that there is a linear relationship between the super size and surface structure of the working frequency, the structure size increases by \( n \) times, working frequency is reduced by \( n \) times.
Figure 5: Underwater low frequency acoustic metasurface effect diagram (working frequency $f_0=2500\text{Hz}$)

At last, an arbitrary low frequency underwater acoustic metasurface is designed by introducing the curly labyrinth into the layered medium theory. Assuming given metasurface height $h=0.25\text{m}$, the sound speeds of two media are $c_1=409.1\text{m/s}$, $c_2=1295.5\text{m/s}$, unite cell width $d=1\text{m}$, working wavelength $\lambda_0=0.7d$. These physical and geometrical parameters are substituted into into layered media theory to calculate deflection angle $\theta=44.4^\circ$. The heights of the two media are calculated by phase change equation, based on the curly labyrinth model, six labyrinth microstructures are introduced to equalize the medium of different heights. Taking the medium 2 in the second slit as an example, the geometric parameters of the equivalent labyrinth microstructure are as follows: $t_2=206\text{mm}, l_2=32\text{mm}, d_2=88\text{mm}, w_2=30\text{mm}$, its structure is shown in Fig. 6. (slit 1 fills background medium water, slit width $w=0.2\text{m}$, rigid structure width $q=0.05\text{m}$).

When the microstructures are combined up and down, in order to ensure the circulation of the background medium, an offset occurs along the $x$ direction, so that the width of the unit cell becomes $d=1.5\text{m}$. The unit cell are arrayed four times along the $x$ direction in Fig. 7 to obtain arbitrary underwater metasurface, as shown in Fig. 7(a). The finite element simulation results are shown in Fig. 7(b).

Figure 6: Geometric diagram of labyrinth microstructures (medium 2 equivalent in the second slit)

Figure 7: (a) Underwater acoustic metasurface unit cell structure; (b) Underwater acoustic meta surface effect diagram.
The results of Fig. 7 (b) show that the deflection angle of the underwater acoustic metasurface is 44.5° when the working frequency is \( f_0 = 1429 \text{Hz} \), while the results of the theoretical model calculation are that working frequency is \( f_0 = 1429 \text{Hz} \), and the deflection angle is 44.4°. The results of simulation and theoretical calculation are in good agreement, it is proved that the combination of the layered media and the equivalent media model can not only achieve the acoustic wave manipulation of underwater acoustic metasurface with any size, but also can greatly broaden its low-frequency application.

The labyrinth acoustic metasurface can not only manipulate the direction of sound waves, but also the attenuation of sound waves is significant. In order to further analyze the attenuation ability of labyrinth structure, the transmission coefficients of metasurface of layered media (Fig. 1a) and curly labyrinth metasurface (Fig. 4b) in air are calculated respectively, as shown in Fig. 8.

![Figure 8: (a) Layered medium metasurface transmission coefficient curve; (b) Curly labyrinth metasurface transmission coefficient curve.](image)

It can be found that transmission coefficient of layered medium acoustic metasurface is 0.9587 at working frequency \( f_0 = 5176 \text{Hz} \), and the average transmission coefficient is 0.959 in the wide range of 50Hz studied. When the labyrinth structure is equivalent to layered medium, the transmission coefficient of the acoustic metasurface of curly labyrinth is 0.76 at frequency \( f_0 = 5300 \text{Hz} \), the average transmission coefficient in the 50Hz frequency band is 0.7592. The transmission coefficient of the two kinds of metasurface decrease with the increase of frequency. Average acoustic energy attenuation of layered medium acoustic metasurface is only 8%, while that of labyrinth metasurface is up to 42%, the sound attenuation ability increased by 4.25 times. It can be seen that the introduction of the curly labyrinth structure obviously enhances the attenuation ability of the metasurface.

4. Conclusion

In this paper, a curled labyrinth structure was introduced into the aeroacoustic metasurface of the layered media. The acoustical metasurface that can be used to achieve sound wave manipulation and acoustic isolation in the air was designed and simulated; based on the layered medium model, an acoustic metasurface which can be used for underwater acoustic wave designed and the corresponding relationship between underwater metasurface size and working frequency is studied. Finally based on the layered medium and the curly labyrinth model, an underwater acoustic metasurface with arbitrary size is designed. The theory and simulation prove that the underwater metasurface structure is simple and can be used to manipulate and attenuate the low frequency underwater acoustic wave.

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References


