To get higher energy storage in flywheel, the rotor may become slimmer. A composite flywheel stored energy of 54MJ in length of 1800mm and diameter of 450 mm was proposed. Elastic connection element is used to keep the composite cylinder for energy storage rotor in rigid state without bending mode for high speed rotation. The vibration characteristics of two composite cylinder with elastic connection and supports are studied. Supposing the cylinders are rigid, the Lagrange vibration equation is established to learn the low frequency modal characteristics. Both modal frequency and vibration mode are solved, and then the variation of modal frequency and mode attenuation coefficient with damping are discussed. The rotor support system is also calculated by the transfer matrix-polynomial method and the bending mode of the flywheel rotor with bellow connection is obtained. The modal frequencies calculated by the two methods are basically the same, the first four order forward precession are 4.3Hz, 8.3Hz, 20.5Hz and 533.3Hz. Considering mode frequencies varies with speed, corresponding critical speeds are 4.3 rps, 8.9 rps and 26.8rps under the rated speed of 500 rps. The vibration behavior was revealed when the damping of the support changes. The special vibration mode switching in such low support stiffness and overdamped rotor system are discussed.

Keywords: flywheel energy storage

1. Introduction

Energy storage is deemed as one of the solutions for stabilizing the supply of electricity with renewable energy such as solar and wind energy due to their fluctuation performance [1]. There are many kinds of EES technologies, including pumped hydro-electricity storage (PHS), compressed air energy storage (CAES), superconducting magnetic energy storage (SMES), and super capacitor energy storage (SCES), flywheel energy storage (FES) and different types of batteries [2].

Flywheel energy storage (FES) works by accelerating a rotor (flywheel) to a very high speed and maintaining the energy in the system as rotational energy. When energy is extracted from the system, the flywheel's rotational speed is reduced as a consequence of the principle of conservation of energy; adding energy to the system correspondingly results in an increase in the speed of the flywheel. FES is composed by rotor in large rotational inertial, motor/generator, bearing, sealing chamber and power electronic interface and running monitoring instruments [3].

Flywheel energy storage has many merits such as high power density, long cycling using life, fast response, observable energy stored and environmental friendly performance. The advantageous features make FES a very suitable option for different applications such as braking energy storage and reusing in transportation, uninterrupted power supply, frequency regulation for grid and wind power smoothing [4].
A single flywheel stored energy of 0.5-130 kWh in charging or discharging with power of 0.3-3000kW in industry application or experimental development of FES. The kinetic energy stored in a flywheel is proportional to the inertia and the square of its rotating speed designed as high as possible.

The vibration problems such as synchronous vibration from mass unbalance, passing through vibration resonance, sub-harmonic or super-harmonic vibration and stability of the rotor-bearing system become serious for high speed machine whose rotational speed is up to 10000 rpm higher than multiple critical speeds [5]. Bearings technology is a key to FES. An optimal control system is proposed by incorporating cross-coupling technology into the control architecture to improve the synchronization performance of the rotor in the radial direction. The balancing of rotor for high speed machine is important and necessary [6].

To get higher energy storage in flywheel, the rotor may become slimmer. The blending mode of the rotor may locate under the rated running speed. The wide running speed of the flywheel motor make the rotor pass through the critical speed frequently, which is not recommended.

Elastic connection element is used to keep the composite cylinder for energy storage rotor in rigid state without bending mode for high speed rotation. The vibration characteristics of two composite cylinder with elastic connection and supports are studied. Supposing the cylinders are rigid, the Lagrange vibration equation is established to learn the low frequency modal characteristics. Both modal frequency and vibration mode are solved, and then the variation of modal frequency and mode attenuation coefficient with different damping are discussed.

2. Configuration of flywheel rotor

2.1 Configuration

The flywheel rotor analysed in this paper is a three-layer composite flywheel, which structure is shown in figure 1.

![Figure 1: Structure of flywheel rotor.](image)

The rotor consists of two composite flywheels, connected by bellows as flexible connection element in the middle. Each single flywheel is a 3-layer composite flywheel.

Size parameters for single flywheel are as follows: The length is 900mm, inner diameter is 270mm and outer diameter is 450mm. The thickness of each layer is 30mm. Flywheel material parameters are shown in Table 1.

<table>
<thead>
<tr>
<th>Category</th>
<th>Layer 1</th>
<th>Layer 2</th>
<th>Layer 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber material</td>
<td>GF-Sglass</td>
<td>T700+Sglass 1:1</td>
<td>CF-T700</td>
</tr>
<tr>
<td>Matrix material</td>
<td>Epoxy</td>
<td>Epoxy</td>
<td>Epoxy</td>
</tr>
<tr>
<td>Fiber volume content</td>
<td>65%</td>
<td>65%</td>
<td>65%</td>
</tr>
<tr>
<td>Thickness/mm</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Ring modulus/GPa</td>
<td>56.5</td>
<td>103.9</td>
<td>150.7</td>
</tr>
</tbody>
</table>
From above parameters, the mass and the moment of inertia of single flywheel can be calculated, and values are 162.9 kg and 5.5 kgm², respectively. The stored energy in the flywheel (length being 1800 mm, diameter being 450 mm) is 54MJ at the rated speed of 30000 rpm with tip speed of 707 m/s.

### 2.2 Strength Analysis

Strength analysis on the composite flywheel is calculated at 500 rps, using MATLAB program and the results are shown in figure 2.

![Figure 2: Hook stress and radial stress in different radial positions.](image)

As shown in figure 2, the minimal hook stress is 370MPa and the maximum is 780MPa. The maximum radial stress is 5MPa in layer 3 (Tensile stress) and -7MPa in the junction of layer 1 and 2 (Compressive stress). The maximum radial deformation is 1118μm in the inner sidewall, while the minimal is 970μm in the outer side. Calculation result show that structure of multi-layer composite flywheel can effectively reduce stress levels. The stress in lower much than the strength both at the radial and circumferential direction.

### 3. Rotordynamics analysis

#### 3.1 Model configuration

To avoid the blending mode vibration from slimmer rotor, the dynamic model of the composite flywheel rotor mentioned above is mainly composed of two composite flywheels connected by bellow as elastic element, upper and lower supports and lower damper, as shown in figure 3 (in horizontal expression diagram). However, the flywheel is in vertical support bearing system configuration.

The following assumptions are made to simplify the rotor-bearing system: only lateral vibration (in X and Y directions in figure 3) is considered and the amplitude is small; the flywheels are rigid; the bellow is flexible and the height and lateral shear motion are ignored; the lower damper is simplified as mass, stiffness and damping and the others are just simplified as stiffness. The parameters in figure 3 are illustrated in table 2.
The kinetic energy of the rotor-bearing system in figure 3 is

$$ T = \frac{1}{2} \left[ \frac{m_1}{4} + \frac{J_1}{l_1^2} \right] (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} \left[ \frac{m_2}{4} + \frac{J_2}{l_2^2} \right] (\dot{x}_2^2 + \dot{y}_2^2) + \frac{1}{2} \left[ \frac{m_3}{4} + \frac{J_3}{l_3^2} \right] (\dot{x}_3^2 + \dot{y}_3^2) + \frac{1}{2} \left[ \frac{m_4}{4} + \frac{J_4}{l_4^2} \right] (\dot{x}_4^2 + \dot{y}_4^2) $$

(1)

The potential energy of the rotor-bearing system is

$$ U = \frac{1}{2} k_{01} (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} k_{12} \left[ \frac{x_1 - x_2}{l_1} - \frac{x_3 - x_4}{l_2} \right]^2 + \frac{1}{2} k_{23} \left[ \frac{y_1 - y_2}{l_1} - \frac{y_3 - y_4}{l_2} \right]^2 $$

(2)

The consumed energy of the rotor-bearing system is

$$ Z = \frac{1}{2} c_0 (\dot{x}_4^2 + \dot{y}_4^2) $$

(3)

According to above assumptions, the height of bellow is ignored, thus $r_2=r_3 (r_1=x_1+y_1)$, which adds another constraint to the motion equation. Using the Lagrange’s equation

$$ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} + \frac{\partial Z}{\partial q_i} = \ddot{q}_i $$

(4)

The vibration equation of the rotor-bearing system is conducted as

$$ [M] \ddot{\mathbf{q}} + [C] \omega \mathbf{q} + [H] \mathbf{q} + [K] \mathbf{q} = \mathbf{F} $$

(5)

Where, $M$, $C$, $H$, $K$ are mass, damping, gyroscopic and stiffness matrix.
\[ \{r\} = \{r_1, r_2, r_3, r_4\}^T, \quad r_i = x_i + iy_i, i = 1, 2, 3, 4. \] \{\dot{r}\} \] and \{\ddot{r}\} respectively represent the first and the second derivatives of \{r\}.

The complex frequencies \(\{\lambda\}\) and the complex vibration amplitude \{r\} can be obtained by solving the vibration equation

\[
\begin{align*}
\lambda = & n + i\omega \\
r_i = & x_i + iy_i, i = 1, 2, 3, \ldots, 8
\end{align*}
\]

\(n\) represents the modal attenuation coefficient, \(\omega\) is the modal frequency.

### 3.2 Modal frequency and mode shape

The working speed range of the flywheel rotor is from 0 to 500 rps. The campbell diagram is shown in figure 4.

![Campbell diagram](image1.png)

Figure 4: Campbell diagram.

According to calculation results, the modal frequencies for the first fourth forward precession are 4.3Hz (1F), 8.3Hz (2F), 20.6Hz (3F), 533.1Hz (4F) and mode shapes are shown in figure 5. The critical frequency is defined when the rotor speed is the same as the modal frequency, so there exist three critical frequencies: 4.3Hz (1F), 8.9Hz (2F), 26.8Hz (3F).

![Mode shapes](image2.png)

Figure 5: Mode shapes.

As shown in figure 5, the 1st mode shape is cylindrical and the 2nd mode shape is conical, both the 1st and 2nd mode shape are mainly determined by the upper and lower stiffness. The 3rd mode shape is decided by the stiffness of bellow, which can be seen from that the rotor is bent at the bellow. For the 4th mode shape, it is obvious that the lower displacement is much larger, which means that it is mainly affected by both the stiffness of support shaft and the lower stiffness.

### 3.3 Damping analysis

In order to study the effect of damping on the system, the range for damping value varies from 0 to 1000 Ns/m while keeping the other parameters unchanged, and then calculate the modal frequency and modal attenuation coefficient of the first four forward precession. The original equivalent damping of lower damper is 10 Ns/m, so the ratio ranges from 0 to 100. Calculation results are shown in figure 6.

![Damping analysis](image3.png)
It is obvious the modal frequencies of the first three forward precession vary little with damping before 600 Ns/m while the frequencies of the fourth forward precession decrease as damping increase. There exists three mode exchange in the damping ratio changing from 60 to 70 and the high-order modal frequencies keep the downward trend until exchange with adjacent low-order modal frequencies. After mode exchange, the modal frequencies of the 1st forward precession decrease to 0 as damping increase while the other three hardly change with damping. Which means that the “fourth damper mode” disappear after the damping coefficient larger than 670 Ns/m due to “overdamped”. The modal attenuation coefficients increase with damping and the values are all positive, which means stable. The larger the values, the more stable the system.

4. Transfer Matrix-Polynomial analysis

4.1 Calculation method

To get the blending mode vibration feature, the transfer matrix-polynomial analysis is presented. The rotor of rotating machinery is an elastomer with continuous mass distribution, which means that the number of degrees of freedom is infinite. To simplify the dynamics model, the lumped mass method is used to concentrate the mass and moment of inertia of the rotor on a limited number of nodes, which are connected by elastic shafts with no mass and equal cross section. In this way, the rotor is simplified as a model with several lumped mass and lumped moment of inertia.

By means of dynamics methods, the transfer relationship of state vector between the two ends of typical parts such as disk, shaft and support are established. So the transfer relationship of state vector between any section and the initial section can be obtained. Based on boundary conditions, searching frequencies which satisfy the remaining quantity, thus the critical frequencies of each order are calculated.

The transfer matrix-polynomial method avoids the iterative search process, for the frequency equation or the characteristic determinant is actually a polynomial. Compared with traditional transfer matrix method, the improvement is to introduce an unknown quantity s in the continuous multiplication of the transfer matrix, and coefficients of the characteristic polynomial can be solved during only once recursion, thus machine time is saved [7].

In the progress of solving the characteristic polynomial equation, general iterations such as Newton, Bairstow and Muller iterative methods are not used, instead the QR decomposition method which can solve the eigenvalues of the adjoint matrix of the characteristic polynomial is used. To prevent numerical overflow during the process of computation, specific multipliers are introduced and the order of adjoint matrix are reduced to reduce the numerical morbid characteristics of the adjoint matrix. Also, the method of similar transformation is used to balance matrix to improve accuracy of calculation [8].

Considering the anisotropic stiffness and damping of damper and support, the x component of the natural vibration can be expressed as:

\[ x = |x| \cdot e^{i\xi} \cos(\omega t + \psi) = R_x \left\{ (x_c + x_s) e^{i\theta} \right\} \]  (7)
The transfer matrix-polynomial method avoids the iterative search process, for the frequency equation or the characteristic determinant is actually a polynomial. Compared with traditional transfer matrix method, the improvement is to introduce an unknown quantity $s$ in the continuous multiplication of the transfer matrix, and coefficients of the characteristic polynomial can be solved during only once recursion, thus machine time is saved.

Where, $|x|$ is amplitude, $\psi$ is phase angle, $\omega$ is natural frequency, $\lambda$ is relative damping index and $s$ is complex frequency.

The flywheel rotor is considered as a combination of equivalent plates and elastic rod elements, taking mass, moment of inertia, shear effect and so on into account. For the lumped system with $N$ nodes, the transfer relationship of section vector between the terminal cross and the initial cross section is:

$$\begin{align*}
[z]_{n+1} & = [U] \{z\}_1, \\
[U] & = [T]_1 [T]_2 \ldots [T]_i \ldots [T], \\
[T]_k & = [T^{(0)}]_k [T^D], \\
[T^{(0)}]_n & = [T]_n + [T^{(1)}]_n s + [T^{(2)}]_n s^2 + \ldots + [T^{(k)}]_n s^k
\end{align*}$$  

(8) (9) (10) (11)

In the process of calculating the entire transfer matrix $U$, the sparsity of the element transfer matrix is used and noticing that each of these is a quadratic polynomial with an unknown quantity of $s$, which means that can be achieved by multiplying numbers and adding matrices.

So the matrix multiplication with unknown quantity $s$ is transformed into matrix polynomial coefficient reorganization and the addition of matrix. The computation time for obtaining the entire transfer matrix will be greatly reduced.

The frequency equation is obtained by introducing boundary conditions:

$$\det[D]_{4	imes 4} = 0$$  

(12)

$[D]_{4	imes 4}$ is a combination of some elements in $[U]$.

$$d_y = d_y^{(0)} + d_y^{(1)} s + d_y^{(2)} s^2 + \ldots + d_y^{(k)} s^k$$  

(13)

Because $d_y$ is a polynomial of $s$, the characteristic polynomial of system can be obtained by multiplying $d_y$ with polynomials according to the algebraic complement formula.

$$c_0 + c_1 s + c_2 s^2 + \ldots + c_{8N} s^{8N} = 0$$  

(14)

The real part of the roots of the above equation represents the attenuation coefficient of free vibration, and its imaginary part represents the natural frequency of free vibration.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Reference value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{01}$</td>
<td>Equivalent lateral stiffness of upper support</td>
<td>200000 N/m</td>
</tr>
<tr>
<td>$k_{12}$</td>
<td>Equivalent bending stiffness of bellow</td>
<td>1000000 N/m/rad</td>
</tr>
<tr>
<td>$k_{26}$</td>
<td>Equivalent lateral stiffness of support shaft</td>
<td>200000 N/m</td>
</tr>
<tr>
<td>$m_a$</td>
<td>Equivalent mass of lower damper</td>
<td>0.1 kg</td>
</tr>
<tr>
<td>$k_a$</td>
<td>Equivalent lateral stiffness of lower support</td>
<td>120000 N/m</td>
</tr>
<tr>
<td>$c_a$</td>
<td>Equivalent damping of lower damper</td>
<td>0 Ns/m</td>
</tr>
</tbody>
</table>

### 4.2 Results

The transfer matrix-polynomial method is used to calculated modal frequencies with parameters in Table 3 and the Campbell diagram data is calculated out like that in figure 4. The critical frequencies are found to be 3.7 Hz, 8.2 Hz, 26.5 Hz and 672.0 Hz respectively.

For the case of without bellow connection element, corresponding mode shapes are shown in Table 4. The first bending critical speed 321 rps locates in the running speed. So that the bellow is necessary to avoid passing through the first bending critical speed.

Table 4: Modal vibration analysis for flywheel with bellow
5. Conclusions

The vibration characteristics of two composite cylinder with elastic connection and supports are studied. Supposing the cylinders are rigid, the Lagrange vibration equation is established to learn the low frequency modal characteristics. The critical frequencies were found to be 4.3 Hz, 8.9 Hz, 26.8 Hz and 672.0 Hz respectively.

There exists three mode exchange in the damping ratio changing from 60 to 70 and the high-order modal frequencies keep the downward trend until exchange with adjacent low-order modal frequencies. The “the fourth damper mode” disappear after the damping coefficient larger than 670 Ns/m due to “overdamped”.

The rotor support system is also calculated by the transfer matrix-polynomial method and the bending mode is obtained. For the case of without bellow connection element, the first bending critical speed 321 rps locates in the running speed. So that the bellow is necessary to avoid passing through the first bending critical speed.

REFERENCES