The main objective of this paper is to perform design sensitivity analysis of two right angle coupled plates connected by various joint connections, to determine an optimum coupling loss factor (CLF) using optimization in the Statistical Energy Analysis (SEA) framework so the developed theoretical model can be used at the early design stage using the right type of joint. First and second-order sensitivity analysis using theoretical equations of CLFs of two right angle plates coupled to form welded, riveted and bolted connections, in the SEA framework in the audible frequency range are presented. These sensitivities were determined using direct differentiation method and finite difference method. The sensitivities found by the direct method agree well with the finite difference method indicating that the direct differentiation method is sufficient to predict the variation in the CLF and response of coupled plates joined by various joint connections. The sensitivity analysis also gives the feasible region in terms of frequency range for the determination of an optimum value for CLF by selecting the right type of joint at the design stage based on the joint stiffness.

Keywords: Statistical energy analysis, optimization, sensitivity analysis, coupled plates, joints

1. Introduction

Designing quiet machines has become a challenging task in recent times due to environment concerns, hearing conservation and customer requirement of pleasant sounding machines. The extent of reduction that was earlier obtained by using enclosures has limitations and in addition, enclosures obstruct maintenance activity of such machines. Since most machines are constructed using plate elements, the original source of structure borne sound gets amplified many times before it reaches the exterior of the machine. Therefore, the airborne sound from a machine will depend not only on the original source of structure borne sound, but also on how the structural elements that are used for constructing the entire machine interact between themselves. Hence, at the drawing board stage the choice of the geometric dimensions of the structural elements will become important. In addition, the nature of joining elements will play a major role in transferring energy between all the structural elements. If one could determine how sensitive the geometric dimensions of the structural elements and nature of joining elements contribute towards the airborne sound levels of the machine, they would be useful inputs at the drawing board stage of designing a quiet machine by optimizing the design parameters of the structural elements.

Optimization and sensitivity analysis has been used for this purpose in the past but was only limited to lower frequency range (20-100 Hz); numerical methods like finite element method (FEM) and boundary element method (BEM) were used for this purpose. These numerical methods help to reduce noise and vibration by doing modification to the structure by performing various design iterations to the structure by selecting correct design parameter. [1-7] employed FEM to determine design sensitivity of radiated noise to vari-
ous design parameters. The use of BEM to determine design sensitivity was shown by [8-13] who used BEM to perform acoustic shape sensitivity analysis. [14 and 15] used coupled FEM-BEM approach to calculate the sensitivity of a structural-acoustic system with respect to structural design parameters.

Notwithstanding the above work, airborne noise of machines is prominent in the high frequency region, typically more than 500 Hz, in which our ears are most sensitive. The noise and vibration response is very sensitive to small changes in the model geometry due to tolerance in the higher frequency range and therefore the use of SEA (statistical energy analysis) models for predicting noise and vibration response of system based on its time and space averaged information are more relevant compared to FEM and BEM. In order to apply SEA to predict the response of a vibro-acoustic system, it is divided into several coupled subsystems. The input power to the system is then the sum of power dissipated by an individual subsystem due to its damping and power transmitted to other subsystems. To apply SEA, determination of three main parameters is required: (1) modal density which represents number of resonances in a given frequency band for subsystem, (2) damping loss factor which represents damping in the subsystem (3) CLF which defines coupling between subsystems in terms of joint damping. The size of the structural elements and the nature of their joining significantly influence the above parameter and hence the need for optimization.

In the past SEA has been successfully used for studying complex models of automobiles [16-18], aircraft and launch vehicles [19-21], ship structures [22-24]. However, sensitivity analysis in SEA framework was not fully developed. [19] developed an analytical model to predict transmission of noise into an airplane interior through the fuselage sidewall using the SEA method and showed that the sensitivity of predicted levels with an optimum value of the chosen structural loss factor. [25] used SEA to find the optimum damping factor for power flow of a system. [26] implemented design sensitivity analysis in an SEA computer code. [27] used sensitivities to structural tolerances, which are used to compute the variance of response of a SEA model. [28] has developed an optimisation strategy for acoustical characteristics by changing the critical dimensions of a structure. Recent work by [29] presented first and second-order sensitivity analysis of vibro-acoustic systems consisting of three plates joined in Z form in the SEA framework, where equations for computing these sensitivities for a general SEA model were obtained using direct method and adjoint method. They verified that these two approaches led to the same results and the difference between them, if any, was only due to the computational efficiency.

Most of the design sensitivity analysis carried out in the SEA framework thus far has mainly related to the determination of optimum geometric dimensions of structural elements to obtain overall vibration and noise reduction. However, the coupling loss factors used were assumed to be due to a simple connection of plate elements without any scope for optimizing them. Since the authors have carried out detailed theoretical and experimental analysis of the contribution of joining elements to the transmission of structure borne sound between coupled plates, the main objective of this paper is to apply design sensitivity analysis to the calculation of first and second derivatives of objective function in the SEA framework that would not only account for the geometric dimensions of the structural elements but also the nature of joining elements connecting them. A mechanical joint - welded, riveted or bolted - is generally used to connect two or more parts of a structure. These joints have an important effect on the capacity of damping. The determination of accurate dynamic behaviour and in particular, damping, is important to analyse the vibration environment of a structure and its subsystems and ultimately their capability to withstand these vibrations.

Theoretical equations for the determination of CLF for welded, riveted and bolted connection of two right angle plates were developed by [30]. These equations need to be verified to predict the variation in the CLF as well as predicted response of the coupled plates, due to variation in the structural parameters at the design stage using first and second-order sensitivity proposed by [29]. This will help in minimizing experimentation using different setups for plates. Also, it will help in determining an optimum CLF that can be designed using the right type of joint. Once the usefulness of sensitivity analysis is established for a two-plate model, the same can be extended to a multi system model of connected systems. The analytical method i.e. direct differentiation method and numerical method i.e. finite difference method are used to perform the first and second-order sensitivity analysis. The design parameters are second plate thickness, second plate damping and frequency.

2. Optimization formulation

General noise and vibration optimization problems are divided into two types of optimization problems based on the objective functions. First, where structure response frequency itself is defined as the objective function is called narrowband optimization method and second where the
structural response values like sound pressure level, displacement, velocity and acceleration in the specified frequency range are defined as objective function called broadband optimization method. The narrowband optimization method is used for lower frequencies where discrete and tonal excitations are the noise sources, where the optimization algorithm tries to minimize the objective function like response of structure or weight, by moving the fundamental vibration frequency outside the excitation frequency range i.e. to either higher or lower side of the excitation band leading to reduction in the objective function. At higher frequencies, response frequencies are so closely spaced such that there is very little or no scope for frequency shifting to reduce the response of the structure. Therefore at higher frequencies broadband optimization method is used which tries to reduce the response of the structure over the entire range of frequencies. In SEA framework, the frequency range of interest is wide and uses broadband optimization method where objective function is to reduce the magnitude of the response of the structure.

General structural SEA model for \( k \) coupled subsystems is described by a matrix equation as

\[
\begin{bmatrix}
\eta_{1i} + \sum_{j=1}^{k} n_i \eta_{ij} & \cdots & -n_i \eta_{1k} \\
\vdots & \ddots & \vdots \\
-n_i \eta_{ki} & \cdots & \eta_{ij} + \sum_{j=i+1}^{k} n_i \eta_{ij} \\
\vdots & \ddots & \vdots \\
-n_i \eta_{ik} & \cdots & \eta_{ik} + \sum_{j=i+1}^{k} n_i \eta_{ik}
\end{bmatrix}
\begin{bmatrix}
e_1 \\
e_2 \\
\vdots \\
e_k
\end{bmatrix} = N(x, f) e(x, f) = \left(\frac{1}{\omega}\right) P
\]  

where \( N(x, f) \) is \( k \times k \) system positive definite matrix consisting of modal density \( n_i \), damping loss factor \( \eta_i \) and CLF \( \eta_{ij} \). A vector \( e(x, f) = \{e_1, \ldots, e_k\}^T \) is the subsystem modal energy vector, vector \( P = \{P_1, \ldots, P_k\}^T \) is input power vector and \( \omega = 2\pi f \) is angular frequency in the frequency band centred at \( f \).

From (1), the modal energy vector becomes,

\[
e(x, f) = N(x, f)^{-1} \left(\frac{1}{\omega}\right) P
\]  

The total energy \( E(x, f) \), is the product of subsystem model energies and the subsystem modal density given by

\[
E(x, f) = n(x, f) e(x, f)
\]  

where \( n(x, f) \) is a diagonal matrix with modal densities. Thus, the total energy of any subsystem is a function of modal density, damping loss factor, CLF and frequency. Further, these modal densities and CLFs are functions of the vector of design parameters \( x = \{x_1, x_2, \ldots\}^T \). Therefore, to perform optimization it is required to have a clear understanding of the objective function, constraints, design parameters and their interconnections.

3. Application of optimization formulation to the two right angle coupled plates

In this paper, first and second-order sensitivity analysis were performed for two plates coupled at right angle using welded, riveted and bolted connections. Figure 1 shows the physical model of the
two plates coupled in right angle. Plate 2 is coupled to plate 1 at right angle through welded, riveted and bolted connection along the edge in the \( Z \)-direction.

![Diagram of two plates coupled in right angle](image)

**Fig. 1** Physical model of two plates coupled in right angle

The material and geometrical data of the model are presented in Table 1 and the parameters for variation are shown in Table 2 and the joint properties are shown in Table 3.

**Table 1** Material properties and Geometrical data of two coupled plate model

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Property</th>
<th>Value</th>
<th>Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Material</td>
<td>Mild Steel</td>
<td>Plates 1 and 2</td>
</tr>
<tr>
<td>2</td>
<td>Density</td>
<td>7816 kg/m(^3)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Young’s Modulus</td>
<td>210 GPa</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Surface area</td>
<td>0.09 m(^2)</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Thickness</td>
<td>0.004 m</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Damping loss factor</td>
<td>0.01</td>
<td></td>
</tr>
</tbody>
</table>

**Table 2** Parameters for variation

<table>
<thead>
<tr>
<th>Thickness variation (( h_z ))</th>
<th>Damping variation (( \eta_z ))</th>
<th>Frequency (( f ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_z ), mm</td>
<td>1-21</td>
<td>4</td>
</tr>
<tr>
<td>( \eta_z )</td>
<td>0.01</td>
<td>0-0.021</td>
</tr>
<tr>
<td>( f ), Hz</td>
<td>3000</td>
<td>3000</td>
</tr>
</tbody>
</table>

**Table 3** Joint Properties

<table>
<thead>
<tr>
<th>Joint</th>
<th>Stiffness</th>
<th>Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weld</td>
<td>100 GN/m</td>
<td>1.8 kNs/m</td>
</tr>
<tr>
<td>Rivet</td>
<td>4 GN/m</td>
<td>1.9 kNs/m</td>
</tr>
<tr>
<td>Bolt</td>
<td>8 kN/m</td>
<td>0.8 kNs/m</td>
</tr>
</tbody>
</table>
With the objective of minimizing energy in the second plate, the following sensitivity analyses were carried out.

First-order sensitivity analysis
i. Second plate thickness as the design variable
ii. Damping loss factor as the design variable

Second-order sensitivity analysis
i. Second plate thickness as the design variable
ii. Damping loss factor as the design variable
iii. Frequency as the design variable

First-order sensitivity of CLF
i. First-order sensitivity to second plate thickness
ii. First-order sensitivity to second plate damping
iii. First-order sensitivity to frequency

Second-order sensitivity of the CLF
i. Second-order sensitivity to second plate thickness
ii. Second-order sensitivity to second plate damping
iii. Second-order sensitivity to frequency

First-order sensitivity of the narrowband objective function
i. First-order sensitivity to second plate thickness
ii. First-order sensitivity to second plate damping
iii. First-order sensitivity to frequency

Second-order sensitivity of the narrowband objective function
i. Second-order sensitivity to second plate thickness
ii. Second-order sensitivity to second plate damping
iii. Second-order sensitivity to frequency

First-order sensitivity of the broadband objective function
i. First-order sensitivity to second plate thickness
ii. First-order sensitivity to second plate damping

Second-order sensitivity of the broadband objective function
i. Second-order sensitivity to second plate thickness
ii. Second-order sensitivity to second plate damping

The results of the sensitive analyses show that the energy of the second plate does show significant variation with respect to the parameters listed above. Due to constraints of space all of these results could not be reproduced here. Hence they are summarized in the next section.

4. Conclusions

An optimization procedure using theoretical equations for CLFs of two right angle plates coupled to form welded, riveted and bolted connections in the SEA framework, in the audible frequen-
Frequency range is presented. The first and second-order sensitivities of CLF were obtained using direct method and finite difference method for variation in second plate thickness, second plate damping and frequency.

The sensitivity analysis for CLF has provided useful information for finding a feasible region to determine optimum values of CLF. For very low stiffness of the joint, more variation in the CLF is observed for lower thickness of second plate. But overall variation in the CLF is less for a second plate thickness greater than 8 mm. It is observed that if there is reasonable damping in the second plate, CLF will not change significantly due to change in damping. For a joint with very low stiffness value up to 10 kN/m and joint with higher stiffness value greater than 10 GN/m can be used in the middle to higher (2000 Hz to 10000 Hz) frequency range and joint with stiffness ranging from 10 kN/m to 10 GN/m can be used for higher frequency range (6000 Hz to 10000 Hz).

The variation of SEA matrix and CLF using theoretical equations with respect to plate thickness, damping loss factor, frequency was used to further calculate the sensitivities for narrowband and broadband objectives functions for second plate energy of bolted joint using direct method. The results of the direct method were compared with the finite difference method and they show close agreement with each other. It is observed from sensitivity analysis that the variation in the narrowband and broadband objective function is due to variation in the CLF resulting from variation in the second plate thickness. Optimum thickness of the second plate to reduce the response of the second plate of bolted joint is found to be less than 8 mm. It is found that nominal values of damping can reduce the CLF and resulting response of plate significantly.

Based on the approach presented in this paper, it should be possible to use the theoretical equations of CLF by proper selection of stiffness and damping (selecting right type of joint), for practical SEA models having various joint connections to determine the optimum values of CLF and response of system at the early design stage. This will be very useful input at the drawing board stage for eliminating expensive experimentation to arrive at the optimum values of CLF and hence eventually reduce the vibro-acoustic response of large subsystem SEA models of machinery.

REFERENCES