**FUNDAMENTAL STUDY ON GENERATION MECHANISM OF AUTOMOTIVE DISC BRAKE HOT JUDDER BASED ON A SELF-EXCITED VIBRATION**

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In the automotive industry, the silence and ride comfort of the car body have been markedly improved by the development of hybrid and fuel cell vehicles. However, the pursuit of high brake performance has caused complicated frictional vibration in the brake system. On high-speed highways and roads with steep descents, the heat generated in disc brakes when they are applied sometimes causes a vibration problem. This phenomenon, called hot judder, is caused by hot spots on the surface of the disc. The generation mechanism of hot judder has not been clarified in previous work. In this study, hot judder was modeled as self-excited vibration due to the time delay caused by heat deformation of the disc, and the fundamental generation mechanism of hot judder was analyzed with a simple three-degree-of-freedom system. The effects of disc surface wear were considered in the vibration analysis model because it is known that the amount of surface wear is large where hot spots have occurred. It was found that the wear increases in proportion to the amount of disc expansion, and as a vehicle decelerates, the number of hot spots also changes.

**Keywords:** Self-excited vibration, Pattern formation, Disc brake, Hot judder, Wear

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1. **Introduction**

In recently developed disc brake systems, when braking is performed from a high speed, a new vibration problem, called hot judder, is caused by multiple hot spots being generated on the surface of the disc, which deforms it. However, because hot judder is caused by heat and vibration, the generation mechanism is still unknown and countermeasures have not yet been developed. Previous studies [1], [2] reported that hot judder can be treated as a problem caused by static instability due to thermal expansion of the disc. Few studies have conducted vibration evaluation and quantified the number of hot spots [3]. Therefore, in this study, the generation mechanism of hot judder was analyzed as self-excited vibration due to the time delay caused by heat deformation of the disc.

2. **Temperature calculation model of the disc**

The temperature calculation model is shown in Fig. 1(a). The contact width of the brake pad in the circumferential direction is assumed to be a point on the disc. To consider the distribution of the disc
temperature, the disc is modeled by \( n \) discrete elements in disc thickness direction \( z \), as shown in Fig. 1(b). The temperature \( T_i' \) of the \( i^{th} \) element at time \( t \) for each element is calculated. In Fig. 1(b), dummy points, which are the \( 0^{th} \) to \( n^{th} \) elements at both outside disc surfaces, are assumed to calculate the temperature using the heat flux quantities.

The heating part of the disc brake is the contact point between the disc and the pad, and the part without any contact is the cooling part. The temperature of the disc in the thickness direction is calculated by the discretized one-dimensional unsteady heat equation shown in Eq. (1), where \( \kappa \) is the thermal diffusion rate.

\[
T_i^{t+1} = T_i^t + \frac{\kappa A \Delta t}{A_x^2} \left( T_{i+1}^t - 2T_i^t + T_{i-1}^t \right) \quad (i = 1 - n - 1). \tag{1}
\]

The parameters for the calculations are same as those used in Ref. [4], and their explanation is omitted here. Figure 2 shows the result of the disc temperature calculation of inner side surface \( T_1' \). The abscissa shows time \( t \), and the ordinate shows temperature \( T_1' \). In Fig. 2, two processes, one for the temperature increase and the other for the decrease, are shown. In addition, the average temperature of the disc and the thickness of the disc due to temperature increase were obtained from Eqs. (2) and (3), and the material is assumed to be cast iron.

\[
T_{ave} = \frac{\sum_{i=1}^{n-1} T_i}{n - 1} \tag{2}
\]

\[
H' = (T_{ave} - 293) \times H \times a + H. \tag{3}
\]
Figure 3 shows the average values of the disc temperature. Figure 4 shows the variation of the disc thickness. Figures 2 to 4 confirm that air cooling has a small effect on the disc because the flux quantity of heat loss in the cooling part is small. Specifically, the loss is 0.397% of the heat flux quantity in the heating part when the angular velocity $\omega$ is 251 rad/s. Therefore, it is supposed that the effect of cooling is small.

The expansion rate $\alpha$ due to the frictional heat of the pad and the shrinkage rate $\beta$ due to air cooling are expressed as Eq. (4).

$$\alpha = \frac{H_{\text{out}}'}{H_{\text{in}}'}, \quad \beta = \frac{H_{\text{in}}^{\omega+T}}{H_{\text{out}}'},$$

where $H_{\text{in}}'$ and $H_{\text{out}}'$ are the disc thicknesses just before the disc enters the contact area with the pad at the leading edge and just after the disc passes through the contact area with the pad at the trailing edge, respectively, and $H_{\text{in}}^{\omega+T}$ is the disc thickness $H_{\text{in}}'$ after one disc rotation period $T$. The expansion rate $\alpha$ and the shrinkage rate $\beta$ are approximately 1 for all rotation speeds of the disc, and at all rotation speeds, $\alpha$ and $\beta$ must always satisfy the conditions $\alpha>1$ and $\beta<1$.

3. Analysis model

This section describes the modeling of an automotive disc brake to analyze the fundamental mechanism of hot judder. Hot judder was treated as a self-excited vibration system with a time delay caused by heat. The relations between the number of disc revolutions, the expansion rate $\alpha$, and the shrinkage rate $\beta$, which were obtained in Section 2.3, were used. In the analysis, the feedback effect of the amount of thermal expansion was considered. Furthermore, the effects of disc surface wear were considered because it is known that the amount of surface wear is large at the position where hot spots occur.

3.1 Analysis model and equation of motion

An analysis model consisting of a disc and caliper with three degrees of freedom is shown in Fig. 5. It was assumed that the disc and the pad contact each other at one point. The caliper was modeled by inner and outer rigid body blocks, which can vibrate in the bouncing mode in the out-of-plane directions $x_1$ and $x_2$. The out-of-plane displacement of the disc is $x_3$. Pads on the inner and the outer sides were modeled by contact points that have stiffness and damping.

The disc expansions on the inner side and outer side immediately after the pad exits the contact point are defined as $u(t)$ and $u(t)$, respectively. Feedback from these expansions were modeled as forced...
Table 1 Parameter values in the analysis model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_1)</td>
<td>2.2 kg</td>
</tr>
<tr>
<td>(c_i)</td>
<td>53.9 N·s/m</td>
</tr>
<tr>
<td>(m_2)</td>
<td>0.22 kg</td>
</tr>
<tr>
<td>(c_o)</td>
<td>53.9 N·s/m</td>
</tr>
<tr>
<td>(m_3)</td>
<td>58.4 kg</td>
</tr>
<tr>
<td>(\zeta_1)</td>
<td>0.06369</td>
</tr>
<tr>
<td>(k_1)</td>
<td>(1.9 \times 10^6) N/m</td>
</tr>
<tr>
<td>(\zeta_2)</td>
<td>0.01</td>
</tr>
<tr>
<td>(k_2)</td>
<td>(3.6 \times 10^7) N/m</td>
</tr>
<tr>
<td>(\zeta_3)</td>
<td>0.039</td>
</tr>
<tr>
<td>(k_3)</td>
<td>(1.81 \times 10^8) N/m</td>
</tr>
<tr>
<td>(\nu)</td>
<td>(8.5 \times 10^{-10}) m·N·rev</td>
</tr>
<tr>
<td>(k_i)</td>
<td>(6.62 \times 10^6) N/m</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>(1.0 \times 10^{-10}) m·N·rev</td>
</tr>
<tr>
<td>(k_o)</td>
<td>(6.62 \times 10^6) N/m</td>
</tr>
<tr>
<td>(f_1)</td>
<td>140 Hz</td>
</tr>
<tr>
<td>(c_1)</td>
<td>273.8 N·s/m</td>
</tr>
<tr>
<td>(f_2)</td>
<td>2135 Hz</td>
</tr>
<tr>
<td>(c_2)</td>
<td>114.8 N·s/m</td>
</tr>
<tr>
<td>(f_3)</td>
<td>280 Hz</td>
</tr>
<tr>
<td>(c_3)</td>
<td>8,019 N·s/m</td>
</tr>
<tr>
<td>(f_0)</td>
<td>400 Hz</td>
</tr>
</tbody>
</table>

Fig. 5 Analysis model for the disc and caliper displacements \(\beta u(t-T)\) and \(\beta u_o(t-T)\) on the contact surface after one rotation of the disc (after a rotation period of \(T\) seconds). In other words, this analysis model treats expansion as a time delay system. In addition, the caliper mass on the inner side, the caliper mass on the outer side, and the disc mass are represented by \(m_1\), \(m_2\), and \(m_3\), respectively. The spring constants and the damping coefficients are represented by \(k_1\), \(k_2\), \(k_3\), \(k_i\), and \(k_o\) and \(c_1\), \(c_2\), \(c_3\), \(c_i\), and \(c_o\), respectively. In addition, \(\nu\) is the thermal expansion displacement per unit of frictional force in one rotation of the disc, and \(\gamma\) is the amount of wear per unit of contact force in one rotation of the disc; both parameters were assumed to be constant. The equations of motion and thermal expansion immediately after the pad exits from the contact point can be expressed as follows:

\[
m_i \ddot{x}_i + c_i \dot{x}_i + k_i x_i + c_i \{ \dot{x}_i - \dot{x}_3 - \beta u_i(t-T)\} + k_i \{x_i - x_3 - \beta u_i(t-T)\} + c_2 (\dot{x}_i - \dot{x}_2) + k_2 (x_i - x_2) = 0 \tag{5}
\]
\[
m_2 \ddot{x}_2 + c_o \{ \dot{x}_2 - \dot{x}_3 + \beta u_o(t-T)\} + k_o \{x_2 - x_3 + \beta u_o(t-T)\} + c_2 (\dot{x}_2 - \dot{x}_1) + k_2 (x_2 - x_1) = 0 \tag{6}
\]
\[
m_3 \ddot{x}_3 + c_3 \dot{x}_3 + k_3 x_3 + c_i \{ \dot{x}_3 - \dot{x}_1 + \beta u_i(t-T)\} + k_i \{x_3 - x_1 + \beta u_i(t-T)\} + c_o \{ \dot{x}_3 - \dot{x}_2 - \beta u_o(t-T)\} + k_o \{x_3 - x_2 - \beta u_o(t-T)\} = 0 \tag{7}
\]
\[
u u_i(t) = \beta u_i(t-T) - \alpha \nu \{c_i \{ \dot{x}_i - \dot{x}_3 - \beta u_i(t-T)\} + k_i \{x_i - x_3 - \beta u_i(t-T)\}\] + \[\gamma \{c_o \{ \dot{x}_3 - \dot{x}_1 + \beta u_o(t-T)\} + k_o \{x_3 - x_1 + \beta u_o(t-T)\}\}
\]
\[
u u_o(t) = \beta u_o(t-T) + \alpha \nu \{c_o \{ \dot{x}_2 - \dot{x}_1 + \beta u_o(t-T)\} + k_o \{x_2 - x_1 + \beta u_o(t-T)\}\] - \[\gamma \{c_i \{ \dot{x}_3 - \dot{x}_2 + \beta u_i(t-T)\} + k_i \{x_3 - x_2 + \beta u_i(t-T)\}\]. \tag{9}

Variable transformations \(\tau = \omega t\) and \(\omega = 2\pi/T\) and the Laplace transform were applied to these equations. Then, by assuming the coefficient matrix \(A\) of the obtained equation, the characteristic equation can be expressed as follows:

\[\det A = 0.\] \tag{10}

The characteristic value satisfying characteristic equation in Eq. (10) can be expressed \(s = \sigma + iN\), and the solution is positive infinity. If the root has a positive real part, then hot judder occurs. The imaginary part \(N\) of the root corresponds to the number of hot spots.

### 3.2 Numerical calculation result

The numerical parameters used in the calculation were same as those used in Ref. [4], and their explanation is omitted here. The natural frequencies and natural modes of the system were also same as
those used in Ref. [4] and not described here. However, we note that the natural frequency of the second mode is 400 Hz and the natural mode is that the caliper and the disc vibrate out of phase with each other.

Figure 6 shows the relation between $\sigma \omega$ and the rotation speed of the disc. In this figure, $\sigma \omega$ represents the growth speed of unstable vibration, and $n$ represents the nearest integer of the imaginary part of the characteristic root and is equal to the number of hot spots. If the real part of the characteristic value is positive, then the system is unstable and hot judder occurs; if all the real parts of the characteristic roots are negative, then the system is stable.

In addition, from Fig. 6, the range of rotation speed is confirmed to become unstable for different numbers of hot spots. The peak of $\sigma \omega$ is located at a different rotation speed for each number of hot spots. The peak values of $\sigma \omega$ for each number of hot spots are smaller than those of the result in Ref. [4] because the contact forces determined in Eqs. (5) through (7) become small as a result of the consideration of disc surface wear in Eqs. (8) and (9). The relation between the generation frequency of hot judder, the rotation speed of the disc, and the number of hot spots satisfies the equation of (generation frequency of hot judder) = (rotation speed of disc) × (number of hot spots) $\approx$ (natural frequency of brake system) [5] [6]. This is the characteristic of the pattern formation phenomenon. It has been reported that hot judder occurs at a rotation frequency of around 40 Hz. In Fig. 6, when the rotation speed is 46 Hz at point 1, the real part of the characteristic root of 9 hot spots reaches a maximum. In this case, hot judder occurs at 414 Hz. The unstable vibration frequency is almost identical to the second natural frequency of 400 Hz [4]. The complex mode for the unstable vibration obtained at a rotation speed 46 Hz at point 1 for 9 hot spots is shown in Table 2. The caliper of the inner side and the outer side vibrate approximately at the same amplitude and phase, and the disc vibrates with small amplitude and out of phase with the caliper, as shown in Table 2. In addition, the complex mode of the unstable vibration is the same as the second natural mode, and the quantity of the expansion of the inner side $u_i(t)$ and the quantity of the expansion of the outer side $u_o(t)$ have the almost same amplitude and are out of phase with each other. These complex modes of unstable vibration for other numbers of hot spots located at different rotation speeds have the same vibration modes as those for 9 hot spots. In this calculation, unstable vibration does not occur at the first mode. Because the variation of the contact force in the first mode is smaller than that in the second mode, only the second mode becomes unstable.

<table>
<thead>
<tr>
<th>Displacement</th>
<th>Amplitude ratio</th>
<th>Phase (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>1.000</td>
<td>0</td>
</tr>
<tr>
<td>$x_2$</td>
<td>0.852</td>
<td>0.653</td>
</tr>
<tr>
<td>$x_3$</td>
<td>$6.627 \times 10^{-2}$</td>
<td>-177.2</td>
</tr>
<tr>
<td>$u_i$</td>
<td>0.133</td>
<td>-223.9</td>
</tr>
<tr>
<td>$u_o$</td>
<td>0.114</td>
<td>-43.31</td>
</tr>
</tbody>
</table>

Fig. 6 Relation between real part $\sigma \omega$ and rotation speed

### 3.3 Simulation result for constant rotation speed

The numerical integral calculation using the Runge-Kutta-Gill method was performed on Eqs. (5) through (9). The simulation was conducted with 46 Hz rotation speed and 9 hot spots, as shown at point 1 in Fig. 6. In the calculation, a minute displacement of the (1,0) mode is given as the initial condition for the quantities of expansion $u_i(t)$ and $u_o(t)$ for one revolution period. The initial conditions are shown in Fig. 7. In addition, the quantities of wear on the inner side $a_i(t)$ and outer side $a_o(t)$ of the disc surface are given by Eqs. (11) and (12):
The actual wear shape of the inner and outer disc surfaces is obtained by integrating the quantities of the wear $a_i(t)$ and $a_o(t)$ at time $t$, and each is defined as $d_i(t)$ and $d_o(t)$, respectively. Figure 8 shows the vibration waveform of $x_2$. Figure 9 shows the frequency analysis for the waveform between 9.2 s and 10.2 s in Fig. 8.

From the calculation results, the frequency component of the generation frequency of hot judder is confirmed to be 414 Hz, which is close to the second natural frequency of the system.

Figure 10 shows the expansion fluctuations of inner side $u_i(t)$ and outer side $u_o(t)$ for one revolution period at 10.2 s. From Fig. 10, even though the initial condition of the shape of the disc surface is given as the (1,0) mode, it can be confirmed that the vibration amplitude of 9 hot spots grows. Moreover, it is confirmed that the amount of expansion fluctuates out of phase with the inner and outer sides.

Figure 11 shows the wear shape fluctuations of inner side $d_i(t)$ and outer side $d_o(t)$ for one revolution period at 10.2 s. Figures 10 and 11 show that the expansion and wear shape are out of phase. This means that the contact force becomes large where the amount of expansion is increased by a hot spot, and the amount of wear also increases in proportion to it.

Figure 12 shows the temperature distributions on the inner and outer sides of the disc at 10.2 s. In Fig. 12, the temperature distribution was calculated by the function that describes the relationship between the amount of temperature rise from the normal temperature obtained in Fig. 2 and the amount of expansion examined in Fig. 4 and obtained in Section 2. Figure 12 confirms 9 hot spots on the disc surface. In addition, the large amount of expansion increases the temperature, and the maximum temperature increases to more than 900 K at 10.2 s. Moreover, the temperature difference is about 300 K between the locations where hot spots occur and do not occur.
The above results confirm that the fluctuation of the expansion due to the fluctuation of the contact force causes feedback that further increases the fluctuation of the contact force. Furthermore, the hot spots occur where the disc expansion is the largest, and these hot spots cause the hot judder of the disc.

### 3.4 Simulation result for deceleration

Next, simulation was performed for deceleration, where the rotation speed of the disc changes from 46 Hz at point 1 to 41.4 Hz at point 2, as shown in Fig. 6. Under this condition, it was expected that the number of hot spots would change from 9 to 10. For simplicity, the calculation was first conducted for 5.1 s at a constant rotation speed of 46 Hz, then the calculation was conducted again for 5.1 seconds at a constant rotation speed of 41.4 Hz. The initial conditions of the displacements $x_1$, $x_2$, and $x_3$; expansions $u_i(t)$ and $u_o(t)$; and wear shapes $d_i(t)$ and $d_o(t)$ in the calculation at 41.4 Hz were used those of the final points of the simulation results at 5.1 s in the first calculation at 46 Hz. To compare the results with the simulation results for constant rotation speed in Section 3.3, the total braking time was limited to 10.2 s.

Figures 13, 14, and 15 show the expansion $u_o$ for one revolution period on the outer side at 5.1 s, 6.8 s, and 10.2 s, respectively. Figure 13 confirms that the vibration amplitude of 9 hot spots grows the same as that in Fig. 10. By changing the rotation speed to 41.4 Hz, Fig. 14 shows that the vibration amplitude of 9 hot spots stabilized, and the number of hot spots changed from 9 to 10. Moreover, in Fig. 15, it is confirmed that the vibration amplitude of 10 hot spots grows. However it does not enough grow and is much smaller than the results for a constant rotation speed in Fig. 10 and the influence of the vibration amplitude of 9 hot spots disappears.

Figure 16 shows the temperature distributions on the outer side of the disc at 10.2 s. The small temperature fluctuations of 10 hot spots can be seen on the disc surface in Fig. 16. However the expansion does not enough grow. Therefore, although the maximum temperature reaches more than 760 K for 10.2 s, the temperature difference between the locations where hot spots occur and do not occur is only 6 K.
The above results confirm that the growth rate of the vibration amplitude of the system and the amounts of expansion are reduced under deceleration in comparison with the results for a constant rotation speed.

4. Conclusions

This study clarified the generation mechanism of hot judder caused by thermal deformation in disc brakes. This paper concludes the following:

(1) Fluctuation of the contact force between the disc and pad due to vibration of the brake system causes fluctuation of the disc surface temperature, and this temperature fluctuation causes a larger fluctuation of the contact force in one disc rotation with a time delay. Thus, hot spots and hot judder are created by self-excited vibration with a time delay.

(2) Only the second mode of the system becomes unstable. Thus, the generation frequency of hot judder is close to the second natural frequency of the brake system.

(3) The contact force becomes large where the amount of expansion is increased by hot spots, and the amount of wear is also increased proportionately.

(4) When the vehicle speed changes, the number of hot spots also changes.

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