AN ANALYTICAL MODEL OF BURIED PLASTIC GAS PIPE FOR ACOUSTIC DETECTION

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Inaccurate location of buried pipes has been a worldwide problem for many years. To address this problem, vibro-acoustic detection method has been developed and shown to be promising in locating buried plastic pipes. By applying acoustic excitation on one part of the pipe and analyzing the ground surface vibration, the location of the buried pipe can be determined. Whilst there are existing studies relating to plastic water pipes in this field, limited research has been carried out on buried gas pipes detection. Based on the model of buried fluid-filled pipe, an analytical model of buried plastic gas pipe will be studied in this paper. The analytical solution and numerical simulation results of the ‘gas-dominated wave’ will be presented, and its characteristics will be analyzed. Following this, the ground surface vibration due to this mode wave is illustrated. In our proposed model, soil loading pressure on the pipe wall is taken into consideration. Moreover, the ground surface vibration directly above the pipe will be expressed as the result of radiated conical waves from ‘gas-dominated wave’ and reflected waves from ground surface. Therefore, far field approximation of Hankel function is not required. This work provides theoretical insight into acoustic detection techniques for locating buried plastic pipes.

Keywords: acoustic detection; analytical model; buried gas pipes
1. Introduction

Buried piping systems play a very important role in modern societies, especially in transporting fluid and gas for everyday use. Within these systems, plastic pipes have been widely used in recent decades because of their relatively low failure rates compared to other types of pipes in service [1]. Unfortunately, records of the location of these underground utilities are often inaccurate or incomplete. Problems associated with inaccurate location of buried pipes are exacerbated by the presence of rapid development of urbanization, which increases both the difficulty and economic costs of excavation work [2]. In response to this, trenchless technologies have been developed including ground penetrating radar technique, quasi-static field method and acoustic technologies [3]. In particular, the vibro-acoustic detection method is very promising in locating plastic pipes [4].

Correspondingly, acoustic behaviours of axisymmetric waves within buried pipes have been the subject of numerous studies [5],[6],[7],[8], providing theoretical insight and guidance to acoustic detection techniques used for leakage detection and pipe location determination. Previous investigations have shown that below pipe ring frequency, four wave types are responsible for most of the energy transfer within buried water piping systems including fluid-dominated wave, shell-dominated wave, torsional wave and flexural wave [9], among which the fluid-dominated axisymmetric wave is the primary vibrational energy carrier for leakage noise (normally below 1000 Hz) in buried water pipes [10]. Furthermore, the pipe location can be determined from analysing ground surface vibration due to propagation and radiation of this mode wave [11]. Therefore, the fluid-dominated wave is the object of this paper. In the process of data interpretation, abrupt phase changes commonly occur when frequency-unwrapping the phase information at a single location directly above the pipe. To account for this phenomenon, an analytical model of ground surface vibration due to fluid-dominated wave motion has been proposed where it was found that radiated compressional and shear waves can interfere with each other, where abrupt reversal of the phase happen [12].

Previous research has been primarily confined to buried water pipes and only limited studies have been conducted on buried gas pipes. Jette and Parker investigated the surface displacements resulting from wave motion within a buried steel gas pipe [13]. Since the pipe material is steel, the soil loading pressure was ignored. However, for buried plastic pipe, ignoring soil loading may lead to inaccurate solutions. Also, plane wave treatment and approximation of Hankel function were employed when calculating the ground surface displacements, which is a possible reason for the disagreement between experimental data and theoretical prediction [12]. This paper is concerned with setting up an analytical model of buried plastic gas pipe and investigating the characteristics of ‘gas-dominated wave’ and corresponding surface displacements. Here we focus on the analytical solutions and the excitation method is beyond the scope of this paper. The details of experimental rig and excitation method can be found in [14]. In our model, soil loading pressure on the pipe wall is taken into consideration based on independent expressions for the pressure coefficients of soil loading [5]. In addition, incident waves from radiation of ‘gas-dominated wave’ and reflected waves from ground surface are expressed as conical waves instead of plane waves, which means that far field approximation of Hankel function is not required.

2. Analytical model of buried plastic gas pipe

In this section, an analytical model of buried plastic gas pipe is presented. Firstly, with reference to the expression of simplified dispersion for fluid-dominated axisymmetric wave within buried water pipe in [8], the gas inside the pipe is treated as liquid with gas property and the dispersion curve of fluid-dominated wave within buried gas pipe will be presented. Since the medium within the pipe is gas, this mode wave is named ‘gas-dominated wave’ in this case, corresponding to the fluid-dominated wave for the water pipe in previous studies. Following this is the modelling of the displacements of pipe wall. To
simplify the model and correlate with the earlier work in [13], the properties of the gas are set the same as air and the surrounded soil is treated as fluid, which can sustain both compressional and shear waves. Here, the expression of potential coefficients with inclusion of soil loading pressure is derived, which will be used for calculating the ground surface displacements. Finally, the conical wave treatment is employed and the analytical solutions to surface vibration due to the propagation of ‘gas-dominated wave’ are derived.

2.1 Analysis of ‘gas-dominated wave’

Consider a plastic pipe surrounded by infinite medium, as shown in Fig.1(a). The internal medium could be water or gas. Three directions of cylindrical coordinate are shown in the $x$(axial), $r$ (radial) and $\theta$ (circumferential). The displacements of pipe and external medium are represented by $u_p$, $v_p$, $w_p$ and $u_m$, $v_m$, $w_m$ in these three directions, respectively. Since only the axisymmetric wave is considered, the variations with respect to circumference vanish. The pipe has a radius $a=3$cm, wall thickness $h=4.76$mm, and $h/a<1$, which satisfies the condition of thin-shell, where Donnell-Mushtari shell equation can be adopted [15].

![Figure 1: The coordinate system for a buried medium-filled pipe: (a) Surrounded by infinite medium; (b) Ground surface included.](image)

The simplified solution to the complex wavenumber of internal medium-dominated (fluid-dominated) wave in imperfect bonding condition is given in [8] as

$$k_s^2 = k_{in}^2 \left( 1 + \frac{\beta}{1 - \Omega^2 + \alpha} \right)$$

(1)

Therefore, the velocity of the internal medium-dominated wave is

$$c_n = \omega/Re(k_s)$$

(2)

where $\alpha$ and $\beta$ are the measures of external medium loading and internal medium loading on the pipe wall, which are $\alpha = -SL_{22} - \sigma_p^2$ and $\beta = 2c_{in}^2\rho_{in}a(1 - \sigma_p^2)/E_ph$, respectively, and $SL_{22} = -\mu_m a(1-\sigma_p^2)/E_p h \left\{ 2 - (k^2_a a^2 - 2k^2_a a^2)H^{(2)}(k_{dr}r) \right\} - 4k^2_a r^2 a \mu_m a(1-\sigma_p^2)/E_p h \left\{ 2 - (k^2_a a^2 - 2k^2_a a^2)H^{(2)}(k_{dr}r) \right\} [8]$. $H^{(l)}_n$ is Hankel function of the $l^{th}$ kind and $n^{th}$ order, and $\omega$ is angular frequency.

Parameters related to the pipe are described as $\rho_p$, $E_p$, $\sigma_p$, denoting density, Young’s modulus and Poisson’s ratio, respectively. $\Omega$ is normalized ring frequency equal to $a\omega^2 \rho_p (1 - \sigma_p^2)/E_p$. Parameters related to the external medium are described as $\rho_m$, $\mu_m$, $\lambda_m$ corresponding to density, shear modules and Lamé elastic constant, respectively. The propagation wave speeds are $c_d = \sqrt{(\lambda_m + 2\mu_m)/\rho_m}$ and $c_r = \sqrt{(\lambda_m + 2\mu_m)/\rho_m}$ for P-wave and S-wave, respectively. The axial wave numbers are $k_d = \omega/c_d$.
and $k_r = \omega / c_r$. The radial wave numbers are $k_{ds}^r = \sqrt{k_d^2 - k_s^2}$ and $k_{rs}^r = \sqrt{k_s^2 - k_d^2}$ related to the P-wave and S-wave. For a medium-filled (gas-filled) pipe, $\rho_{in}$, $c_{in}$ are density and free field wave speed of the internal medium and axial wave number is $k_{in}$. The values of parameters used in the simulation work are shown in Table 1.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Plastic pipe</th>
<th>External medium</th>
<th>Internal medium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density $\rho$ (kg/m$^3$)</td>
<td>2000</td>
<td>2000</td>
<td>1.29</td>
</tr>
<tr>
<td>Young's modulus $E$ (GN/m$^2$)</td>
<td>5.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Poisson's ratio $\sigma$</td>
<td>0.4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Compressional wave speed $c_d$ (m/s)</td>
<td>-</td>
<td>259</td>
<td>340</td>
</tr>
<tr>
<td>Shear wave speed $c_r$ (m/s)</td>
<td>-</td>
<td>164</td>
<td>-</td>
</tr>
</tbody>
</table>

Since the wavenumber of the ‘gas-dominated wave’ cannot be determined in a straightforward manner, Newton-Raphson numerical method is adopted to find the numerical solution to Eq. (1). In this paper, the desired mode wave is ‘gas-dominated wave’ within the buried plastic gas pipes. The velocity of this mode wave obtained from Eq. (2) is shown in Fig. 2.

![Figure 2: Wavespeed of ‘gas-dominated wave’ in a buried plastic gas pipe.](image)

As Fig. 2 shows, the wave speed decreases slightly with frequency increasing. However, the overall variation of wave speed within the considered frequencies is very small and can be neglected. Hence, the ‘gas-dominated wave’ can be treated as non-dispersive at low frequencies. This conclusion is illustrated in next section and the gas within the pipe conducts a plane wave with velocity 340m/s when calculating ground surface displacements.

### 2.2 Displacements of pipe wall (Inclusion of soil loading in radial direction)

The calculation method of ground surface vibration is based on the buried steel gas pipe model proposed in 1980 [13], where soil loading pressure on the pipe wall was ignored since the pipe material is steel. In this paper, soil loading pressure on the plastic pipe is taken into consideration and is expressed in terms of radial pipe wall displacement [5].

Neglecting the shearing stress from soil loading to pipe wall, displacements of pipe wall are given by [13]:

$$v_p = (a^2 / E_p h)(p_0 - p_m), \quad u_p = -(ia\sigma_p / k_s E_p h)(p_0 - p_m)$$

(3)
where $p_0$ is pressure from internal medium, which equals to $P_0e^{i(\omega t-k_s x)}$ and $p_m$ is soil loading pressure in radial direction. Setting $p_m$ equal to 0 represents the condition where soil loading pressure is neglected, where we can get:

$$v_p = (a^2/E_p h)p_0, \quad u_p = -(i\alpha\sigma_p/k_s E_p h)p_0$$

The vibration of pipe wall will generate wave motion that can radiate into external medium. In this way, the acoustic field in the external medium can be described in terms of compression and shear wave potentials as:

$$\phi_m = A_m H_0^{(2)}(k_{ds}^r r)e^{i(\omega t-k_s x)}, \quad \psi_m = B_m H_0^{(2)}(k_{rs}^r r)e^{i(\omega t-k_s x)}$$

where $A_m$ and $B_m$ are potential coefficients which can be determined by boundary conditions. The soil displacements can be described in terms of wave potentials by:

$$u = \frac{\partial \phi}{\partial x} - \frac{\partial^2 \psi}{\partial r^2} - \frac{\partial \psi}{r \partial r}, \quad v = \frac{\partial \phi}{\partial r} + \frac{\partial^2 \psi}{\partial r \partial r}$$

Applying elastic wave theory and velocity continuity between pipe wall and the external medium in radial direction at pipe-soil interface, soil loading can be expressed in terms of pipe wall as [5]:

$$p_m = -\omega^2 \rho_m \frac{H_0^{(2)}(k_{ds}^r a)}{k_{ds}^r H_1^{(2)}(k_{ds}^r a)} v_p - \omega^2 \rho_m \frac{H_0^{(2)}(k_{rs}^r a)}{k_{rs}^r H_1^{(2)}(k_{rs}^r a)} v_p$$

Substituting Eq. (7) into Eq. (3), the displacements of pipe wall are obtained as:

$$v_p = (a^2/E_p h)\left(\frac{1}{1-Ga^2/E_p h}\right)p_0, \quad u_p = -(i\alpha\sigma_p/k_s E_p h)\left(\frac{1}{1-Ga^2/E_p h}\right)p_0$$

where $G = \omega^2 \rho_m \left\{\frac{H_0^{(2)}(k_{ds}^r a)}{k_{ds}^r H_1^{(2)}(k_{ds}^r a)} + \frac{H_0^{(2)}(k_{rs}^r a)}{k_{rs}^r H_1^{(2)}(k_{rs}^r a)}\right\}$. Comparing Eq. (8) to Eq. (4), it is noted that the effect of inclusion of soil loading pressure is to add a constant $C = \omega^2 \rho_m / E_p h$ in the denominator of Eq. (8). Setting $C$ equal to zero represents the condition where the soil loading pressure is neglected.

By setting the displacements of the soil to be identical to the displacements of pipe wall, both radially and axially, at pipe-soil interface $r=a$, the potential coefficients are obtained as:

$$A_m = \frac{-aP_0}{E_p h(1+C)}\left\{a(k_{rs}^r)^2 H_0^{(2)}(k_{ds}^r a) - \sigma_p k_{rs}^r H_1^{(2)}(k_{ds}^r a) - \sigma_p k_{rs}^r H_1^{(2)}(k_{ds}^r a) + k_s^2 k_{rs}^r H_1^{(2)}(k_{ds}^r a)H_0^{(2)}(k_{ds}^r a) + k_s^2 k_{rs}^r H_1^{(2)}(k_{ds}^r a)H_0^{(2)}(k_{ds}^r a)\right\}$$

$$B_m = \frac{-aP_0}{E_p h(1+C)k_s}\left\{\sigma_p k_{rs}^r H_1^{(2)}(k_{ds}^r a) + ak_s^2 H_0^{(2)}(k_{ds}^r a) + ak_s^2 H_0^{(2)}(k_{ds}^r a)H_1^{(2)}(k_{ds}^r a) + k_s^2 k_{rs}^r H_1^{(2)}(k_{ds}^r a)H_0^{(2)}(k_{ds}^r a) + k_s^2 k_{rs}^r H_1^{(2)}(k_{ds}^r a)H_0^{(2)}(k_{ds}^r a)\right\}$$

2.3 Ground surface vibration due to ‘gas-dominated wave’ within the pipe

Consider a plastic gas pipe buried at depth $z = 0.76m$, as shown in Fig. 1(b). In addition to the incident waves radiated from pipe, reflected waves are generated because of the presence of ground surface. Jette and Parker [13] and Yan Gao et al [12] treated these waves as plane waves and adopted the first term of asymptotic expansion of Hankel function. The corresponding expressions of ground surface displacements can be found in [13] (given in their work as Eq. (26)), representing the condition where plane wave treatment is adopted. In this paper, both incident waves and reflected waves are expressed as conical waves, using zero order of Hankel function of second kind and first kind, respectively. In this way, there is no need to employ far field approximation of Hankel function.

The resultant acoustic field due to conical waves radiating outward from the pipe and reflected waves from ground surface can be expressed as:

$$\phi_{all} = A_m H_0^{(2)}(k_{ds}^r r)e^{i(\omega t-k_s x)} + A_m r H_0^{(1)}(k_{ds}^r r)e^{i(\omega t-k_s x)}$$
\[ \psi_{all} = B_m H_0^{(2)}(k_{rs}r) e^{i(\omega t - k_x x)} + B_{mr} H_0^{(1)}(k_{rs}r) e^{i(\omega t - k_x x)} \] (10)

where \( A_{mr} \) and \( B_{mr} \) are potential coefficients of reflected waves which can be obtained through boundary conditions. The coefficients \( A_m \) and \( B_m \) have been determined in Eq. (9). The normal and tangential stresses can be expressed in terms of wave potential by:

\[
\sigma_{rr} = \mu_m \left( \frac{\partial v}{\partial r} - \frac{\partial u}{\partial \theta} \right), \quad \sigma_{rx} = \lambda_m \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \phi}{\partial r} \right) + \frac{\partial^2 \phi}{\partial \theta^2} \right] + 2 \mu_m \frac{\partial v}{\partial \theta}
\] (11)

For free surface at \( r=a+z \), both normal and tangential stresses vanish. Therefore, potential coefficients \( A_{mr} \) and \( B_{mr} \) can be obtained by:

\[
(A_{mr} \\
B_{mr}) = -T^{-1}_{m2} T_{m1} \begin{pmatrix} A_m \\ B_m \end{pmatrix}
\] (12)

where \( T_{mn} = \begin{bmatrix}
-2\mu_m (k_{ds}^r)^2 \left( H_0^{(1)}(k_{ds}^r) - \frac{\mu_m^*(k_{ds}^r)}{k_{ds}^r} \right) - \lambda_m k_x^2 H_0^{(2)}(k_{ds}^r) \\
2i\mu_m k_x k_{ds}^r H_0^{(1)}(k_{ds}^r) - \mu_m k_x^2 (k_{ds}^r)^2 H_0^{(2)}(k_{ds}^r)
\end{bmatrix} \right|_{r=a+z}

The displacements of ground surface where \( r=a+z \) can be given as:

\[
\begin{pmatrix}
u_m \\
v_m
\end{pmatrix} = \begin{bmatrix}
-ik_x H_0^{(2)}(k_{ds}^r) (k_{ds}^r)^2 H_0^{(2)}(k_{ds}^r) & (k_{ds}^r)^2 H_0^{(1)}(k_{ds}^r) \\
-ik_x H_1^{(1)}(k_{ds}^r) & ik_x k_{ds}^r H_0^{(1)}(k_{ds}^r)
\end{bmatrix} \begin{pmatrix} A_m \\ B_m \end{pmatrix} + \begin{bmatrix}
-ik_x H_0^{(1)}(k_{ds}^r) (k_{ds}^r)^2 H_0^{(1)}(k_{ds}^r) \\
-ik_x H_1^{(1)}(k_{ds}^r) & ik_x k_{ds}^r H_1^{(1)}(k_{ds}^r)
\end{bmatrix} \begin{pmatrix} A_{mr} \\ B_{mr} \end{pmatrix}
\] (13)

Substitute Eq. (12) into Eq. (13), the displacements of ground surface can be expressed in terms of \( A_m \) and \( B_m \).

3. Numerical results and discussion

In this part, displacements of ground surface from propagation of ‘gas-dominated wave’ within a buried plastic gas pipe are illustrated. Three different conditions are presented: 1) neglecting soil loading pressure with plane wave treatment; 2) including soil loading pressure with plane wave treatment, and 3) including soil loading pressure with conical wave treatment. In this way, the effect of inclusion of soil loading pressure and conical wave treatment can be demonstrated.

The theoretical displacements of ground surface determined from Eq. (9) combined with plane wave treatment (which can be found in Eq. (26) in [13]) are illustrated in Fig. 3(a) and Fig. 3(b) for two conditions 1) and 2) where soil loading pressure is neglected (i.e. \( C = 0 \)) and included, respectively. Both amplitude and phase information are presented.

As Fig. 3(a) shows, neglecting the soil loading pressure can decrease the amplitude of the vertical displacement of the ground surface. This effect on amplitude is accentuated with increasing frequency, whereas the phase information remains the same. This can be easily identified in Eq. (9), where only a small negative constant \( C \) is added to the denominator compared to Eq. (16) in [13], resulting in an increased absolute value of potential coefficients and indirectly increasing the absolute value of ground surface displacements. This is exactly the reason why inclusion of soil loading pressure increases the amplitude of the vertical displacement while phase information does not change.

A similar phenomenon can be found in Fig. 3(b), where the horizontal displacements of ground surface are demonstrated. The effect of soil loading pressure increases the amplitude of the displacement. However, the abrupt phase reversals happen at around 1500 Hz and 2000 Hz, which coincide with some minima of horizontal displacement. This phenomenon has been mentioned in [12] and [13]. The existence of these anomalies was explained in detail in [12].

The ground surface displacements calculated from Eq. (9) combined with Eq. (13) are presented in Fig. 4(a) and Fig. 4(b), where the red solid line represents the method in which both radiated waves and
reflected waves are treated as conical waves, and far field approximation of Hankel function is not employed. To compare the results with those determined from plane wave treatment, the blue dotted curves from Fig. 3 are also provided as reference. It is evident that conical treatment decreases the amplitudes of ground surface displacements in both radial (vertical) and horizontal directions, and this effect is more distinct radially. In addition, the overall pattern of the amplitude and phase information appear to be similar for both cases, with some shift along the frequency range. The amplitude difference may account for discrepancies between the predictions and measured data about ground surface in [12].

![Figure 3: Displacements of ground surface with plane wave treatment: (a) Vertical; (b) Horizontal.](image)

![Figure 4: Displacements of ground surface inclusion of soil loading pressure: (a) Vertical; (b) Horizontal.](image)

4. Conclusion

In this research, the characteristics of ‘gas-dominated wave’ in buried plastic gas pipes have been analysed, and the ground surface vibration due to this mode wave has been presented. It has been shown that the ‘gas-dominated wave’ can be treated as plane wave without dispersion behaviour as in the free field. For plastic pipe, neglecting the soil loading pressure can lead to the underestimation of the amplitudes of ground surface displacements. Furthermore, employing far field approximation of Hankel function affects both amplitude and phase information of ground surface vibration. The overall simulation results agree with the results presented in [13] for a buried steel gas pipe. This paper has shown the feasibility of locating buried plastic gas pipe using the vibro-acoustic method, and has provided theoretical insight to relevant acoustic detection techniques.
REFERENCES


