THE STUDY OF DISCHARGE COEFFICIENT OF AEROSTATIC THRUST BEARINGS WITH INHERENT ORIFICES BASED ON FLUENT

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Abstract: Aerostatic bearings with orifices adopt external pressure to provide additional load capacity in order to overcome the poor load capacity with low viscosity of gas lubrication. In general, the investigation of the performances of aerostatic bearings is based on the solution of Reynolds equation and the discharge coefficient in the analysis is always constant 0.8 which may influence the accuracy of solution of Reynolds equation. In the present study, a method which combines Fluent and Reynolds equation is proposed to obtain the discharge coefficient in the aerostatic thrust bearings with inherent orifices. The discharge coefficients are calculated by comparing the mass flow rate obtained by Fluent and Reynolds equation simulations. The results indicate that the discharge coefficients are not constant and change with the flow and geometry parameters. Moreover, the bearing capacity from solution of Reynolds equation with discharge coefficient is agreement with that from Fluent, which means the discharge coefficient is suitable and valid for solution of Reynolds equation.

Keywords: aerostatic thrust bearing; discharge coefficient; Reynolds equation; Fluent

1. Introduction

Compared with conventional bearings, such as oil bearing and ball bearings, gas bearings have superior advantages resulting from their lubrication low friction, high accuracy, high-speed abilities, and long working life and have a wider application in precision machines and the high-speed turbomachinery [1-6]. Additionally, compared with self-acting gas bearings, aerostatic bearings with external pressure can overcome the shortcoming of the low load capacity due to the lower viscosity of gaseous lubrication. There exist abundant papers about the aerostatic bearing performances published in the past years and the performances of the aerostatic bearing are generally studied by calculation of the Reynolds equations. There exist three type of boundary conditions which are coincidence boundary, atmosphere boundary and the mass flow balance boundary that is the key boundary in the solution for the Reynolds equations of aerostatic bearing [7]. The mass inflow rate to pass the orifice can be obtained by formula 7 [8], while the crux of the calculation of mass inflow rate is the value of discharge coefficient $C_d$ which is the key parameter connected the real mass inflow rate with the ideal mass inflow rate of the ideal Laval nozzle. Generally, in the analysis for mass inflow rate of feeding orifices of aerostatic bearing, the feeding orifice is treated as ideal Laval nozzle. With the discrepancy between the ideal Laval nozzle with the realistic feeding orifices of aerostatic bearing, the actual mass inflow rate of realistic feeding orifices of aerostatic bearing is equal to the product of the mass flow rate of ideal Laval nozzle and loss factor which is called discharge coefficient $C_d$ [9]. Usually, the value of discharge coefficient is taken as constant 0.8, which
may influence the accuracy of solution of Reynolds equation. In fact, the discharge coefficient is affected by the geometry and flow parameters such as orifice diameter, bearing radius, supply pressure and so on. Belforte et al. [10] studied the discharged coefficients experimentally. In the end, he obtained that discharge coefficients could be computed by an empirical equation based on aerostatic bearing geometry and Reynolds number. Although Belforte [10] proposed an empirical equation to compute discharge coefficients, the discharge coefficients were obtained by comparing the actual mass inflow rates measured in the experiments with that calculated by the supply pressure and downstream pressure which is the maximum pressure in the pressure depression region. So, the discharge coefficients computed by the empirical equation may not suit for the solution of Reynolds equation using finite difference method (FDM). In view of the defect of discharge coefficients in the reference [10], by comparing the load capacity calculated by Reynolds equation using FDM and CFD simulations, Chang [11, 12] investigated the influence of the geometry and flow parameters on the discharge coefficient. As mentioned previously, the discharge coefficient in solution for Reynolds equation involves the mass flow rate balance boundary and that is why the discharge coefficient proposed by Chang may be not suitable for the solution of the Reynolds equation of aerostatic bearings.

In the paper, the discharge coefficient is obtained by a method which combines Fluent and Reynolds equation in the aerostatic thrust bearings with inherent orifices. The discharge coefficients are calculated by comparing the mass flow rate obtained by Fluent and Reynolds equation simulations. In this paper, the definition of discharge coefficient is not the same to the general definition of the discharge coefficient mentioned before. The general definition of the discharge coefficient is based on the mass flow rate of ideal Laval nozzle, while the definition of the discharge coefficient in the paper is based on the actual mass inflow rate of orifices calculated by Fluent. Moreover, the effects of geometry and flow parameters on the discharge coefficient are studied. In the end, the bearing capacity from solution of Reynolds equation with the \( C_d \) at the constant 0.8 and obtained in the paper are compared with the results calculated by the Fluent and results show that the bearing capacity with the discharge coefficients in the paper are more agreement with that from Fluent, comparing with the bearing capacity with constant \( C_d \) 0.8. This means the discharge coefficient is suitable and valid for solution of Reynolds equation.

2. Mathematical model of the discharge coefficient

In this section, the calculation of aerostatic bearing based on the commercial software Fluent is introduced firstly and then the solution of Reynolds equation of aerostatic bearing is described, while in the last, the method combined the Fluent and Reynolds equation is proposed to study the discharge coefficient based on the mass flow rate balance.

2.1 Solution of aerostatic bearing with single inherent orifice based on Fluent

The Fig. 1 is the schematic of aerostatic bearing with single inherent orifice with supply chamber and without chamber. The commercial software Fluent is used to study the performance of the aerostatic bearing. In order to save computing resource and time, the 1/4 model of aerostatic bearing seen Fig. 1 were built based on the assumption that the gas is isothermal, ideal gas, seen Fig. 2. In order to study the effect of flow regime on the performance of the bearing, the flow regime is assumed laminar and Transition SST turbulence model. There exist several boundaries, i.e. 1/4 cycle boundary, inlet and outlet pressure boundary (supply pressure and atmosphere pressure). The comparison results of laminar model, turbulence model and paper [10] are shown in the Fig. 3 and the bearing capacity, mass flow rate of laminar and turbulence model are listed in the table 1. As a whole, the results of Fluent simulations show good agreement with the results published in the paper [10], which means the boundary conditions and model of Fluent simulation are valid. In addition, the effect of laminar and turbulence model can be ignored in the aerostatic thrust bearing with single inherent orifice. The effect of supply chamber on the
performance of the aerostatic bearing with single inherent orifice is shown in the Fig. 4, which the simulation model is laminar model. The results show that the supply chamber has litter influence on the performance of the aerostatic thrust bearing with single inherent orifice. To sum up, in this paper, the simulation model of the aerostatic thrust bearing with single inherent orifice can be assumed as laminar model without supply chamber.

Fig. 1 Aerostatic bearing with inherent orifice

Fig. 2 Mesh for aerostatic thrust bearing with single orifice

Fig. 3 Comparison of laminar solution, turbulent solution, experimental data in [10]

Fig. 4 Effect of the supply chamber on pressure distribution in the air film

Table 1 The comparison of bearing capacity and mass inflow rate between the laminar solution and turbulent solution

<table>
<thead>
<tr>
<th>Film thickness/μm</th>
<th>Bearing capacity /N</th>
<th>Mass flow rate/kg/s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>laminar model</td>
<td>turbulence model</td>
</tr>
<tr>
<td>9</td>
<td>99.45</td>
<td>99.48</td>
</tr>
<tr>
<td>14</td>
<td>59.17</td>
<td>59.33</td>
</tr>
</tbody>
</table>

2.2 Reynolds equation

The Reynolds equation of aerostatic thrust bearing with single central orifice can be solved analytically, shown as Eq. (1) [13]. Then mass outflow rate \( \dot{m}_{\text{out}} \) can be computed by Eq. (2), while mass inflow rate \( \dot{m}_{\text{in}} \) can be computed by Eq. (3). As shown in the left section of Fig. 5, for given geometry and flow parameters, the solution of Eq. (1) to Eq.(3) goes on with adjusted \( p_r \) until the convergence of \( \dot{m}_{\text{out}} \) and \( \dot{m}_{\text{in}} \).
\[ p = \sqrt{p_r^2 + (p_a^2 - p_r^2) \frac{\ln \frac{r}{r_0}}{\ln \frac{r_a}{r_0}}} \]  
\[ \dot{m}_{\text{out}} = -\frac{\pi r_a h_0}{12 \mu R_g T} p_r \ln \frac{r_a}{r_0} \]  
\[ \dot{m}_{\text{in}} = C_d A_p \left( \frac{2p_a}{p_a} \phi(p_r) \right) \]  

where

\[
\phi(p_r) = \begin{cases} 
\frac{\kappa}{2} \left[ \left( \frac{2}{\kappa + 1} \right)^{\frac{\kappa + 1}{\kappa - 1}} \right]^{1/2}, & p_r \leq \left( \frac{2}{\kappa + 1} \right)^{\frac{\kappa}{\kappa - 1}} \\
\frac{\kappa}{\kappa - 1} \left[ \left( \frac{p_r}{p_s} \right)^{\frac{2}{\kappa}} - \left( \frac{p_r}{p_s} \right)^{\frac{\kappa + 1}{\kappa}} \right]^{1/2}, & p_r > \left( \frac{2}{\kappa + 1} \right)^{\frac{\kappa}{\kappa - 1}} 
\end{cases}
\]

where \( p_r \) is the pressure of orifice in the solution of Reynolds equation, \( r_a \) is the bearing radius, \( r_0 \) is orifice radius, \( \rho_a \) is the gas density in atmosphere, \( \kappa \) is ratio of specific heats of a gas, \( \mu \) is dynamic viscosity, \( R_g \) is gas constant.

### 2.3 The method for the discharge coefficient

As mentioned before, the definition of the discharge coefficient in the paper is based on the actual mass inflow rate of orifices calculated by Fluent. So, in order to calculate discharge coefficient, as shown in Fig. 5, for given parameter, \( C_d \) is changed until \( \dot{m}_{\text{real}} \) converging with \( \dot{m}_{\text{in}} \), where \( \dot{m}_{\text{real}} \) is computed by Fluent solution and \( \dot{m}_{\text{in}} \) is calculated by Reynolds equation.

Fig. 5 Flow chart of solution of discharge coefficients

Fig. 6 Comparison of pressure distribution determined by Fluent simulations and Reynolds equation

\( (d_0 = 0.3 \, \text{mm}, \, r_a = 20 \, \text{mm}, \, l = 0.3 \, \text{mm} \, \text{and} \, p_s = 0.6 \, \text{MPa}) \).
3. Results and discussion

By solution of the flow chart Fig. 5, the comparison pressure distribution of Fluent, Reynolds equation with constant \( C_d \) 0.8 and the \( C_d \) proposed in the paper is shown in the Fig. 6. Except for pressure depression region, the results shown that the pressure distribution of Reynolds equation with proposed \( C_d \) is more agreement with the results obtained by Fluent than the pressure distribution of Reynolds equation with constant \( C_d \) 0.8. In the next, the influences of geometry and flow parameters on the discharge coefficient are studied orderly. The simulation variable parameter and corresponding reference parameters are listed in the table 2. Moreover, in order to verify feasibility of the discharge coefficient, the Reynolds equation is solved with the proposed \( C_d \) and constant \( C_d \) 0.8 to obtain the bearing capacity which are compared with the results from Fluent simulations. The detail results are shown in section 3.1 and 3.2.

Table 2 The parameter of influences of geometry and flow parameters on the discharge coefficient

<table>
<thead>
<tr>
<th>number</th>
<th>name of Simulation condition</th>
<th>variable parameter</th>
<th>corresponding reference parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Effect of film thickness</td>
<td>( h=2,6,10,14,18,22\mu m )</td>
<td>( d_0=0.3mm, l=0.3mm, r_a=20mm, p_s=6atm )</td>
</tr>
<tr>
<td>2</td>
<td>Effect of supply pressure</td>
<td>( p_s=2,4,6,8,10,12atm )</td>
<td>( d_0=0.3mm, l=0.3mm, r_a=20mm, h=14\mu m )</td>
</tr>
<tr>
<td>3</td>
<td>Effect of orifice diameter</td>
<td>( d_0=0.2,0.3,0.4,0.5mm )</td>
<td>( h=14\mu m, l=0.3mm, r_a=20mm, p_s=6atm )</td>
</tr>
<tr>
<td>4</td>
<td>Effect of bearing radius</td>
<td>( r_a=20,30,40,50mm )</td>
<td>( h=14\mu m, l=0.3mm, d_0=0.3mm, p_s=6atm )</td>
</tr>
<tr>
<td>5</td>
<td>Effect of orifice length</td>
<td>( l=0.3,1,1.7,2.4,3mm )</td>
<td>( h=14\mu m, l=0.3mm, r_a=20mm, p_s=6atm )</td>
</tr>
</tbody>
</table>

Where atm is the atmosphere pressure.

3.1 Effect of geometry and flow parameters on the \( C_d \)

According to the table 2, the effect of geometry and flow parameters on the \( C_d \) are studied by solution of flow chart \( C_d \) seen Fig. 5. As shown Fig. 7, the \( C_d \) rises firstly and then decreases with increasing film and \( C_d \) decreases with increasing supply pressure, while it ascends with increasing orifice diameter. In addition, the \( C_d \) is not sensitive to the variation of bearing radius and orifice length, which is the results that effects of bearing radius and orifice length on the mass inflow rate can be ignored. As a whole, the discharge coefficient is most sensitive to film thickness, while supply pressure and orifice diameter is followed by the film thickness. In general, the traditional \( C_d \) is based on the mass flow rate of ideal Laval nozzle, which makes the traditional \( C_d \) is always lower than 1. However, the \( C_d \) proposed in this paper may be larger than 1, shown as Fig. 7. The \( C_d \) proposed in the paper is obtained by comparing actual mass flow rate \( \dot{m}_{\text{real}} \) with mass flow rate in solution for Reynolds equation \( \dot{m}_{\text{in}} \) and it is the coefficient that make \( \dot{m}_{\text{in}} \) be equal to \( \dot{m}_{\text{real}} \). Thus, when the \( \dot{m}_{\text{in}} \) is lower than \( \dot{m}_{\text{real}} \), the discharge coefficient from this paper is larger than 1. When the \( \dot{m}_{\text{in}} \) is higher than \( \dot{m}_{\text{real}} \), the discharge coefficient from this paper is less than 1. Therefore, it that the discharge coefficient from this paper is larger than 1 is not a validation against the traditional \( C_d \) which is always less than 1.

![a) effect of film thickness on the \( C_d \)](image1)

![b) effect of supply pressure on the \( C_d \)](image2)
3.2 Feasibility study on the $C_d$

Following the previous section, in order to verify feasibility of the discharge coefficient, the Reynolds equation is solved with the proposed $C_d$ and constant $C_d$ 0.8 to obtain the bearing capacity which are compared with the results from Fluent simulations. The results show that bearing capacity calculated by the Reynolds equation with the proposed $C_d$ is more agreement with the results obtained by Fluent simulations than that with constant $C_d$ 0.8, as shown in Fig. 8. This means that the proposed $C_d$ is more accurate than the constant $C_d$ 0.8 and suitable for solution of Reynolds equation.
4. Conclusions

In this paper, a method combined Fluent and Reynolds equation is proposed to calculate discharge coefficient based on the actual mass flow rate from Fluent. The definition of discharge coefficient proposed here is not the same to the traditional discharge coefficient that based on the mass flow rate of ideal Laval nozzle. The results show that the $C_d$ is affected by film thickness, supply pressure, orifice diameter and is not sensitive to bearing radius and orifice length. Moreover, the $C_d$ in this paper is more acute than the constant $C_d 0.8$ in solution of Reynolds equation. Meanwhile, it is also suitable to be used in the solution of Reynolds equation.

Acknowledgments

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REFERENCES


