INFLUENCE OF APERTURE RATIO ON AMPLITUDE AND PHASE OF INPUT IMPEDANCE IN A HALF-STOPPED HORN

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French horn players often tune sound volume and pitch by changing the aperture ratio of the cross-sectional area of the bell with a right-hand palm. This technique is called half-stopped horn. In this study, we numerically analyse the half-stopped horn by a one-dimensional model with multiple short tubes based on the transmission-matrix method, and we investigate the influence of the aperture ratio on the amplitude and the phase of input impedance for the model of half-stopped horn with the amplitude and phase modulations. As a result, the amplitude modulation has the largest peak at the aperture ratio of 18%, and the phase modulation has the largest peak at the ratio of 25%.

Keywords: numerical analysis, half-stopped horn, transmission matrix method, amplitude modulation, phase modulation

1. Introduction

French horn players often tune sound volume and pitch by changing the aperture ratio of the cross-sectional area of the bell with a right-hand palm. This technique is called half-stopped horn. In this study, we numerically analyse the half-stopped horn by a one-dimensional model with multiple short tubes based on the transmission-matrix method, and we investigate the influence of the aperture ratio on the amplitude and the phase of input impedance for the model of half-stopped horn with the amplitude and phase modulations. As a result, the amplitude modulation has the largest peak at the aperture ratio of 18%, and the phase modulation has the largest peak at the ratio of 25%.

Stopped horn’s pitch and tone are studied by Backus [1] and Ebihara et al. [2], respectively. Backus shows that stopped horn’s pitch is low by input impedance curves. Ebihara et al. shows that stopped horn’s tone is changed due to nonlinear sound propagation along the bore. However, details of half-stopped horn have not been studied.

Stopped horn need only closing the bell. However, half-stopped horn needs to adjust the aperture ratio for each pitch. Players use sensuously half-stopped horn because they have no clear aperture ratio indicator that is most effective for each music note. It is expected that this technique will be easier to handle when the half-stopped horn will be quantitatively evaluated. Numerical analysis of the acoustic characteristics of wind instruments has been studied as a response of input impedance calculated by the transmission-matrix method (TMM) [3], [4].
In this study, we evaluate the amplitude and phase modulations of input impedance with the aperture ratio varying in the range of 0.01 to 1.0. We numerically analyse the half-stopped horn by a one-dimensional model with multiple short tubes based on the TMM. Especially we investigate the effect of the aperture ratio on the amplitude and the phase of input impedance for the model. Finally, we show aperture ratios at maximum modulations of the amplitude and phase, respectively.

2. Numerical model

A numerical model of the half-stopped horn is composed of a straight pipe section (a), a spacer insertion section (b), and a conical corn section (c) as shown in Fig 1. We calculate its input impedance by the TMM. The method approximates the shape of the horn with a sequence of multiple short tubes, and derives the input impedance of the approximate model by calculating from the radial end side. The length of a straight pipe section, a spacer insertion section, and a conical corn section are 2380 mm, 50 mm, and 180 mm, respectively. Each section is divided into 1000 stages. The sound source is a pure sound changing from 200 Hz to 900 Hz. We examine the response of input impedance when the aperture ratio range is from 0.01 to 1.0.

The input impedance of N-stage cascaded tubes, $Z(f)$, is represented by

$$ Z(f) = \frac{p_{\text{in}}(f)}{v_{\text{in}}(f)} = \frac{A(f) + B(f)/Z_r}{C(f) + D(f)/Z_r}. $$

(1)

Here, $Z_r$ shows radiation impedance [5]. The relationship between the sound pressure and the particle velocity of the tube is defined as

$$ \begin{bmatrix} p_{\text{in}}(f) \\ v_{\text{in}}(f) \end{bmatrix} = H(f) \begin{bmatrix} p_{\text{out}}(f) \\ v_{\text{out}}(f) \end{bmatrix} = \begin{bmatrix} A(f) & B(f) \\ C(f) & D(f) \end{bmatrix} \begin{bmatrix} p_{\text{out}}(f) \\ v_{\text{out}}(f) \end{bmatrix}. $$

(2)

Here, $p_{\text{in}}(f)$ and $v_{\text{in}}(f)$ are the input sound pressure and the input particle velocity, and $p_{\text{out}}(f)$ and $v_{\text{out}}(f)$ are the output sound pressure and the output particle velocity. Transmission matrix of N-stage cascaded tubes, $H(f)$, is defined as

$$ H(f) = \prod_{i=1}^{N} T_i(f). $$

(3)
Transmission matrix showing the relationship between sound pressure and particle velocity in one short tube, \( T_i(f) \), is defined as

\[
T_i(f) = \begin{bmatrix}
\cosh(\Gamma L_i) & Z_c \sinh(\Gamma L_i) \\
\frac{1}{Z_c} \sinh(\Gamma L_i) & \cosh(\Gamma L_i)
\end{bmatrix}.
\]

Here, \( L_i \) shows the length of one short tube, \( Z_c \) shows the complex element impedance, and \( \Gamma \) shows the propagation wave number. \( Z_c \) and \( \Gamma \) are approximated by Mapes-Riordan [4] as followings:

\[
Z_c = R_0 \left[ (1 + 0.369r_v^{-1}) - j0.369r_v^{-1} \right],
\]

\[
\Gamma = k \left[ 1.045r_v^{-1} + j(1 + 1.045r_v^{-1}) \right],
\]

\[
r_v = \sqrt{\frac{\rho \omega S}{\eta \pi}},
\]

\[
R_0 = \frac{\rho c}{S},
\]

\[
k = \frac{\omega}{\omega'},
\]

where, \( \rho \) is the equilibrium gas density, \( \omega \) is the radian frequency, \( \eta \) is the shear viscosity coefficient, \( j \) is the complex unit, and \( S \) is the sectional area of one short tube.

3. The modulation of input impedance

We introduce the modulation of input impedance, which represents the modulation of acoustic characteristic. This section defines the amplitude modulation and phase modulation.

3.1 Amplitude modulation

The amplitude of input impedance, was calculated by the TMM, with the aperture ratio varying in the range 0.01 to 1.0. Depending on the aperture ratio, the amplitude of input impedance is modulated as shown in Fig 2. The amplitude at the aperture ratio of 0.18 is almost same as that of an aperture ratio of 1.0 in the range from 200 Hz to 500 Hz, whereas it becomes very different in the range from 500 Hz to 600 Hz, and it becomes a little larger in the range from 600 Hz to 900 Hz. Figure 3 shows the minimum amplitude, \( A_{\text{min}} \), and the maximum amplitude, \( A_{\text{max}} \), of input impedance.

![Figure 2: Comparison of amplitude between aperture ratios of 1.0 and 0.18.](image1)

![Figure 3: Minimum and maximum amplitude of input impedance.](image2)
The amplitude modulation, $M_A$, is defined as

$$M_A = \frac{A_{\text{max}} - A_{\text{min}}}{A_{\text{max}} + A_{\text{min}}}. \quad (10)$$

### 3.2 Phase modulation

The phase of input impedance, was calculated from complex amplitude, in the same range. Depending on the aperture ratio, the phase of input impedance is also modulated as shown in Fig 4. The phase at an aperture ratio of 0.25 is almost same as that of an aperture ratio of 1.0 in the range from 200 Hz to 550 Hz, whereas it sharply shifts to the lower frequency in the range from 550 Hz to 600 Hz, and it slightly shifts to the lower frequency in the range from 600 Hz to 900 Hz. Figure 5 shows the minimum phase, $P_{\text{min}}$, and the maximum phase, $P_{\text{max}}$, of input impedance.

![Figure 4: Comparison of phase between aperture ratios of 1.0 and 0.25.](image)

![Figure 5: Minimum and maximum phase of input impedance.](image)

The phase modulation of input impedance, $M_p$, is defined as

$$M_p = \frac{P_{\text{max}} - P_{\text{min}}}{P_{\text{max}} + P_{\text{min}}}. \quad (11)$$

### 4. Results

This section shows the change in the amplitude and phase modulations obtained from numerical analysis. We also explain the differences in the change of modulations. The amplitude modulation rapidly increases in the range from 0.01 to 0.18, and reaches a peak at 0.18, and slowly decreases from 0.18 to 1.0. The phase modulation rapidly increases in the range from 0.01 to 0.25, and reaches a peak at 0.25, and slowly decreases from 0.18. As a result, it was found that the amplitude modulation and the phase modulation had peaks at different aperture ratios.
5. Conclusion

In this study, we numerically analyzed the half-stopped horn by a one-dimensional model with multiple short tubes based on the transmission-matrix method. We especially investigated the influence of the aperture ratio on the amplitude and phase modulations of input impedance. As a result, the amplitude modulation was the largest when the aperture ratio is 18 %, and the phase modulation becomes largest when the aperture ratio is 25 %. Also, it was found that the amplitude modulation and the phase modulation had peaks at different aperture ratios.

REFERENCES