DESIGN AND EVALUATION OF THE VIBRATION DAMPING CONTROLLER FOR A PUSHCART CONSIDERING THE BEHAVIOR OF AN UNGROUNDED WHEEL

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Pushcarts have been widely used to convey various packages. However, they experience shocks and vibrations during package conveyance, which needs to be reduced especially for vibration-sensitive packages. Therefore, the authors developed a low-shock and low-vibration pushcart based on the center of percussion and an active vibration-damping controller. The controller suppresses the residual vibration by using a voice coil motor placed between the cart loading platform and bearing support block. Simulation results of the closed loop system of the controller indicate effective suppression of the sharp resonance peaks. However, the experimental results obtained when the cart was passed over a bump were inadequate because the wheel was not in contact with the ground after passing over the bump. To solve this problem, an additional accelerometer was attached to the bearing support block to restrict the sharp change in behavior of the bearing support block when the wheel was not in contact with the ground. The design and effectiveness of the above-mentioned controller are presented in this paper.

Keywords: vibration control, H-infinity control theory, sky hook controller, pushcart, ungrounded wheel

1. Introduction

Pushcarts have been widely used to convey various packages. However, they experience considerable shocks and vibrations when they pass over bumps, because of which they are not suitable to convey vibration-sensitive packages. Therefore, Authors have developed a low shock and vibration cart that can convey vibration-sensitive packages [1], as shown in Fig. 1. The cart has a caster that is designed based on the center of percussion (COP), to reduce the shocks caused by passing over bumps. Additionally, the cart is equipped with an active vibration damping controller and a voice coil motor (VCM) to suppress residual vibrations. Fig. 2 depicts the mechanism of the caster, which comprises of wheel, swing arm,
VCM, bearing support block, springs, and loading platform. The bearing support block is guided by a linear guide and forced by the VCM while the weight of the pushcart is balanced by the springs. In addition, the wheel is connected to the swing arm while the swing arm is connected to the bearing support block via the bearing.

An active controller has been developed. However, the controller gain is usually tuned by trial and error rather than systematically. Thus, a single-input and single-output (SISO) vibration-damping controller was designed using $H_{\infty}$ control theory [2]. The controller damped the residual vibration induced by the disturbance directly acted on the loading platform of the cart. However, some experiments clarified that this controller was not able to damp the residual vibration that occurs when the pushcart passes over bumps. This was because the swing arm pushes the bearing support block downward when the wheel comes in contact with the ground, thereby compressing the spring of the load compensation. However, after the wheel passes over the bump, it momentarily loses contact with the ground. At this moment, the compressed spring is released such that the force pushes the support-bearing block upward. Consequently, the wheel hits the ground with this force and induces a shock vibration. Thus, an additional accelerometer was attached to the bearing support block to restrict the sharp behavior change of the bearing support block when the wheel is ungrounded. Therefore, a new multi-input and single output (MISO) controller is designed in this paper using $H_{\infty}$ control theory, which accepts the accelerations of the loading platform and the bearing support block. The details of the controller design and its effects are also described.

![Picture of a low-impact cart.](image1.png)

**Figure 1:** Picture of a low-impact cart.

![Mechanism of the caster.](image2.png)

**Figure 2:** Mechanism of the caster.

## 2. Modeling

Fig. 3 depicts the mechanical model of the low-impact cart. Two types of transfer functions from $u$ to $\ddot{y}$ and $\ddot{y}_Q$ were required to design the controller and were formulated as follows: (i) the linearized equation of motion was formulated from Fig. 3. (ii) the linearized equation of motion was then transformed by Laplace transformation to establish Eq. (1), where $s$ is a complex variable. Symbols used in Eq. (1) are denoted below of Eq. (1).

$$
\begin{bmatrix}
(m + m_G)s^2 + (1 - \lambda)^2 G_H - (1 + \mu) K_{EG} & -m_G s^2 + \lambda(1 - \lambda) G_H -(1 + \mu) C_{EQ} & -(1 - \lambda) G_H \\
ms^2 + (1 - \lambda) G_H - K_{EG} & m_G s^2 + G_s + \lambda G_H - C_{EQ} & -(G_s + G_H) \\
m - K_{EG}/s^2 & m_G s^2 - C_{EQ}/s^2 & M
\end{bmatrix}
\begin{bmatrix}
Y_G \\
Y_Q \\
Y
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
-F_u \\
0
\end{bmatrix}
$$

where, $\lambda = e/b$, $\mu = a/b$, $m_G = I_G/b^2$, $K_{EG} = -(\mu + 1)(K_T + sC_T)$, $C_{EQ} = \mu(K_T + sC_T)$.

$y$: Height of the loading platform from the floor.

$y_Q$: Height of the bearing support block from the floor.

$y_G$: Height of the gravity center of the swing arm from the floor. 

$m$: Mass of the swing arm. $m_Q$: Mass of the bearing block. 

$M$: Mass of the loading platform of the pushcart.
\(a\): Length between point \(P\) to point \(G\). \(b\): Length between point \(G\) to point \(Q\). \(e\): Length between point \(G\) to point \(A\). \(I_G\): Inertia moment of the swing arm about the gravity center. \(K_T\): Stiffness coefficient of the wheel rubber. \(C_T\): Viscosity coefficient of the wheel rubber. \(F_u\): The force generated by VCM.

(iii) the following transfer functions were developed by solving \([Y_G \ Y_Q \ Y]^{T}\):

\[
P_{yu} = \frac{s^2 Y}{u} = \frac{s^2(n_{yu4}s^4 + n_{yu3}s^3 + n_{yu2}s^2 + n_{yu1}s + n_{yu0})}{d_6s^6 + d_5s^5 + d_4s^4 + d_3s^3 + d_2s^2 + d_1s + d_0},
\]

\[
P_{Qu} = \frac{s^2 Y}{u} = \frac{s^2(n_{Qu4}s^4 + n_{Qu3}s^3 + n_{Qu2}s^2 + n_{Qu1}s + n_{Qu0})}{d_6s^6 + d_5s^5 + d_4s^4 + d_3s^3 + d_2s^2 + d_1s + d_0}
\]

The coefficients of these transfer functions are given in Table 1.

<table>
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<th>(d_6)</th>
<th>(d_5)</th>
<th>(d_4)</th>
<th>(d_3)</th>
<th>(d_2)</th>
<th>(d_1)</th>
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<td>1.7553 (\cdot) 10^7</td>
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<td>1.8484 (\cdot) 10^{12}</td>
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<td>(n_{yu3})</td>
<td>(n_{yu2})</td>
<td>(n_{yu1})</td>
<td>(n_{yu0})</td>
<td>(n_{Qu4})</td>
<td>(n_{Qu3})</td>
</tr>
<tr>
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<td>-1.1602 (\cdot) 10^3</td>
<td>-7.0134 (\cdot) 10^5</td>
<td>7.2923 (\cdot) 10^6</td>
<td>3.8970 (\cdot) 10^9</td>
<td>6.023 (\cdot) 10^9</td>
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### 3. Controller design

#### 3.1 Design of single-input and single-output (SISO) controller

The SISO controller design and the experimental results are described in this sub-section before describing the design of the new MISO controller. A reduced ordered plant model \(P_{yuL}\) was utilized in the SISO controller design. This was because the order of the full order model \(P_{yu}\) given by Eq. (2) was too high to identify the correct parameters of the plant, thereby making the order of the controller to become high. Thus, a second order model \(P_{yuL}\) that represents the only first vibration mode of \(P_{yu}\) was utilized.
In the controller design. However, it needs to prevent the unstable spillover caused by the higher dynamics of $P_{yu}$. Therefore, the SISO controller was designed based on $H_{\infty}$ control theory to realize robust stability. Meanwhile, $P_{yuL}$ was formulated as follows:

$$P_{yuL} = \frac{0.9661s^2}{s^2 + 1.877s + 262.5}.$$  \hspace{1cm} (4)

where the parameters of $P_{yuL}$ were identified using least mean squares method in the frequency domain. Fig. 4 shows the Bode gain diagram and the multiplicative perturbation of the plant. $P_M$ denotes the measurement frequency response of the plant, and $\Delta$ denotes the multiplicative perturbation between $P_{yuL}$ and $P_M$. This figure indicates that $P_{yuL}$ agrees with the actual first vibration mode. Fig. 5 depicts a closed loop system utilized in the controller design, where $w_1$ and $w_2$ are the exogenous inputs, $z_T$ and $z_S$ are the controlled outputs, $W_T$, $W_S$, and $W_N$ are the weighting functions, $u$ is the control input, $y$ is the output, and $C$ is the controller. The figure indicates that $W_T$ is required for the robust stability of the system and $W_S$ characterizes $P_{yuL}S$, where $S$ is the sensitivity function. However, $W_N$ was introduced to satisfy the assumption of a standard $H_{\infty}$ control problem. Hence, the following weighting functions were determined:

$$W_T = \frac{17.5s^2 + 461.8s + 3.385 \cdot 10^4}{s^2 + 35.19s + 4.836 \cdot 10^4},$$  \hspace{1cm} (5)

$$W_S = \frac{0.5s^2 + 7.54s + 28.42}{s^2 + 6.032s + 56.85},$$  \hspace{1cm} (6)

$$W_N = 0.05.$$  \hspace{1cm} (7)

Fig. 6 shows the Bode gain diagram of $W_S^{-1}$, closed loop system $SP_{yu}$, $SP_{yuL}$, and $P_{yu}$. This figure indicates that $W_S^{-1}$ shapes the closed loop system such that the sharp resonance peak of the first vibration mode is suppressed.

Moreover, two kinds of experiments were conducted to evaluate the vibration damping performance of the designed controller, given below:

Condition A: The disturbance directly acted on the cart loading platform.
Condition B: The disturbance was induced when the cart passed over a bump.

In these experiments, the acceleration of the loading platform was compared when the control was considered and when the control was not considered. Figures 7 and 8 depict the experimental results for conditions A and B, respectively. The result based on condition A indicates that the designed controller effectively suppresses the residual vibration. On the other hand, the result based on condition B indicates

![Figure 4: Bode gain diagram and the perturbation of the plant.](image1)

![Figure 5: Closed loop system utilized in SISO controller design.](image2)
the designed controller did not work effectively. This was because the bearing support block was not suppressed when the wheel was ungrounded, as discussed in Section 1. However, this problem will be solved using MISO controller, which is described in the next sub-section.

3.2 Design of multi-input and single-output (MISO) controller

A MISO controller that determines the control input \( u \) from \( \ddot{y} \) and \( \ddot{y}_Q \) was designed to solve the problem of SISO controller, using \( P_{Qul} \) (the lower order model of \( P_{Qu} \)) and \( P_{yuL} \). These lower order plant models were developed as follows:

\[
P_{yuL} = \frac{-1.08s^2 + 1.013s + 0.00741}{s^2 + 2.103s + 247.3}, \tag{8}
\]

\[
P_{Qul} = \frac{-1.353s^2 + 1.344s + 0.0249}{s^2 + 2.103s + 247.3}. \tag{9}
\]

These transfer functions were developed by the following procedure:

1. A state space equation of the plant was established using Eqs. (2) and (3).
2. The above-mentioned state space equation was then changed into a diagonal canonical form.
3. The states were removed except the lowest vibration mode.
4. The lower order state equation was then re-established from the lowest mode.
5. The nominal parameter of the plant was modified such that the squared error between the frequency response determined by the model and the response measured experimentally was minimized.

Fig. 9 depicts the frequency responses and the multiplicative perturbation, where the left and right figures are the results of \( P_{yuL} \) and \( P_{Qul} \), respectively. Fig. 9 also depicts the multiplicative perturbations \( \Delta_{yu} \) and \( \Delta_{Qu} \), where their gains are lower than 0 dB at around the frequency of the first resonance peak. This indicates that \( P_{yuL} \) and \( P_{Qul} \) effectively represent the dynamics of the first vibration mode.

Figure 10 depicts the closed loop system utilized in the MISO controller design, where \( w_1, w_2, \) and \( w_3 \) are the exogenous inputs, \( z_T, z_{S1}, \) and \( z_{S2} \) are the control outputs, \( W_T, W_{S1}, W_{S2}, W_{N1}, \) and \( W_{N2} \) are the weighting functions, \( C_1 \) and \( C_2 \) are the controllers \( u \) is the control input, and \( y \) is the output. A weighting function \( W_{S2} \) that operates on the relative acceleration \( \ddot{y} - \ddot{y}_Q \) was introduced to provide a viscous damping effect between the loading platform and the bearing support block, and to suppress the sharp behavior change of the bearing support block. From Fig. 10, the transfer function matrix from the exogenous inputs to controlled output was formulated as Eq. (10).
Figure 9: Bode gain diagram of the plant: lower order model, multiplicative perturbation of the plant, and $W_T$.

![Bode gain diagram](image)

Figure 10: Closed loop system utilized in the MISO controller design.

The transfer function matrix was then used to determine the weighting functions.

\[
\begin{bmatrix}
-z_{s1} \\
-z_{s2} \\
z_T
\end{bmatrix} = \frac{1}{D_P} \begin{bmatrix}
-W_{S1}W_{N1}C_1P_{yu} & -W_{S1}W_{N2}C_2P_{yu} & W_{S1}P_{yuL} \\
W_{S2}W_{N1}C_1(P_{Qu} - P_{yu}) & W_{S2}W_{N2}C_2(P_{Qu} - P_{yu}) & W_{S2}(P_{yuL} - P_{QuL}) \\
W_TW_{N1}C_1 & W_TW_{N2}C_2 & W_T(C_1P_{yuL} + C_2P_{QuL})
\end{bmatrix} \begin{bmatrix}
w_1 \\
w_2 \\
w_3
\end{bmatrix}
\]

\[D_P = 1 + C_1P_{yuL} + C_2P_{QuL}\]

Meanwhile, the control system needs to prevent the unstable spillover caused by the higher dynamics. Therefore, the gain of $W_T$ must cover the both gain of $\Delta_{yu}$ and $\Delta_{Qu}$ to ensure robust stability. Hence, $W_T$ is given as follows:

\[
W_T = \frac{\omega_{NT1}^2\omega_{NT4}^2(s^2 + 2\zeta_{T2}\omega_{NT2}s + \omega_{NT2}^2)(s^2 + 2\zeta_{T3}\omega_{NT3}s + \omega_{NT3}^2)}{\omega_{NT2}^2\omega_{NT3}^2(s^2 + 2\zeta_{T1}\omega_{NT1}s + \omega_{NT1}^2)(s^2 + 2\zeta_{T4}\omega_{NT4}s + \omega_{NT4}^2)}
\]

\[\beta\]
The parameters of $W_T$ were: $\omega_{nT1} = 2\pi \cdot 1$, $\zeta_{T1} = 0.7$, $\omega_{nT2} = 2\pi \cdot 2$, $\zeta_{T2} = 0.7$, $\omega_{nT3} = 2\pi \cdot 5$, $\zeta_{T3} = 0.5$, $\omega_{nT4} = 2\pi \cdot 33$, and $\zeta_{T4} = 0.04$. Fig. 9 shows that the Bode gain diagram of $W_T$ tightly covers the gains of $\Delta y_u$ and $\Delta Q_u$. On the other hand, Eq. (10) indicates that $W_{S1}$ operates on $P_{yul}/D_p$ while $W_{S2}$ operates on $(P_{yul} - P_{Qul})/D_p$. Thus, the gains of $W_{S1}$ and $W_{S2}$ must be high at the resonance frequency of the plant to suppress the resonance peak gain. Hence, $W_{S1}$ and $W_{S2}$ were determined as following notch filter characteristics:

$$W_{S1} = K_{WS1}\frac{S^2 + 2\zeta_{S12}\omega_{ns12}s + \omega_{ns12}^2}{S^2 + 2\zeta_{S11}\omega_{ns11}s + \omega_{ns11}^2},$$ (12)

$$W_{S2} = K_{WS2}W_{S1}.$$ (13)

The parameters of $W_{S1}$ and $W_{S2}$ were $\omega_{ns11} = \omega_{ns12} = 2\pi \cdot 1.8$, $\zeta_{S11} = 0.5$, $\zeta_{S12} = 0.7$, $K_{WS1} = 1.1$, and $K_{WS2} = 0.8$. Moreover, $W_{N1}$ and $W_{N2}$ were introduced to satisfy the assumption of a standard $H_{\infty}$ control problem[3]. Since they are required to be small numbers, they were determined as follows:

$$W_{N1} = W_{N2} = 0.005.$$ (14)

The MISO controller was synthesized using MATLAB robust control toolbox. Fig. 11 depicts the Bode diagram of the controller while Fig. 12 depicts the Bode gain diagram of the plant, two kinds of closed loop system $R_1$ and $R_2$ including the designed controller, and $W_{S1}^{-1}$ and $W_{S2}^{-1}$. $R_1$ and $R_2$ were defined as $P_{yul}/D_p$, $(P_{yul} - P_{Qul})/D_p$, respectively. The closed loop gain is smaller than the gain of $W_{S1}^{-1}$ and $W_{S2}^{-1}$, indicating that $W_{S1}$ and $W_{S2}$ shape the closed loop system such that the designed controller can suppress the sharp resonance peak of the plant.

![Bode diagram of the MISO controller](image-url)

Fig. 11 Bode diagram of the MISO controller.
Fig. 12 Comparison of the open loop system and the closed loop system of the designed controller.

4. Conclusions

The authors have developed a pushcart that can convey vibration-sensitive packages. To achieve this goal, the center of percussion was utilized in the cart mechanical design. However, a vibration damping controller was required to suppress the residual vibration. Thus, two kinds of vibration controllers were described in this paper. The first is a SISO controller that accepts only the acceleration of the cart loading platform, while the other is a MISO controller that accepts both acceleration of the cart loading platform and the bearing support block. The SISO controller could not suppress the residual vibrations induced when the cart passes over bumps because the bearing support block was not suppressed when the wheel was not in contact with the ground. Thus, the MISO controller was designed to suppress the behavior of the bearing support block, thereby solving the problem associated with SISO controller. The impact of the MISO controller will be demonstrated by experiments during the conference.

REFERENCES