AN ANALYTICAL INVESTIGATION ON STIFFNESS - ERROR PHASING OF EQUALLY SPACED PLANETARY GEAR

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Planetary gear with equally spaced planets is mostly used to achieve vibration and noise reduction. To investigate the internal mechanism of planet phasing, a time varying mesh stiffness - static transmission error planet phasing model was proposed. The mesh stiffness and transmission error were expressed as Fourier series, and the mesh excitation of the central component was derived to reveal the relationship among mesh stiffness phasing term, transmission error phasing term and mesh excitation phasing term. Meanwhile, the mesh torque phasing theory of traditional planet phasing theory was modified by stating that vibration is suppressed when the mesh torque harmonic is zero. Numerical simulations were implemented to verify the modified planet phasing model. The results indicated that it agrees well to the traditional one except for the case that the mesh torque harmonic is zero. The proposed model incorporates the actual mesh stiffness and transmission error into planet meshing, which improves the compatibility between the study of planet phasing theory and the study of the internal mesh characteristics.

Key words: planetary gear, planet phasing, time-varying stiffness, transmission error, mesh excitation

1. Introduction

Planetary gears own compact structure, large transmission ratio, strong bearing capacity and high transmission efficiency. It has been more and more popular with transmission machinery in recent years.

Seager [1] studied the phase tuning theory by using the static transmission error dynamic excitation model, and revealed the mapping relationship between the number of central gear teeth, the number of planetary gear and the vibration mode of the system. Kahraman [2] and Blankenship [3] use the static transmission error dynamic excitation model to study the mapping relationship between the phase characteristics of the excitation force and the vibration mode of the system. Parker [4,5] takes meshing excitation as the starting point, expresses meshing excitation as Fourier series expansion form, deduces the force situation of each central gear, and gives the mapping relationship between the number of planetary gears, the number of teeth of the central gear, the harmonic order and the force characteristics of the central gear.

In the process of dynamic analysis of planetary gears, in order to reflect the actual meshing state of each gear pair more truly, the meshing excitation between gear pairs often takes both time varying meshing stiffness and error excitation into account [6-7]. However, the existing theory of phase tuning of
planetary gears is not consistent with the current method of dealing with the internal meshing characteristics of planetary gears, which leads to poor compatibility between the theory of phase tuning and the study of the internal meshing characteristics of planetary gears.

Planetary gear transmission mostly adopts planetary gear uniform structure to achieve better vibration and noise reduction engineering purpose [8]. Therefore, taking equally spaced planetary gears as the research object, starting from the actual time-varying meshing stiffness-static transmission error meshing excitation, the meshing stiffness component and transmission error component in meshing excitation are respectively expressed as Fourier series expansion form. At the same time, the model takes the time-varying meshing stiffness into account, which improves the compatibility between phase tuning theory and the study of internal meshing characteristics of planetary gears.

2. Force analysis of central gear

The force analysis of the central component of planetary gear train is carried out, taking the central gear sun gear as an example, as shown in Figure 1. $O_{ij}$ is the fixed coordinate system, $Oe_i'e_2'$ is the local coordinate system, which is connected to the $i$ planet gear. $Z_j$ is Number of sun gear teeth, $F_i$ is the meshing frequency of sun and $i$ planet gear, $F_{i1}$ and $F_{i2}$ is the meshing excitation of the meshing gear pair which is projected along the radial and tangential direction of the sun gear, $F_i$ can be expressed in $Oe_i'e_2'$ and $O_{ij}$ as follows [4]:

$$F_i = F_{i1}e_i + F_{i2}e_{2}'$$

$$F_i = F_{i1}i + F_{i2}j$$

$$\begin{bmatrix} F_{ix} \\ F_{iy} \end{bmatrix} = \begin{bmatrix} \cos \psi_i \\ -\sin \psi_i \end{bmatrix} \begin{bmatrix} F_{i1} \\ F_{i2} \end{bmatrix}$$

where $\psi_i$ is the angle between $Oe_i'e_2'$ and $O_{ij}$, and $\psi_i = 2\pi(i-1)/N$, $N$ is the number of planet gears.

Through the transformation of coordinate system, the expressions of lateral excitation and torsion moment of the sun gear acting on the central component of the planetary gear are as follows:

$$\begin{bmatrix} F_{ix} \\ F_{iy} \\ T_{sun} \end{bmatrix} = \sum_{i=1}^{N} \begin{bmatrix} \sin \psi_{si} \\ -\cos \psi_{si} \\ r_{sun} \end{bmatrix} F_i$$

Figure 1: Mesh force at the sun-planet gear pair
where \( \psi_{si} = \psi_i - \alpha_s \).

3. Time varying meshing stiffness-static transfer error phase tuning model

In transverse meshing excitation, the tuning mechanism of \( y \) direction meshing excitation is the same as that of \( x \) direction meshing excitation. Therefore, only \( x \) direction meshing excitation is taken as an example for theoretical deduction, and \( y \) direction can be deduced by the same method. In classical theory, the derivation of transverse meshing excitation is carried out in the following local coordinate system \( Oe'_1, e'_2 \). In order to reduce the complexity and redundancy of the derived equations, a meshing excitation refinement model based on time-varying meshing stiffness and static transfer error is established in a fixed coordinate system \( Oij \), and the time varying meshing stiffness and static transfer error are expressed as Fourier series expansion forms respectively.

The expression of the transverse meshing excitation of the sun gear in \( x \) direction is:

\[
F_x (t) = \sum_{i=1}^{N} k_{si} (t) e_{si} (t) \sin \psi_{si} \tag{4}
\]

where \( k_{si} (t) \) is the time-varying meshing stiffness between the sun gear and the \( i \) planet gear; \( e_{si} (t) \) is the static transmission error between the sun gear and the planet gear, the difference of the static transmission error between the sun gear and the planet gear is neglected. The expansions of Fourier series of \( k_{si} (t) \) and \( e_{si} (t) \) are as follows [7]:

\[
k_{si} (t) = \bar{k}_{si} \left[ 1 + \sum_{l_i=1}^{l} k_{si}^l \sin \left( l_1 2 \pi f_m t + l_1 Z_s \psi_i + l_1 \gamma_{ke} + \phi_{si}^l \right) \right] \tag{5}
\]

\[
e_{si} (t) = \sum_{l_i=1}^{l} e_{si}^l \sin \left( l_2 2 \pi f_m t + l_2 Z_s \psi_i + \phi_{si}^l \right) \tag{6}
\]

where \( \bar{k}_{si} \) is the average meshing stiffness between the sun gear and the planet gear; \( k_{si}^l \) is \( l \)-th harmonic dimensionless amplitude of time-varying meshing stiffness between sun and planet gears; \( e_{si}^l \) is \( l \)-th harmonic second magnitude of static transmission error between sun and planet gears; \( \gamma_{ke} \) is the initial phase of the time-varying meshing stiffness and the transfer error function respectively. \( f_m \) is mesh frequency; \( \gamma_{ke} \) is the phase of the time-varying meshing stiffness and the transfer error function.

Substitution of equations of (5) and (6) into equation (4), The transverse meshing excitation in the \( x \) direction of the sun gear in the form of double Fourier series expansion multiplication can be obtained as follows:

\[
F_x (t) = \sum_{i=1}^{N} \bar{k}_{si} \sin \psi_{si} \sum_{l_1=1}^{l_1} e_{si}^l \sin \left( \theta_1 + l_1 Z_s \psi_i \right) + \sum_{i=1}^{N} \bar{k}_{si} \sin \psi_{si} \sum_{l_1=1}^{l_1} k_{si}^l \sin \left( \theta_2 + l_1 Z_s \psi_i \right) \sum_{l_2=1}^{l_2} e_{si}^l \sin \left( \theta_2 + l_2 Z_s \psi_i \right) \tag{7}
\]

where \( \theta_1 = l_1 2 \pi f_m t + l_1 \gamma_{ke} + \phi_{si}^l, \theta_2 = l_2 2 \pi f_m t + \phi_{si}^l \).

The structure of the term I conforms to Kahraman's average meshing stiffness-static transfer error model [2]. The term II is a time varying meshing stiffness-static transfer error double Fourier series
expansion term. It is a model that includes both meshing stiffness tuning and transmission error tuning. The core of this theory is to elaborate the relationship between stiffness tuning, error tuning and actual meshing excitation on the basis of refinement of traditional phase tuning model.

Definition of meshing stiffness tuning term $k_1 = \text{mod} \left( l_i Z_s / N \right)$, the transfer error tuning term $k_2 = \text{mod} \left( l_i Z_s / N \right)$, the meshing excitation tuning term $k = \text{mod} \left( l Z_s / N \right)$.

Substitution of equation $\psi_{si} = \psi_i - \alpha_s = 2 \pi \left( i - 1 \right) / N - \alpha_s$ into term I yields:

$$I = \frac{1}{2} k_{sp} \sum_{l_1=1}^{L_1} \sum_{l_2=1}^{L_2} e_{sp}^{l_1} e_{sp}^{l_2} \left\{ \sin \theta_2 \sum_{i=1}^{N} \sin A_N^{i} + \cos \theta_2 \sum_{i=1}^{N} \cos B_N^{i} \right\}$$

(8a)

$$\sin A_N^{i} = \sin \left( \frac{2 \pi \left( i - 1 \right) \left( k_2 + 1 \right)}{N} - \alpha_s \right) - \sin \left( \frac{2 \pi \left( i - 1 \right) \left( k_2 - 1 \right)}{N} + \alpha_s \right)$$

(8b)

$$\cos B_N^{i} = \cos \left( \frac{2 \pi \left( i - 1 \right) \left( k_2 - 1 \right)}{N} + \alpha_s \right) - \cos \left( \frac{2 \pi \left( i - 1 \right) \left( k_2 + 1 \right)}{N} - \alpha_s \right)$$

(8c)

The following identities hold for integer values of $m$:

$$\sum_{i=1}^{N} \sin \left[ \frac{2 \pi \left( i - 1 \right) m}{N} + \alpha_s \right] = \begin{cases} 0, & \frac{m}{N} \neq \text{integer} \\ N \sin \alpha_s, & \frac{m}{N} = \text{integer} \end{cases}$$

(9)

$$\sum_{i=1}^{N} \cos \left[ \frac{2 \pi (i - 1) m}{N} + \alpha_s \right] = \begin{cases} 0, & \frac{m}{N} \neq \text{integer} \\ N \cos \alpha_s, & \frac{m}{N} = \text{integer} \end{cases}$$

(10)

The phase tuning law of the term I is consistent with the classical phase tuning theory[4,5]. The meshing stiffness is constant, so it can be considered that the meshing excitation tuning term is equal to the transmission error excitation tuning term, $k = k_2$.

The term II is the Fourier series expansion term of the time varying meshing stiffness-static transfer error function, which is an additional part of the traditional phase tuning theory after considering the time-varying meshing stiffness. Similar to the term I, Substitution of equation $\psi_{si} = \psi_i - \alpha_s = 2 \pi \left( i - 1 \right) / N - \alpha_s$ into term II yields:

$$II = \frac{1}{2} k_{sp} \sum_{l_1=1}^{L_1} \sum_{l_2=1}^{L_2} k_{sp}^{l_1} e_{sp}^{l_2} \left\{ \sin \theta_2 \sin \theta_1 \sum_{i=1}^{N} \cos \left( \frac{2 \pi \left( i - 1 \right) k_1}{N} \right) \sin A_N^{i} + \cos \theta_2 \sin \theta_1 \sum_{i=1}^{N} \sin \left( \frac{2 \pi \left( i - 1 \right) k_1}{N} \right) \sin A_N^{i} \\
+ \cos \theta_2 \cos \theta_1 \sum_{i=1}^{N} \cos \left( \frac{2 \pi \left( i - 1 \right) k_1}{N} \right) \cos B_N^{i} + \sin \theta_2 \cos \theta_1 \sum_{i=1}^{N} \cos \left( \frac{2 \pi \left( i - 1 \right) k_1}{N} \right) \cos B_N^{i} \right\}$$

(11)

Fourier series expansion term II of time-varying meshing stiffness-static transfer error, can be divided into the following four situations:

(1) When $k_1 \neq 0$ and $k_2 = 1$ or $N - 1$

(1.1) If $k_1 \neq 0$ and $k_2 = 1$
(a) while $k_1 \neq 2, N - 2$:

$$\Pi = 0$$  \hspace{1cm} (12)

(b) while $k_1 = 2$:

$$\Pi = -\frac{1}{4} N k_{sp}^l h_e^l \left[ \sin \alpha_s \cos (\theta_1 - \theta_2) + \cos \alpha_s \sin (\theta_1 - \theta_2) \right]$$  \hspace{1cm} (13)

(c) while $k_1 = N - 2$:

$$\Pi = \frac{1}{4} N k_{sp}^l h_e^l \left[ \sin \alpha_s \cos (\theta_1 + \theta_2) + \cos \alpha_s \sin (\theta_1 + \theta_2) \right]$$  \hspace{1cm} (14)

(1.2) If $k_1 \neq 0$ and $k_2 = N-1$

(a) while $k_1 \neq 2, N - 2$:

$$\Pi = 0$$  \hspace{1cm} (15)

(b) while $k_1 = 2$:

$$\Pi = \frac{1}{4} N k_{sp}^l h_e^l \left[ \sin \alpha_s \cos (\theta_1 + \theta_2) + \cos \alpha_s \sin (\theta_1 + \theta_2) \right]$$  \hspace{1cm} (16)

(c) while $k_1 = N - 2$:

$$\Pi = -\frac{1}{4} N k_{sp}^l h_e^l \left[ \sin \alpha_s \cos (\theta_1 - \theta_2) - \cos \alpha_s \sin (\theta_1 - \theta_2) \right]$$  \hspace{1cm} (17)

For situation (1), when $N = 4$, all additive terms in the term $\Pi$ can be converted into constants. The specific expressions are as follows:

$$\begin{cases} 
\Pi = -\frac{1}{4} N k_{sp}^l h_e^l \left[ \sin \left( \theta_1 + \theta_2 - \alpha_s \right) + \sin \left( \theta_1 - \theta_2 + \alpha_s \right) \right] & k_2 = 1 \\
\Pi = \frac{1}{4} N k_{sp}^l h_e^l \left[ \sin \left( \theta_1 + \theta_2 + \alpha_s \right) + \sin \left( \theta_1 - \theta_2 - \alpha_s \right) \right] & k_2 = N - 1 
\end{cases}$$  \hspace{1cm} (18)

For other three cases: (2) $k_1 = 0$ and $k_2 \neq 1$ and $N - 1$ (3) $k_1 = 0$ and $k_2 = 1$ or $N - 1$ (4) When $k_1 \neq 0$ and $k_2 \neq 1$ and $N - 1$ are consistent with the deduction logic of the case (1), it is no listed in detail.

Based on the discussion of the above four tuning conditions, the general laws of transverse meshing excitation tuning are summarized as follows:

(L1) When $k_1 \neq 0$ and $k_2 = 1$ or $N - 1$, translational response excited;

(L2) When $k_1 = 0$ and $k_2 \neq 1$ and $N - 1$, translational response suppressed;

(L3) When $k_1 = 0$ and $k_2 = 1$ or $N - 1$, translational response excited;

(L4) When $k_1 \neq 0$ and $k_2 \neq 1$ and $N - 1$: If $k_1 \neq k_2 + 1$ and $N - k_2 - 1$, translational response suppressed; If $k_1 = k_2 + 1$ or $N - k_2 - 1$, translational response excited.

According to the deduced conclusion of phase tuning of time varying meshing stiffness-static transfer error, it can be found that the characteristic frequency under the excitation of vibration is as follows:
The relationship between meshing excitation tuning term, time-varying meshing stiffness tuning term and error phase tuning term represents:

\[ k = \text{mod}\left(\frac{IZ_s}{N}\right) \]

\[ = \text{mod}\left(\frac{l_1 + l_2}{N}Z_s\right) \text{ or } \text{mod}\left(\frac{l_1 - l_2}{N}Z_s\right) \text{ or } \text{mod}\left(\frac{l_2Z_s}{N}\right) \text{ or } 0 \]  \hspace{1cm} (19)

Simplification of the above formula can be obtained:

\[ k = \text{mod}\left(\frac{k_1 + k_2}{N}\right) \text{ or } \text{mod}\left(\frac{|k_1 - k_2|}{N}\right) \text{ or } \text{mod}\left(\frac{l_2Z_s}{N}\right) \text{ or } 0 \]  \hspace{1cm} (20)

According to the relations among the three tuning terms, the relationship between the tuning terms of transverse meshing excitation and the general tuning law of transverse meshing excitation in Section 2 is summarized:

(1) \( k \neq 1, \ N - 1 \) suppression of transverse vibration in corresponding (L2) and (L4);
(2) \( k = 1 \text{ or } N - 1 \) excitation of transverse vibration in corresponding (L1), (L3) and (L4);

4. Numerical simulation of stiffness-error phase tuning model

Taking a single-stage four-planetary gear transmission system as an example, its working speed range is 500-1750 r/min. The basic parameters of planetary-sun gear pair are listed in Table 1.

<table>
<thead>
<tr>
<th>Category</th>
<th>modulus /mm</th>
<th>Tooth number</th>
<th>Tooth width /mm</th>
<th>pressure angle /°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun gear</td>
<td>1.75</td>
<td>29</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>Planet gear</td>
<td>43</td>
<td>35</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figures 2-5 are the transverse force spectrum of the central component sun gear under 500-1750 r/min working speed combined with different transverse meshing excitation harmonics.

Figure 2: Force of central component with \( l_1 = 1, l_2 = 1 \)

Figure 3: Force of central component with \( l_1 = 1-2, l_2 = 1 \)
Situation $l_1 = 1$, $l_2 = 1$ in figure 3 is consistent with the results of transverse meshing excitation tuning (L1), and the characteristic frequency of transverse meshing excitation is meshing fundamental frequency. Situation $l_1 = 1$, $l_2 = 1$ in figure 4 and situation $l_1 = 1$, $l_2 = 1$ in figure 5 is consistent with the transverse vibration excitation in the conclusion of translational meshing excitation tuning (L1), which excites the fundamental frequency of meshing and the triple frequency of meshing. Situation $l_1 = 1$, $l_2 = 1$ in figure 6. It coincides with the results of translational meshing excitation tuning (L1) and (L3) which excites translational vibration, and at the same time excites the fundamental, triple and quintuple frequencies of meshing. In agreement with the classical phase tuning results, the correctness of the phase tuning model of transverse meshing excitation is verified. Considering the space problem, the numerical simulation of torque tuning in torsion direction is omitted in this paper.

5. Conclusion

In this paper, a time-varying meshing stiffness-static transmission error phase tuning model of planetary gear transmission system is established. The phase tuning relationship between time varying meshing stiffness, static transmission error and actual meshing excitation is deeply analysed. By analytic deduction, the calculation model of meshing excitation under different stiffness-error tuning conditions is given. Through numerical simulation, the consistency of the proposed theory of time-varying meshing stiffness-static transfer error type phase tuning with the traditional theory of phase tuning is verified. The traditional theory of phase tuning is modified and improved to make the calculation theory of phase tuning of planetary gear meshing excitation more accurate and perfect. It improves the compatibility between the phase tuning theory and the internal meshing characteristics of planetary gears, lays a theoretical foundation for further refinement of the meshing characteristics of planetary gears, and has important significance for guiding the vibration reduction and noise reduction of planetary gears.

Acknowledgement

This research work is supported by National Natural Science Foundation of China (Grant No. 51805106) and the Marine Low-Speed Engine Project-Phase I.
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