EXPERIMENTAL STUDY ON THE NONLINEAR VIBRATIONS OF A CIRCULAR CYLINDRICAL SHELLS: EFFECTS OF THERMAL GRADIENTS.

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In this paper, an experimental study on the large amplitude vibrations of a thin polymeric cylindrical shell subjected to a thermal gradient across the thickness is presented. The effects of the temperature gradient on the shell dynamic behavior are investigated. The present study has the aim of providing a deeper contribution to the experimental literature on the shell structures. Test have been carried out in controlled environment condition thanks to a climatic chamber and a heater cartridge placed inside the shell. The shell carries a top mass and an electro-dynamic shaker has been used in order to excite, with a harmonic load, the test specimen in the longitudinal direction. The harmonic forcing load consists of a stepped-sine sweep with frequency band limits containing the resonance frequency of the first axisymmetric vibration mode. Four different excitation amplitude levels and two different thermal gradients have been considered. The experimental results are presented and discussed in detail by means of frequency response analysis and bifurcation analysis.

Keywords: shells, experimental, nonlinear vibrations, thermal gradients

1. Introduction

Shells are widely used in the field of structural engineering as they allow to create strong and, at the same time, lightweight structures. However, it is well-known that under particular loading conditions these structures have a strongly non-linear behavior.

The research on the subject in the first half of the twentieth century has been significant (for example, see Ref. [1]). Nowadays, in many engineering applications, thin-walled structures are made of composite or hyper-elastic materials (Ref. [2]) and are subjected not only to mechanical loads but also to thermal loads or to fluid interactions. Such matters emphasize the nonlinear dynamics of the shells and make it even more difficult analyzing and predicting the response of the structure.

A short and absolutely non-exhaustive literature review on the subject covered in this article is reported below.

Mallon et al. [3] and Pellicano et al. [4] experimentally investigated the dynamics of a polymeric cylindrical shell with attached top mass. When the structure is axially excited by a resonant forcing, parametric excitations arisen due to the top mass-shell interaction. That leads to strong saturation of the top mass motion and large radial response of the shell has been observed.
Liew et al. [5] presented a numerical study on the linear and nonlinear vibrations of a functionally graded cylindrical panel subjected to a temperature gradient across the thickness. The influence of several parameters, such as the aspect ratio or the vibration amplitude, on the dynamic behavior of the panel has been analyzed for different temperature conditions.

Strictly related to the present paper are the works of Zippo et al. [6-7]: the dynamics of a polymeric circular cylindrical shell subjected to extreme temperature conditions has been experimentally investigated. Results pointed out the complex relationship between the temperature and parameter such as damping and stiffness and how these effects emphasize the nonlinear behavior of the shell.

In this paper, an experimental study focusing on the nonlinear vibrations of a polymeric circular cylindrical shell subjected to a thermal gradient across the thickness is presented.

2. Test set-up

A comprehensive description of the test set-up can be found in [7].

A polymeric circular cylindrical shell made of polyethylene terephthalate (PET) is mounted vertically on an electro-dynamic shaker: an aluminum cylindrical mass is glued on the top edge of the shell; the bottom edge is clamped to the shaker vibrating base.

All the tests are performed in a controlled environment under steady-state conditions of temperature distribution using a climatic chamber surrounding the test article and a finned heat cartridge placed inside the shell.

The test specimen is excited in the longitudinal direction through a harmonic load and an open-loop control is used to avoid interaction between the control system and structure under study [8].

<table>
<thead>
<tr>
<th>Table 1- Material properties and dimensions of the testing article</th>
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<tbody>
<tr>
<td><strong>Shell</strong></td>
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<tr>
<td>Material</td>
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<tr>
<td>Mass density</td>
</tr>
<tr>
<td>Young’s modulus</td>
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<td>Poisson’s ratio</td>
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<td>Length (effective)</td>
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<td>Radius</td>
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<td>Thickness</td>
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<table>
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<tr>
<th><strong>Top mass</strong></th>
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<tbody>
<tr>
<td>Material</td>
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<tr>
<td>Mass density</td>
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<tr>
<td>Weight</td>
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</table>

The harmonic forcing load consists on a stepped-sine sweep of frequency band limits containing the resonance frequency of the first axisymmetric vibration mode of the structure.

The frequency range 300 - 730 Hz is analyzed with a frequency step of 2.5 Hz and a step duration of 10 s. Both upward and downward frequency variations are considered.

Top mass vibrations are measured by means of three triaxial accelerometers; conversely, a laser vibrometer and a laser telemeter are used to measure, respectively, the lateral velocity and lateral displacement of the shell.
Two different temperature gradients across the shell thickness ($T_{in} = 48^\circ C / T_{ext} = 0^\circ C$; $T_{in} = 48^\circ C / T_{ext} = 20^\circ C$) and four drive excitation amplitude levels (0.1 V, 0.2 V, 0.3 V, 0.4 V) are considered.

![Fig. 1-detailed view of the shell with the attached cylindrical top mass](image)

3. **Experimental results**

In this section, are reported the experimental results of the tests carried out for upward frequency variation and amplitude of the drive excitation equal to 0.4 V. For the sake of clarity, the following notation is here adopted:

- Case A: $T_{in} = 48^\circ C / T_{ext} = 0^\circ C$;
- Case B: $T_{in} = 48^\circ C / T_{ext} = 20^\circ C$

The amplitude-frequency diagram of the top mass acceleration is shown in Fig. 2(a): a severe saturation occurs when the frequency of the forcing load approaches the natural frequency of the first axisymmetric mode of vibration of the structure. In this region, the shell dynamics is governed by the parametric excitation resulting from the top mass-shell interaction: this leads to energy exchange between directly and non-directly excited vibration modes, in fact, the shell exhibits a large radial response (Fig. 2(b,c)).
In Table 2 are reported the maximum value of the radial velocity (RMS) of the shell. The reduction of stiffness resulting from the higher mean temperature (case B), leads to an increment of the radial response of the shell even if the external excitation is nearly the same (Fig. 2(d)).

Table 2 – Maximum radial velocity of the shell

<table>
<thead>
<tr>
<th>Condition</th>
<th>Max Vel (RMS) [m/s]</th>
<th>f [Hz]</th>
</tr>
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<tbody>
<tr>
<td>$T_{in} = 48^\circ C$/ $T_{ext} = 0^\circ C$ (black line)</td>
<td>0.0534</td>
<td>445</td>
</tr>
<tr>
<td>$T_{in} = 48^\circ C$/ $T_{ext} = 20^\circ C$ (red line)</td>
<td>0.0905</td>
<td>440</td>
</tr>
</tbody>
</table>

In order to define a clear view of the complex dynamic scenario of shells, the frequency response analysis alone is not exhaustive.

For such mechanical systems, which are characterized by strong nonlinear behavior, the bifurcation analysis is mandatory.
The bifurcation diagrams of the top mass acceleration are shown in Fig. 3. A region of unstable motion can be observed in Fig. 3(a) between 417.5 - 542.5 Hz. When the mean temperature increases, Fig. 3(b), a larger amount of scattering can be observed in the instability region, that in case B is bounded between 410 - 525 Hz.

Taking advantage of what has been pointed out by the bifurcation diagrams, an in-depth analysis is presented using time histories, Fourier’s spectra and Poincaré maps.

Case A results are shown in Fig. 4.

Outside the saturation region, the shell behavior is linear, thus the top mass motion is periodic with the same frequency of the drive excitation and no large amplitude vibrations of the shell can be observed.

Contrariwise, inside the instability frequency range, the structure exhibits a really complex dynamics. For \( \omega_{\text{drive}} = 500 \times 2\pi \) rad/s, the top mass motion is subharmonic: in the Fourier spectrum (Fig. 4(b)) can be observed few well-pronounced peaks, in correspondence of the odd harmonics \((1 \times (\omega/\omega_{\text{drive}}), 3 \times (\omega/\omega_{\text{drive}}), 5 \times (\omega/\omega_{\text{drive}}), 7 \times (\omega/\omega_{\text{drive}}), 9 \times (\omega/\omega_{\text{drive}}))\), and side-bands, for\((\Delta \omega/\omega_{\text{drive}}) = 0.4\); three distinct sets are shown by the Poincaré map (Fig. 4(c)).

![Fig. 4 - 48°C/0°C (Case A): (a)time history; (b)spectrum; (c) Poincaré map.](image)

Fig. 5 shows the results relative to the case B. The response of the shell is different respect to what has been observed in Fig. 4. For \( \omega_{\text{drive}} = 500 \times 2\pi \) rad/s, the top mass acceleration spectrum, Fig. 5(b), shows an increased contribution of the 5th harmonic. Furthermore, the Poincaré map (Fig. 5(c)) shows a closed set of five distinct points, which suggest a quasi-periodic motion where the ratio, between the fundamental frequencies of the top mass response and the fundamental frequency of the shell velocity response, can be written as rational number [9].

![Fig. 5 - 48°C/20°C (Case B): (a)time history; (b)spectrum; (c) Poincaré map.](image)
4. Conclusions

The effect of the temperature gradient across the shell thickness on the nonlinear dynamics of a polymeric cylindrical shell has been experimentally investigated.

Two different temperature gradients (Tin = 48°C/Text = 0°C; Tin = 48°C / Text = 20°C) as well as several load amplitude conditions (0.1V, 0.2V, 0.3V, 0.4V), have been considered. Tests have been carried out under controlled environmental conditions and test specimen, composed by a polymeric shell and an aluminum top mass, has been excited in the longitudinal direction through a harmonic forcing load. Both upward and downward frequency variation of the drive excitation has been analyzed.

Results confirm the strong influence of the temperature on the shell dynamics and a completely different dynamic scenario has been pointed out for different temperature gradient conditions.

REFERENCES