APPLICATION OF AN ADAPTIVE PIEZO-SHUNT ABSORBER FOR VIBRATION CONTROL ON A CANTILEVER BEAM

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This work describes the application of a piezo-shunt absorber for robust vibration suppression on a cantilever beam with two piezoelectric patches. The cantilever beam is submitted to harmonic excitation forces containing several harmonic components. In particular, the implementation of the Resistance-Inductance-Capacitance circuit is employed for vibration absorption at the first four mode-shapes of the beam and, by using the Frequency Response Function of the arrangement, the modal parameters of such dominant mode-shapes can be used to synthesize an adaptive-like piezo-shunt absorber for damping injection at specific modes. Some experimental results are presented to validate the good dynamic performance of the piezo-shunt absorber for adaptive damping injection to the cantilever beam.

Keywords: Damping injection; Euler-Bernoulli beam; Vibration control; Piezo-shunt absorber.

1. Introduction

Vibration control problems in engineering structures can be actively solved by using different types of smart sensors and actuators [1, 2]. Active damping technologies are continuously evolving and improving their dynamical properties. In particular, Piezoelectric (PZT) actuators are quite popular for active vibration control and energy harvesting, with many results and applications reported in the literature [3]. Moreover, PZT actuators can be used as smart vibration absorbers to inject damping at specific vibration mode-shapes. One important and low-cost approach consists of the so-called Piezo-shunt vibration absorbers, mainly because the automatic or adaptive tuning can be realized, in an efficient manner, via shunt circuits and appropriate selection of the coupled impedance electrical parameters to adjust their tuned resonance frequency [4, 5].

On the other hand, an algebraic parametrical identification technique for vibrating mechanical systems has been proposed in [8]. This on-line algebraic parameter identification is based on module theory, differential algebra and operational calculus, where the modal parameters of a mechanical system can be identified in a fast and efficient manner.

This work considers the application of a piezo-shunt absorber for damping injection and vibration control on a cantilever beam with two PZT patches. The Resistance-Inductance-Capacitance circuit is
employed for vibration absorption at the first 4 mode-shapes of the beam and, by using the Frequency Response Function of the arrangement, the modal parameters of some dominant mode-shapes can be used to combine with algebraic identification to synthesize an adaptive-like piezo-shunt absorber for damping injection at specific modes.

2. Cantilever beam with two PZT patches

2.1 System description

The experimental setup consists of an aluminum cantilever beam of total length $L$, tip mass $M$ and two PZT patches, as shown in Fig. 1. The distributed mass is denoted as $m(x) = \rho A$, Young modulus $E$, area moment of inertia $I(x)$, density $\rho$ and cross sectional area $A$. Moreover, $x$ and $y(x,t)$ represent the longitudinal coordinate and lateral deflection of the beam. The PZT patches are from Physik Instrumente® DuraAct™, models P-876.A12 and P-876.A15, which were cemented over the beam on locations determined according to the Gawronski’s approach.

![Figure 1: Cantilever beam with two PZT patches.](image)

2.2 Mathematical model for the cantilever beam: a modal description

A cantilever (thin) beam can be modeled via the Euler-Bernoulli equation, as follows

$$\rho A \frac{\partial^2 y(x,t)}{\partial t^2} + \frac{\partial^2}{\partial x^2} \left[ EI(x) \frac{\partial^2 y(x,t)}{\partial x^2} \right] = f(x,t), \quad 0 < x < t \quad (1)$$

where $f(x,t)$ is the distributed force acting along the beam. The cantilever beam boundary conditions are established as

$$y(0,t) = 0, \quad \frac{\partial y(x,t)}{\partial x} \bigg|_{x=0} = 0 \quad (2)$$

For the free end, considering that there is a concentrated mass $M$ at $x = L$, the boundary conditions are given by

$$EI(x) \frac{\partial^2 y(x,t)}{\partial x^2} = 0, \quad x = L \quad (3)$$
\[ EI(x) \frac{\partial^3 y(x,t)}{\partial x^3} - M \frac{\partial^2 y(x,t)}{\partial t^2} = 0, \quad x = L \] \hspace{1cm} (4)

The application of the Rayleigh’s expansion theorem to Eq. (1), including proportional (structural) damping, results in the modal description

\[ \ddot{q}_r(t) + 2\zeta_\omega r \dot{q}_r(t) + \omega_r^2 q_r(t) = f_r(t), \quad r = 1, 2, \cdots \] \hspace{1cm} (5)

where \( \omega_r \) and \( \zeta_r \) denote the natural frequency and modal damping associated to the \( r \)-th mode.

2.3 Experimental modal analysis

An experimental modal analysis on the cantilever beam with two PZT patches was performed to get the inherent modal parameters of the beam. The corresponding Frequency Response Function (FRF) is shown in Fig. 2.

![Figure 2: Frequency Response Function of the cantilever beam describing its first four modes (0.7172 Hz, 5.96 Hz, 17.41 Hz and 34.93 Hz).](image)

3. Application of Piezo-shunt absorbers for active damping injection

PZT shunt circuits were computed and applied to inject damping to the first four vibration modes of the cantilever beam. The PZT shunt circuits were designed and adjusted by using virtual inductors based on differential operational amplifiers and voltage-controlled resistors via a JFET transistor and analog multiplier [6].

The integration of the experimental setup is described in Fig. 4 (see [7]).

4. Experimental results

The PZT shunt absorber can be applied to inject damping at specific frequencies associated to the first analyzed four mode-shapes of the cantilever beam. For instance, the experimental evaluation without and with PZT shunt circuit is shown in Fig. 5. In this case, by comparing the resulting modal parameters in Table 1 one can observe that the PZT shunt absorber is indeed able to inject damping to the first mode-shape up to 80.64\%, increasing also the effective damping on other mode-shapes.
Figure 3: PZT shunt circuit using virtual inductors and voltage-controlled resistance with JFET.

Table 1: Modal parameters of the first mode-shape at 0.8392 Hz without/with PZT shunt circuit.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency $\omega_i$ [Hz]</th>
<th>Damping $\zeta_i$ without PZT shunt</th>
<th>Damping $\zeta_i$ with PZT shunt</th>
<th>Variation %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8392</td>
<td>0.0062</td>
<td>0.0112</td>
<td>80.64</td>
</tr>
<tr>
<td>2</td>
<td>6.2714</td>
<td>0.0089</td>
<td>0.0100</td>
<td>12.36</td>
</tr>
<tr>
<td>3</td>
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<td>0.0020</td>
<td>0.0037</td>
<td>85.00</td>
</tr>
<tr>
<td>4</td>
<td>35.0879</td>
<td>0.0025</td>
<td>0.0033</td>
<td>32.00</td>
</tr>
</tbody>
</table>

5. On-line algebraic identification techniques for adaptive-like PZT shunt absorbers

The on-line algebraic identification methodology, described in detail in [8], can be applied to compute fast parameter estimation for two different situations:

- Estimation of the modal parameters (natural frequencies and damping) of the primary mechanical structure with PZT shunt absorbers to synthesize an adaptive-like and robust tuning of the PZT
Figure 4: Schematic diagram of the PZT shunt integration for active damping injection on the cantilever beam.

shunt circuits.
- Estimation of the parameters associated to the excitation signals, such as their amplitudes, frequencies and phases.

The modal description of the primary structure in Eq. (5) can be used to synthesize the on-line algebraic identification algorithms. A schematic diagram is depicted in Fig. 6.

6. Concluding remarks

This work deals with the synthesis and application of adaptive-like PZT shunt absorbers for active damping injection on primary mechanical structures and on-line algebraic parameter identification techniques. A case study of a cantilever beam is considered for experimental evaluation of the proposed approach.
Figure 5: Experimental FRF. (a) Mode 1 without PZT shunt circuit, and (b) Mode 1 with PZT shunt circuit.

Figure 6: Schematic diagram of adaptive-like PZT shunt absorbers using algebraic identification techniques.
REFERENCES


