NONLINEAR DYNAMIC ANALYSIS OF WIND TURBINE ROTOR BLADE

Praveen Shakya, Mohammed Rabius Sunny and Dipak Kumar Maiti

Department of Aerospace Engineering, Indian Institute of Technology Kharagpur, Kharagpur, India
email: praveen.shakya@aero.iitkgp.ernet.in

Now a days the wind turbines rotor blades are designed as long as 60-80m to maximize the power production (5-10MW). The increased flexibility of these long slender beams leads to complex aeroelastic behavior. The deformation of such blades in different modes like flap and edge wise bending and twisting due to the aerodynamic loads influence the aerodynamic parameters. Because of the high flexibility of these blades the nonlinear geometric effects are predominant in the aeroelastic behavior of these blades. Apart from that, large deformation of these blades may reduce the clearance between the shaft and the blade and the tower and blade tip too. The effect of flap-twist coupling in the aeroelastic behavior has been studied numerically by several researchers. For stall-controlled blades edge wise vibration can become destructive. Few researchers have analyzed the effect of flap-lag coupling for such blades. However, literature dealing with such analysis incorporating the nonlinear geometric effects and coupling of all the different modes such as flap and edgewise bending, twisting and axial deformation is insufficient to the best of the author’s knowledge. The goal of the present study is to perform finite element based modal analysis of large wind turbine blades followed by reduced order nonlinear dynamic analysis coupled with unsteady aerodynamic analysis. The unsteady blade element momentum theory has been used for aerodynamic load calculation. The structure has been modeled using the Euler-Bernoulli assumption with von-Karman strain displacement relationship considering the different deformation modes mentioned above. Mode shapes have been obtained through finite element analysis. Considering the mode shape as the basis functions a set of nonlinear ordinary differential equations (ODEs) have been obtained. The ODEs have been integrated through the beta-Newmark method accounting for the interaction of aerodynamic and structural parameters at each time step.

Keywords: aeroelastic, unsteady, beam, coupling, vibration

1. Introduction

Wind energy is one of the cleanest renewable source of energy. For enhancing the power production, the size of horizontal axis wind turbine (HAWT) has been increased in last few years. Wind turbine blade of 5 m radius produces 50kW power while the blade length of 60-80 m radius produces 5-10 MW. These long blades are made up of glass reinforced polymer (GRP) composite materials. Effect of geometric nonlinearity is significant in these large flexible blades. Wind turbines experience static and dynamic aerodynamic loads which affect the structure of the blades. Therefore, the static and dynamic response of the blade has been studied by the several researchers [1-3]. Severe edgewise vibration produces the longitudinal cracks on the trailing edge of the blade. Dynamic analysis of flap-edge wise coupled wind turbine blade is important to investigate the destructive edgewise vibration [4]. Bend twist coupling is highly desirable in load mitigation of wind turbine blade [5], but it may prevent or favour dynamic instabilities based on the nature of coupling [6]. Flap-edge wise coupled...
large wind turbine blade experiences significant vibration and deflection during operation [7-8]. Centrifugal force acting on the rotating wind turbine blade affects the stiffness of the blade and the overall deflection on the blade decreases due to the centrifugal stiffening [8]. Aeroelastic analysis requires the efficient integration of aerodynamics and structural dynamic models. Different aerodynamic model has been proposed by various researchers to predict the aerodynamic load on wind turbine blade. Simple aerodynamic models such as 2D quasi-steady aerodynamic model, Theoderson’s method for aerodynamic model and classical blade element momentum method (BEM) have been used on wind turbine aeroelasticity [9-11]. The use of unsteady aerodynamic essential for more accurate predictions of the aeroelastic behaviour in wind turbines. Computational fluid dynamics (CFD) techniques also used in wind turbine aerodynamics. But, the computational cost of CFD is very high compared to other methods. Kim et al. [12] has proposed aerodynamic model based on modified strip theory, which considers the unsteady effects that result from structural motion, such as lunging and pitching motion, and dynamic stall. Lee et al. [13] presented the performance and aeroelastic characteristics of wind turbine blades based on flexible multi-body dynamics, aerodynamic model based on modified strip theory, and the fluid-structure interaction. Vortex method based on the potential flow is accurate for the wake dynamics but the computational cost is high [14]. ONERA model are used in the wind turbines for dynamic stall [15]. Unsteady BEM method is fast, efficient and include the dynamic stall and wake model.

The main aim of this research is to study the dynamic behaviour of flap-edge wise bending, twisting and extension of wind turbine rotor blade under aerodynamic load considering nonlinear geometric effects. BEM method has been used to calculate the steady and unsteady aerodynamic loads. Wind turbine blade is modelled as Euler-Bernoulli beam with von Karman strain displacement relationship. Mode shapes and frequency of wind turbine blade has been evaluated using the finite element based modal analysis. To solve the nonlinear equation an iterative approach as Newton-Raphson method has been applied at each time step. The ODEs have been integrated through the beta-Newmark method accounting for the interaction of aerodynamic and structural parameters at each time step.

2. Aeroelastic model of wind turbine blade

Aeroelastic model is a combination of aerodynamic and structure model. Aerodynamic model is used to calculate the steady and unsteady aerodynamic loads on the wind turbine blade. The structure model is used to formulate the linear and nonlinear differential equation of motion of the blade.

2.1 Aerodynamic Model

The Blade element momentum method is presented by Glauert [17] to calculate the steady loads, thrust and power. Unsteadiness in wind due to atmospheric turbulence, wind shear and presence of tower effects the rotor blade of wind turbine. Unsteady BEM method is used to compute these variable behaviours of the wind. The undistributed wind velocity at different cross-section of blade is obtained by transforming the coordinate of the wind velocity in inertial reference system to body system using a transformation matrix [9]. Relative velocity seen by the blade is depicted in Fig. 1 by taking a cross-section of the wind turbine blade. The expression of induced velocity is given in the literature [16]. Relative velocity $V_{rel}$ is the vector sum of wind velocity $V_0$, induced velocity $W$ and rotational velocity $V_{rot}$. Accounting for the structural velocity, relative velocity can be written as: $V_{rel} = V_0 + W + V_{rot} - V_s$, where $V_s$ is the structural velocity. The flow angle ($\phi$) and angle of attack ($\alpha$) are calculated as:

$$\tan \phi = \frac{V_{rel,z}}{-V_{rel,y}} \quad (1)$$

$$\alpha = \phi - (\beta + \theta) \quad (2)$$

The angle $\theta$ is the pre-twist of the wind turbine blade and $\beta$ is the twist angle of the blade. The aerodynamic loads acting on the blade can be determined as:
\[ p_x = L \cos \phi + D \sin \phi \]
\[ p_y = L \sin \phi - D \cos \phi \]

(3)

and

\[ L = \frac{1}{2} \rho_\infty V_{rel}^2 C_l \]
\[ D = \frac{1}{2} \rho_\infty V_{rel}^2 C_d \]

(4)

where \( p_x, p_y \) are normal and tangential loads; \( L, D \) are lift and drag acting on the wind turbine blade respectively; \( c \) is chord length of the aerofoil; \( \rho_\infty \) is the density of air; \( C_l \) and \( C_d \) are coefficient of lift and drag respectively.

Figure 1: Velocities acting at the cross-section of the wind turbine blade

2.2 Structural Model

A large wind turbine blade usually has a slender structure, so the Euler-Bernoulli beam model is adopted to describe the rotating blade. Four deformation modes are considered in this study. Flap wise and Edge wise vibration along the out of plane and in-plane bends respectively. The axial extension is along the elastic axis. Twist is the rotation of blade about the reference line. The coordinate system of the wind turbine blade is defined such that the x-axis corresponds to the undeformed elastic axis, the y-axis lies in the plane of rotation, and the z-axis is perpendicular to the plane of rotation as shown in Fig. 2.

Figure 2: Coordinate system of wind turbine rotor blade

The equation of Euler-Bernoulli beam blade is obtained using Hamilton’s principle

\[ \int_{t_1}^{t_2} [\delta(U - T) - \delta W] dt = 0 \]

(5)
where $U$ is elastic energy, $T$ is kinetic energy and $W$ is work done by external loads on the body and $t_1, t_2$ are initial and final times.

\[
\delta U = \int_0^V \sigma \delta \varepsilon \, dV = \int_0^L \int_A E \varepsilon_{x} \varepsilon_{x} \delta \varepsilon_{x} + G \gamma_{xy} \varepsilon_{y} + G \gamma_{xz} \varepsilon_{z} \, dV \\
\delta T = \rho A \int_0^L \hat{\tau} \cdot \delta \hat{\tau} \, dx \\
\delta W = \int_0^L L_{w} \delta v + L_{w} \delta w + M_{\beta} \delta \beta 
\]

where $\sigma, \varepsilon$ are the stress and strain respectively; $E, G, \rho$ and $A$ are the modulus of elasticity, modulus of rigidity; $\rho$ is the density of the blade and area of cross-section of the blade respectively; $\hat{\tau}$ is the velocity vector in reference frame of the blade, $L_{w}, L_{w}, M_{\beta}$ are the aerodynamic force loads distributed along the length of the blade in the lag, flap and torsion directions respectively.

Von Karman strain displacement has been employed for nonlinear analysis.

\[
\varepsilon_{x} = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \\
\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \\
\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial z} 
\]

where

\[
u = u_{0} - z \frac{\partial w(x)}{\partial x} - y \frac{\partial v(x)}{\partial x} + \frac{\partial \beta}{\partial x} \psi(y, z) \\
\psi = \nu(x) + \beta(x) y \\
w = w(x) - \beta(x) y 
\]

$u_{0}$ is axial displacement; $w(x)$ is transverse displacement; $\nu(x)$ is lateral displacement; $\beta(x)$ is twist angle; $\psi$ is warping rigidity. Substituting Eqs. (6-10) into Eq. (5), we get nonlinear differential equation of motion which is written in matrix form as:

\[
[M] \ddot{\mathbf{X}} + [C] \dot{\mathbf{X}} + [K_{L} + K_{NL}] \mathbf{X} = \mathbf{F} 
\]

where $[M]$ is the mass matrix; $[C]$ is the Rayleigh damping matrix; $[K_{L}]$ and $[K_{NL}]$ are linear and nonlinear stiffness matrix; $[F]$ is the aerodynamic load vector, which is computed using the aerodynamic model; $\{X\}$ represent the displacement vector as $\{X\} = [u_{0} \ w \ \nu]^{T}$; $\{\dot{X}\}$ and $\{\ddot{X}\}$ are acceleration and velocity vector respectively. Wind turbine blade has inherent structural damping which is obtained using the Rayleigh damping matrix as $[C] = \alpha_{1} [M] + \beta_{1} [K]$. The term $\alpha_{1}$ is mass proportional Rayleigh damping coefficient and $\beta_{1}$ is stiffness proportional Rayleigh damping coefficient.

From linear part of the governing differential equations (linearized with respect to the undeformed configuration) the mode shapes and natural frequencies are obtained by applying Finite Element Method (FEM). After assembling the elemental mass and stiffness matrices, we get the global mass matrix $[M]$ and stiffness matrix $[K]$. Free vibration has been analysed by neglecting the Rayleigh damping matrix. The governing differential equation is written as

\[
[M] \ddot{\mathbf{X}} + [K] \mathbf{X} = \mathbf{0} 
\]

Considering the harmonic solution of the above equation, it is converted into the eigenvalue problem to calculate the mode shapes and natural frequencies of the blade. Modal superposition principle has been applied to get response of the system. The overall displacement is calculated by the summation of products of the mode-shape vectors $\phi_{n}$ and the modal amplitude $Y_{n}$ as

\[
\mathbf{X} = \phi_{1} Y_{1} + \phi_{2} Y_{2} + \cdots + \phi_{N} Y_{N} = [\phi]_{n} \{Y\}_{n} 
\]
where $[\phi]$ is the modal matrix of first $n$ coupled rotating modes and $\{\gamma\}$ is the vector of $n$ generalized coordinates in the modal space. Substituting the Eq. (13) into Eq. (11) and pre-multiplied the transpose of modal matrix $[\phi]^T$, the rearranged equation is expressed as

$$
$$

(14)

Nonlinear stiffness matrix is calculated by the Newton-Raphson iterative approach which is given in the Ref. [18]. Modal amplitude is obtained by using the beta-Newmark’s method. Dynamic flapwise-edgewise deflection, axial deflection and twist angle is calculated by substituting the modal amplitude obtained from Eq. (14) into Eq. (13).

3. Results and Discussion

Wind turbine blade data for aeroelastic analysis is taken from NREL 5MW [19]. The general characteristics of wind turbine blade is shown in Table 1. The mode shapes and frequencies of the wind turbine blade are calculated using linear model. Results are validated for frequencies shown in Table 2. Structural model coupled with aerodynamic steady BEM, the aeroelastic response are validated with the numerical results [2, 20].

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present</td>
<td>Hansen[21]</td>
</tr>
<tr>
<td>1st Flapwise</td>
<td>0.69</td>
</tr>
<tr>
<td>1st Edgewise</td>
<td>1.13</td>
</tr>
<tr>
<td>2nd Flapwise</td>
<td>1.99</td>
</tr>
<tr>
<td>1st Torsion</td>
<td>5.81</td>
</tr>
<tr>
<td>Hansen[21]</td>
<td>0.69</td>
</tr>
<tr>
<td>1st Flapwise</td>
<td>0.72</td>
</tr>
<tr>
<td>2nd Flapwise</td>
<td>1.07</td>
</tr>
<tr>
<td>1st Edgewise</td>
<td>1.08</td>
</tr>
<tr>
<td>1st Torsion</td>
<td>2.05</td>
</tr>
<tr>
<td>Meng[22]</td>
<td>Pourazarm[6]</td>
</tr>
<tr>
<td>1st Flapwise</td>
<td>0.64</td>
</tr>
<tr>
<td>2nd Flapwise</td>
<td>1.86</td>
</tr>
<tr>
<td>1st Edgewise</td>
<td>5.62</td>
</tr>
<tr>
<td>1st Torsion</td>
<td>2.02</td>
</tr>
<tr>
<td>Pourazarm[6]</td>
<td>0.69</td>
</tr>
<tr>
<td>1st Flapwise</td>
<td>5.39</td>
</tr>
<tr>
<td>Jonkman[19]</td>
<td></td>
</tr>
<tr>
<td>1st Edgewise</td>
<td>1.08</td>
</tr>
</tbody>
</table>

First flapwise, first edgewise and second flapwise frequencies are close agreement with other studies. First torsion frequencies show close agreement with Meng [22] and Pourazarm [6].

Figure 12 presents the static aeroelastic response at a rated wind speed compared with the numerical results [2, 20]. These steady-state deflections are assumed as a constant inflow and no gravity. As seen in Fig. 3, the flapwise deflections are larger than the edgewise deflections. There are minor offsets in both flapwise and edgewise blade deflections. Flapwise deflection is closer to the Ref. [20]; while edgewise deflection to the Ref. [2]. These distinctions were caused by the differences between the present method and their approaches. Through the comparison of the numerical predictions, the present method proved to be valid for steady-state blade deflections. Steady-state deflection is also
found out by taking the first four mode shape vectors as shown in Fig. 3 as Present (mode shape). They are exactly matching with direct approach.

![Deflection in flap direction - Static](image1)

![Deflection in edge direction - Static](image2)

**Figure 3: Steady-state blade deflections at a rated wind speed of 11.4 m/s**

![Flapwise deflection](image3)

![Edgewise deflection](image4)

![Extension deflection](image5)

![Twist](image6)

**Figure 4: Linear dynamic behaviour of blade at tip**

Linear dynamic behaviour of wind turbine blade at tip with steady BEM, unsteady BEM and unsteady BEM with aerodynamic damping is presented in Figure 4. Due to unsteadiness, loads on the
blade is larger than considering the steady behaviour. Therefore, deflections due to unsteady is larger than the steady BEM. When considering the unsteady BEM with aerodynamic damping (velocities as feedback to unsteady BEM) transient response show the overshoot very less and steady-state reaching fast than without aerodynamic damping. Due to velocities in the feedback, the deflections are larger than the steady and unsteady BEM. Nonlinear dynamic analysis of the wind turbine blade at tip is presented in Fig. 5 using the BEM methods. It is showing the same signature as in Fig. 4. Figure 6 presents the linear and nonlinear dynamic behaviour at the tip of the blade. Deflection in nonlinear analysis is less compared to linear analysis for flapwise direction while for edgewise, it is almost same. Additional stiffness added in the nonlinear analysis gives the less deflection compared to linear case.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{nonlinear.png}
\caption{Non-linear dynamic behaviour of blade at tip}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{linear_nonlinear.png}
\caption{Linear and Non-Linear dynamic behaviour of blade at tip}
\end{figure}

4. **Summary and Conclusion**

Dynamic response analysis of wind turbine blade for linear and nonlinear structural model has been done. Mode summation method has been used to find the dynamic response. FEM is implemented in linear model to find the mode shapes and frequencies. Static deflections have been found out using the fluid-structure interaction for 5MW wind turbine blade. Results are validated with the previous studies. Due to unsteady behaviour of the wind, unsteady BEM model incorporated to find the actual dynamic response. Von Karman strain has been used for nonlinear analysis. Newmark’s
Beta scheme implemented for dynamic analysis. For non-linear analysis, Newton-Raphson iterative method has been used for better convergence solutions. Significant difference between linear and nonlinear response has been observed in the flapwise direction. In lag wise direction the linear and nonlinear response are almost same.

REFERENCES


