STABILITY CHARACTERISTICS OF COUPLING SYSTEM BETWEEN SPEED CONTROL SYSTEM AND PROPULSION SHAFTING TORSIONAL VIBRATION

Zhen Liu, Yunbo Yuan, Yahui Chen, Wanyou Li and Yibin Guo*

Harbin Engineering University, Heilongjiang, Harbin 150001, China

email: guoyibin@hrbeu.edu.cn (*corresponding author)

The coupled oscillation problems between shafting torsional vibration system and speed control system have often occurred due to the increasingly complex marine propulsion system. The propulsion shafting is always assumed to be rigid and the diesel engine is usually modelled as mean value model for general simulation of diesel speed control system. To investigate the stability characteristics between these two systems, a frequency-domain model considering the coupling effects is proposed. In such a coupling model, the shafting was modelled as deformable rather than rigid and the actual cylinder excitations were considered while modelling the diesel engine. The proposed model was verified by test results and they agree with each other very well. The results illustrated that the stability of the coupling system obtained by the proposed model is worse than that obtained by the mean value model, and the stability margin of the coupling system is negatively correlate with the gain coefficients of the speed control system. What’s more, an instability region has been found due to the inappropriate integral coefficient of speed controller, which should be avoided in practical engineering applications.

Keywords: Torsional vibration, Speed control system, Coupling, Stability

1. Introduction

Diesel engine power plant has become the most widely used marine power plant because of its high economy, good maneuverability and reliability. The diesel engine drives the propeller and other subsidiary mechanisms through the transmission system composed of gearbox and transmission shaft. From the dynamic point of view, these parts with mechanical connections constitute a mechanical torsional vibration system. The mechanical torsional vibration system will be coupled with the diesel engine speed control system which keeps the output shaft speed constant, forming a closed-loop self-excited oscillation system.

Such a coupling system can cause stability problems in propulsion systems. When torsional vibration occurs in the shafting, its response must be based on the superposition of some natural modes [1]. The pulsation of torsional vibration components of a certain order will also be superimposed in the speed signal of the diesel engine output shaft. When the speed sensor of the diesel engine fuel regulating system senses the pulsation, the fuel supply will also pulsate. Correspondingly, the output torque of the diesel engine will pulsate, and there will be a phase difference between the torque pulsation and the speed pulsation. Under certain conditions, this phase difference just causes the input energy to the torsional
vibration. When the input energy exceeds the damping of the mechanical system, unstable self-excited oscillation will occur [2].

To solve the stability problem of the coupling system, not only the dynamic model of the diesel engine propulsion system shafting, but also the model of the diesel engine speed control system should be established according to the law of the fuel regulation system. At present, the average model is usually used to analyse and model the performance of diesel engine, which only expresses the comprehensive effect of process and the time average effect of state variables. Hence, in a working cycle of diesel engine, the average value of each sub-parameter in a cycle time is chosen [3-6]. For example, the average value of load torque is substituted for calculation, without considering the actual fluctuation of load torque, and the load is simplified to a single inertia, ignoring the torsional vibration components of shafting.

In order to overcome the shortcomings of existing research models, this paper, referring to the relevant literature [7-8], uses the transfer function method to establish the coupling system model based on frequency domain through Laplace transform, which is used to describe the dynamic performance of the system in time domain. Compared with the traditional average model of diesel engine, the transfer function models of diesel engine control system and propulsion shafting are established respectively. The real situation of diesel engine cylinder excitation is synthesized by frequency domain method, and the accuracy of the shafting model is verified by experiments. Finally, the coupling system model is used to analyse the influence of control parameters on the stability of the coupling system.

2. Mathematical Model

In the stability analysis of the coupling system, the mathematical model of the system is established firstly, which is the mathematical relationship between the input and output of each component. Then the mathematical models based on different physical quantities are transformed into transfer function models in frequency domain. On the whole, the coupling system includes speed control system and shafting system. The speed control system includes four parts: controller, speed regulating motor & transmission mechanism, high pressure oil pump and cylinder. In this paper, the transfer function models of the above parts will be established separately. Finally, the closed-loop model of the coupling system will be combined. The composition of the coupling system is shown in Fig. 1. The meaning of input and output parameters of each link in Fig. 1 is shown in Table 1.

![Figure 1: Coupling system.](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_e$</td>
<td>Target Speed</td>
</tr>
<tr>
<td>$n_c$</td>
<td>Actual Speed</td>
</tr>
<tr>
<td>$\delta n$</td>
<td>Speed Deviation</td>
</tr>
<tr>
<td>$\delta i$</td>
<td>Adjusting Current</td>
</tr>
<tr>
<td>$\delta x$</td>
<td>Rack Displacement</td>
</tr>
<tr>
<td>$\delta q$</td>
<td>Fuel Injection quantity</td>
</tr>
<tr>
<td>$\delta M_g$</td>
<td>Excitation Torque</td>
</tr>
</tbody>
</table>

Table 1: Passing parameters of coupling system
2.1 Control system section

2.1.1 Controller

The diesel engine adopts a PID controller. The input is a deviation of the actual speed of the flywheel and the target speed $\Delta n$. The output is a correction of the current $\Delta i$. The time-domain control law of the PID controller is as follows:

$$i(t) = k_p \Delta n(t) + k_i \int_0^t \Delta n(t) dt + k_d \frac{d\Delta n(t)}{dt}$$ (1)

The frequency domain model is obtained by Laplace transform:

$$G_i(s) = \frac{k_p}{1 + \frac{1}{T_i \cdot s} + T_d \cdot s}$$ (2)

2.1.2 Speed regulating motor and transmission mechanism

The input value is adjustable current $\Delta i$, and the output value is the variation value of the rack displacement $\Delta x$. According to the relevant literature [9], because the step response of motor actuator is similar to that of the same step inertia link, the relationship between them can be established as a first-order inertia link.

$$G_2(s) = \frac{k_2}{T_2s + 1}$$ (3)

where $K_2$ is the gain coefficient, $T_2$ is the time constant, and the two values can be obtained through the measured curve. It takes a certain time to adjust the output shaft angle of the motor, and then to change the rack displacement. Therefore, it is feasible to use the inertial link to show that the output slowly reflects the change rule of the input.

2.1.3 High-pressure oil pump

This part of the input is the variation value of the rack displacement $\Delta x$, and the output is the variation value of the fuel injection $\Delta q$. Assuming that the input varies in a small range, the characteristics of the high-pressure oil pump are approximated by the linear relationship, thus a simplified linear model can be obtained, which can generally be regarded as an inertial link:

$$G_3(s) = \frac{K_3}{T_3s + 1}, \quad K_3 = \frac{1}{X_{\text{max}}} \frac{\pi D^2}{4} \rho \frac{n_c}{120}$$ (4-5)

where $T_3$ is a time constant, $K_3$ is a partial gain of the oil pump, which can be obtained from the following formula, $X_{\text{max}}$ the maximum displacement of the rack, $D$ the cylinder diameter, $s$ the stroke of the diesel engine, $\rho$ the fuel density, and $n_c$ the actual speed of the diesel engine.

2.1.4 Cylinder

In this part, according to the mathematical relationship between fuel injection and cylinder excitation, the transfer function between the variation value of the fuel injection $\Delta q$ and the variation value of the gas torque $\Delta M_g$ is established. Considering the linear relationship between the fuel injection and the gas torque, the proportional link is selected in the transfer function.

$$G_4(s) = \frac{\delta M_g}{\delta q} = k_v e^{-\tau}, \quad k_v = \frac{1}{q_{\text{e}}} M_{g0}, \quad \tau = (1-1.25) \frac{60n}{Nn_c}$$ (6-8)

where $q_{\text{e}}$ is the fuel injection quantity under rated condition, $M_{g0}$ the gas torque of single cylinder under rated condition, and $\tau$ the delay time. Because of the thermal inertia of combustion chamber after the
change of fuel injection quantity, there is an intermediate lag between the instantaneous ignition and the change of the average indicating pressure, which results in the lag of the dynamic change of the effective gas torque. \( n \) is the stroke number, \( N \) the cylinder number and \( n_e \) the actual speed.

For the single cylinder gas torque \( M_{g0} \) under rated working conditions, if the mean value model is adopted, then there are:

\[
M_{g0} = 9550 \frac{P}{n_e} \left( \frac{n}{n_{ed}} \right)^2
\]  

where \( P \) is the rated power, \( n_e \) the running speed and \( n_{ed} \) the rated speed.

In this paper, considering the fluctuation of the gas torque in a single cylinder of a real diesel engine, the frequency domain method is used to synthesize the real excitation situation.

\[
M_{g0} = \sum_{\nu=0.5}^{12} \left[ a_{\nu} \cos(\nu \omega_0 t - \nu \zeta_{k}) + b_{\nu} \sin(\nu \omega_0 t - \nu \zeta_{k}) \right]
\]

Then Laplace transformation is performed to obtain:

\[
M_{g0} = \sum_{\nu=0.5}^{12} \left( a_{\nu} e^{-s \nu \omega_0 / \nu} \frac{s}{s^2 + (\nu \omega_0)^2} + b_{\nu} e^{-s \nu \omega_0 / \nu} \frac{\nu \omega_0}{s^2 + (\nu \omega_0)^2} \right)
\]

where \( \zeta_{k} \) is the angle of the ignition interval between cylinders.

The other excitations include the mass torque for moving parts of diesel engines:

\[
M_f = mR^2 \omega^2 \left( \frac{\lambda}{4} \sin \omega_0 t - \frac{1}{2} \sin 2 \omega_0 t - \frac{3}{4} \lambda \sin 3 \omega_0 t - \frac{\lambda^2}{4} \sin 4 \omega_0 t \right)
\]

### 2.2 Shafting section

The diesel engine propulsion shafting studied in this paper mainly includes torsional vibration damper, diesel engine, elastic coupling, reduction gear box and propeller.

In the stability problem of the coupling system, the mechanical torsional vibration system is mainly composed of low-order modes [10]. In order to simplify the analysis work, the actual deformable shafting is simplified. The original shafting is simplified to 5 inertias by the lumped parameter method, which ensures that the vibration characteristics before and after simplification remain unchanged. The lumped parameter model is shown in Fig. 2.

![Figure 2: Equivalent simplified drawing of propulsion shafting](image)

According to the five inertia model, the differential equations of torsional vibration are established as follows:

\[
J \ddot{\phi} + C \dot{\phi} + K \phi = M
\]

where \( J \) is the inertia matrix, \( K \) is the stiffness matrix, \( C \) is the damping matrix, and \( M \) is the excitation matrix.

In order to establish the transfer function between the excitation torque \( M_g \) and the rotation speed \( n \), the rotation speed \( n \) is used instead of \( \phi \). Considering the fluctuation of the diesel engine output torque,
the differential equation is differentiated at the same time. It is assumed that the working speed of each inertia is \( n_0 \) and the working speed of each inertia is equal before the output torque fluctuates, that is, when there is no torsional vibration. The output working torque is \( M_0 \), and the ratio of fluctuation to working value is expressed by \( \delta \). Then the differential equations (13) can be written as follows:

\[
\begin{align*}
\frac{\delta n_1}{\delta n_5} &= \frac{J_5 s^2 + C_{a5}s + K_4}{K_4}, \\
\frac{\delta n_3}{\delta n_5} &= \frac{J_4 s^2 + C_{a4}s + K_4 + K_3}{K_3}, \\
\frac{\delta n_2}{\delta n_5} &= \frac{J_3 s^2 + C_{a3}s + K_2 + K_3}{K_2}, \\
\frac{\delta n_1}{\delta n_5} &= \frac{J_1 s^2 + C_{a1}s + K_1}{K_1}. \\
\end{align*}
\]

(14-18)

Since the measuring position is at 3 inertia, the transfer function of shafting part should be the relationship between \( \delta n_3 \) and \( \delta M \), and the transfer function of the shafting part should be:

\[
G_s(s) = \frac{n_0 \delta n_1}{M_0 \delta M}
\]

(39)

So far, the transfer functions of the whole coupling system have been established, the open-loop transfer function of the coupling system is as follows:

\[
G_k(s) = G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)
\]

(20)

3. **Shafting Inherent Characteristic verification**

3.1 **Simulation of Shafting Inherent Characteristic**

Before coupling into the control system, it is necessary to ensure the stability of the shafting itself. The most commonly used method to judge stability is to look at the distribution of the system poles, whether they appear at the right end of the imaginary axis or not. The pole distribution of the shafting is shown in Fig. 3. It is concluded that when the propulsion shafting itself is a stable system, the following coupling system modelling and stability determination can be carried out.

![Figure 3: Pole map](image1)

![Figure 4: Bode diagram of shafting](image2)

The frequency characteristics of the shafting transfer function are given in the form of Bode diagram, as shown in Fig. 4. It can be seen from the figure that there are four obvious peaks in the amplitude
diagram and four obvious mutations in the corresponding phase diagram, which indicates that there are four oscillating links in the partial transfer function of the shafting system.

3.2 Test of Shafting Inherent Characteristic

The natural frequency of torsional vibration of diesel engine shafting is tested and analysed. The testing instruments include magnetoelectricity sensor, signal acquisition instrument and computer. The torsional vibration signal of flywheel is collected by magnetoelectricity sensor. The field test results are shown in figs. 5 and 6.

![Figure 5: Sensor arrangement](image)

![Figure 6: Schematic diagram of testing system](image)

The test results are compared with the simulation results in Table 2, and all errors are within 5%. It can be concluded that the transfer function of the shafting system is correct and can reflect the real vibration characteristics of the shafting system.

<table>
<thead>
<tr>
<th></th>
<th>First order</th>
<th>Second order</th>
<th>Third order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculation values/(rad/s)</td>
<td>40.5</td>
<td>92.3</td>
<td>233</td>
</tr>
<tr>
<td>Testing values/(rad/s)</td>
<td>40.4</td>
<td>92.2</td>
<td>233</td>
</tr>
<tr>
<td>Errors/%</td>
<td>0.3</td>
<td>0.1</td>
<td>0</td>
</tr>
</tbody>
</table>

4. Stability Analysis

4.1 Comparison of stability between coupling model and mean value model

Compared with the traditional mean value model, the coupling model established in this paper synthesizes the real cylinder excitation, and takes the deformable shafting into account. Under the same PID control parameters ($k_p = 0.32$, $k_i = 0.26$, $k_d = 15.4$), the stability margin of the coupling system is calculated by using the mean value model and the real cylinder gas torque model established in this paper, respectively. The results are shown in figs. 7 and 8.

![Figure 7: Stability margin of the mean value model](image)

![Figure 8: Stability margin of the coupling model](image)
From the perspective of stability margin, the mean value model has 22.2 dB amplitude margin and 26.8 deg phase margin. The amplitude margin of the coupling model is 8.78 dB and the phase margin is 16.4 deg. It can be found that the stability margin of the model considering the actual diesel engine excitation is smaller than that of the mean value model, and the system stability is worse.

4.2 Analysis of the influence of control parameters on stability

Considering that the mechanical parameters of the diesel engine propulsion system have been fixed if the coupling system is unstable during the actual ship operation, the more effective solution is to adjust some parameters of the diesel engine control system. In this part, the parameters of the controller, the gain of the motor drive mechanism and the gain of the high-pressure oil pump are simulated to study the influence of the control parameters on the stability of the coupling system.

When only considering the control characteristics of diesel engine, the control parameters obtained by parameter tuning are: \( k_p = 0.32, k_i = 0.26, k_d = 15.4, k_2 = 5.47, k_3 = 24.0422 \). Based on these parameters, the stability of the coupling system in a certain range is analysed in this paper. The simulation results of four sets of parameters are shown in Fig. 9-12.

From the simulation results, it can be seen that the controller parameters \( k_p \) and \( k_i \) have great influence on the stability of the coupling system. The stability margin has a linear relationship with the proportional coefficient \( k_p \) in a certain range around the selected value of 0.32, and the amplitude margin and phase margin decrease with the increase of the proportional coefficient \( k_p \), which has a greater impact on the phase margin. The value of integral coefficient \( k_i \) has unstable region, which should be avoided when adjusting parameters. At the same time, \( k_i \) has little effect on the phase margin.

The influence of the parameters of the actuator on the stability of the coupling system is also linear in a certain range before and after the selected value, and the stability margin decreases with the increase of the gain value.
5. Conclusion

In order to analyse the stability of the coupling system, the transfer function models of the diesel engine speed control system and the propulsion shafting are established respectively, and the stability of the system is analysed by using the frequency characteristics of the open-loop transfer function of the coupling system.

Compared with the traditional mean value model, this paper synthesizes the real cylinder gas excitation torque by the frequency domain method, and combines it with the deformable shafting. The simulation results of shafting inherent characteristics are consistent with the test results, so that the torsional vibration of shafting can be truly reflected, and the coupling relationship between the speed control system and shafting can be realized. At the same time, it is found that the stability of the system is worse than that of the mean value model when the torsional vibration factor is taken into account.

The influence of the parameters of the control system on the stability of the coupling system is simulated. The general law of the influence of the parameters of the control system on the stability of the coupling system is obtained. It is found that the parameters of the controller $k_p$ and $k_i$ have a great influence on the stability of the coupling system. The stability margin and the proportional coefficient $k_p$ are negatively correlated, while the selection of the integral coefficient $k_i$ has an unstable region. It is easy to cause speed oscillation in the coupling system. The gain of the motor drive mechanism and the gain of the high-pressure oil pump are also negatively correlated with the stability margin.

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REFERENCES