FUNDAMENTAL STUDY ON OPTIMAL DESIGN OF DYNAMIC ABSORBER TO SUPPRESS SUBHARMONIC VIBRATION OF ORDER 1/2 IN AUTOMATIC TRANSMISSION FOR CARS

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In the torque converter of the automatic transmission for cars, a damper at the lock-up clutch is used to reduce the transmission of torsional vibration due to combustion in the engine. Although a damper with low stiffness reduces the transmission of vibrations, the fitting of low-stiffness springs is difficult because of space limitations. Therefore, a piecewise-linear spring with three stiffness stages is used for the damper in actual automatic transmissions. However, subharmonic vibration of order 1/2 occurs when the spring restoring characteristics of the damper is near the switching point of the piecewise-linear spring. In the subharmonic vibration of order 1/2, the fundamental frequency component is half of the forced vibration frequency, and the subharmonic vibration occurs when the forced vibration frequency range is about twice the natural frequency. In this study, experiments using a two-degree-of-freedom system were conducted to suppress the subharmonic vibration by the dynamic absorber, and the optimal design of the dynamic absorber is discussed. We conducted a theoretical analysis to determine the optimal design of the dynamic absorber to suppress the subharmonic vibration. The effects of the natural frequency and the damping ratio of the dynamic absorber to the vibration suppression are discussed. We found optimum values for the natural frequency and damping ratio of the dynamic absorber that suppress the subharmonic vibration effectively. The natural frequency of the dynamic absorber should not be set close to the excitation frequency but rather near the natural frequency of the system. Thus, subharmonic vibration is completely suppressed by using the optimum dynamic absorber. The experimental results agreed with results of the numerical analyses.

Keywords: vibration of rotating body, forced vibration, nonlinear vibration, automatic transmission powertrain, dynamic absorber
1. Introduction

In the torque converter of automatic transmissions for cars, a lock-up clutch is used to improve fuel economy. When the lock-up clutch is engaged, the converter cover (torque input) and the output shaft to the gear train are connected directly. The engine torque fluctuates due to combustion in individual engine cylinders, and this decreases the comfort of the ride. To solve this problem, the torque converter is equipped with a damper to reduce the transmissibility of the torsional vibration. Although a damper with low stiffness effectively reduces the torsional vibration, it is difficult to use low-stiffness springs because of space limitations. Therefore, a piecewise-linear spring with three stiffness stages is used for the damper in actual automatic transmissions. The piecewise-linear spring can realize a wide range of restoring torque characteristics in a small space. A piecewise-linear spring with three stiffness stages has two switching points. When the spring restoring characteristics of the damper are around the switching point, which has a large stiffness ratio, the subharmonic vibration of order 1/2 (referred to hereinafter simply as the subharmonic vibration) occurs. The fundamental frequency component is half of the forced vibration frequency, and the subharmonic vibration occurs when the forced vibration frequency range is about twice the natural frequency. Even though the natural frequency of the system is outside the engine speed range, large subharmonic vibration occurs in actual vehicles.

In previous studies, we analytically clarified the mechanism that creates subharmonic vibration in automatic transmissions of automobiles and performed experiments and numerical analyses using a simple one-degree-of freedom system with a piecewise-linear spring [1,2]. We also performed theoretical analyses using an analytical vehicle model and clarified the optimum design of the dynamic absorber [3]. Although a number of studies [4-6] have been conducted on the nonlinear vibrations in mechanical systems with piecewise-linear springs, experimental analyses to suppress the subharmonic vibration by using the dynamic absorber have not yet been conducted. In this study, experiments using a fundamental two-degree-of-freedom system were conducted to suppress the subharmonic vibration by the dynamic absorber, and its optimal design was evaluated.

2. Subharmonic vibration in vehicles with automatic transmissions

Figure 1 is a schematic diagram of a torque converter. The red dotted line shows the torque flow when the lock-up clutch is not engaged. The input torque from the pump impeller is transmitted to the turbine runner by the automatic transmission fluid, which causes inefficiency. To overcome this disadvantage, a lock-up clutch is used. The green dashed line shows the torque flow when the lock-up clutch is engaged. The input torque is transmitted to the output shaft directly through the damper.

Figure 2 shows the frequency analysis of the turbine runner rotating speed when subharmonic vibration occurs in the actual vehicle. The frequency component of half of the excitation frequency is large. The subharmonic vibration occurs near the switching point of the piecewise-linear spring characteristics.
3. Experimental equipment

Figure 3 shows the experimental setup. The mass of the main system $m$ is supported by leaf spring A with spring constant $k_1$. A damper with damping coefficient $c$ was added by attaching a sponge to leaf spring A. Leaf spring B with spring constant $k_2$ was set to touch the mass. This spring served as the piecewise-linear spring. The mass was also connected to leaf spring C with spring constant $k$, which was connected to the exciter. The vibration amplitude of the exciter was $u = a \cos \omega t$. The dynamic absorber was attached to the main system mass. The mass, damping coefficient, and the stiffness coefficient of the dynamic absorber are represented as $m_d$, $c_d$, and $k_d$, respectively. The natural frequency $\omega_d = \sqrt{k_d / m_d}$ of the dynamic absorber could be changed by changing the leaf spring and the height of the mass, and the damping ratio $\zeta_d = c_d / 2 \sqrt{m_d k_d}$ was controlled by changing the size of the sponge between the leaf springs.

4. Numerical analysis

4.1 Analytical model

This numerical analysis evaluated the optimum value for suppressing the subharmonic vibration by changing the natural frequency and the damping ratio of the dynamic absorber. Figure 4 shows the analytical model for the experimental setup. The restoring force characteristics are shown in Fig. 5. The equilibrium point is the switching point P. The equation of motion is written as
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\[
m\ddot{x} + c\dot{x} + f(x) + c_d(\dot{x} - \dot{x}_d) + k_d(x - x_d) = ka\cos\omega t
\]

\[
m_d\ddot{x}_d + c_d(\dot{x}_d - \dot{x}) + k_d(x_d - x) = 0
\]

(1)

where \(x\) is the displacement from the equilibrium point. The restoring force of the piecewise-linear spring \(f(x)\) is given by

\[
f(x) = \begin{cases} (k + k_1)x & (x < 0) \\ (k + k_1 + k_2)x & (x \geq 0) \end{cases}
\]

(2)

Table 1: Standard parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m)</td>
<td>0.75 kg</td>
</tr>
<tr>
<td>(k)</td>
<td>440 N/m</td>
</tr>
<tr>
<td>(k_1)</td>
<td>850 N/m</td>
</tr>
<tr>
<td>(k_2)</td>
<td>1290 N/m</td>
</tr>
<tr>
<td>(c)</td>
<td>2.0 N\cdot s/m</td>
</tr>
<tr>
<td>(m_d)</td>
<td>0.075 kg</td>
</tr>
</tbody>
</table>

Table 1 shows the standard parameters used in the model. The excitation frequency, \(f = \omega/2\pi\), was changed from 12 to 22 Hz. The Shooting method was used to solve the equation of motion. During the numerical integration used in the Shooting method, the time at the switching point of the piecewise-linear spring between the time steps was calculated precisely with the Newton-Raphson method.
4.2 Occurrence of the subharmonic vibration

In the fundamental analysis, a two-stage spring was used without dynamic absorber. The stiffness ratio of the first to second stage, $\gamma = (k_1 + k_2 + k)/(k_1 + k)$, was 2. Even though the amplitude of the exciter varied with increased excitation frequency, the amplitude of the exciter $a$ was set at a constant value of 2.7 mm for simplicity.

Figure 6 shows the frequency response curve when the equilibrium point is at switching point P in Fig. 5. The abscissa represents the excitation frequency of the exciter, and the ordinate represents the peak-to-peak amplitude of the displacement of the mass. The black and red lines show the stable and unstable solutions, respectively. In this system, the natural frequencies of the first and second stages of the piecewise-linear spring are 6.60 and 9.33 Hz, respectively. Figure 7 shows the frequency analysis at point X in Fig. 6. Although the excitation frequency $f = 15.4$ Hz, the fundamental frequency is 7.7 Hz, which is half of the excitation frequency. This frequency is close to the first and second natural frequencies. This is the characteristic of the subharmonic vibration.

4.3 Countermeasure against subharmonic vibration by the dynamic absorber

Next, the suppressive effect of the dynamic absorber to the subharmonic vibration is examined. Figure 8 shows the region of subharmonic vibration that is suppressed by the dynamic absorber. The suppressive effect of the dynamic absorber is confirmed by checking whether the flip bifurcations exist or not. The abscissa represents the natural frequency of the dynamic absorber, and the ordinate represents the damping ratio of the dynamic absorber. The subharmonic vibration can be suppressed by the dynamic absorber in the shaded region. In the case of a tunable absorber, such as a centrifugal pendulum absorber, the natural frequency of the dynamic absorber should follow the excitation frequency. In contrast, in the optimum design of a dynamic absorber for a linear system, the optimum tuning value is close to the natural frequency of the system, and the amplitude around the excitation frequency range near the natural frequency of the system should be the focus of attention. According to the result of Fig. 8, the natural frequency of the dynamic absorber, $f_d = \omega_d/2\pi$, should be set close to fundamental frequency of the subharmonic vibration, which is close to the natural frequency of the main system. This damping ratio has the optimum range to suppress the subharmonic vibration.

Figure 9(a) shows the result for a dynamic absorber at point A ($f_d = 7.20$ Hz, $\zeta_d = 0.0348$) in Fig. 8. The natural frequency of the dynamic absorber is in the range of the optimum value, but the damping is less than optimum. The subharmonic vibration occurs in two regions. The unstable solution around $f = 15$ Hz is the quasi-periodic solution.

Figures 9(b) and 9(c) show the results for a dynamic absorber at points B ($f_d = 6.56$ Hz, $\zeta_d = 0.138$) and C ($f_d = 8.19$ Hz, $\zeta_d = 0.146$) in Fig. 8, respectively. In these cases, the damping ratios of the dynamic absorbers are within the optimum range, but the natural frequencies of the dynamic absorber are outside the optimum range. Figure 9(d) shows the result for a dynamic absorber at point D ($f_d = 7.26$ Hz, $\zeta_d = 0.126$) in Fig. 8. In this case, the damping ratio and the natural frequency of the dynamic absorber are within the optimum ranges. The subharmonic vibration is completely suppressed by attaching this dynamic absorber.
5. **Vibration damping experiments**

First, an experiment without any dynamic absorber was conducted under the same condition as used in Fig. 6. Figure 10 shows the frequency response curve when the dynamic absorber is not attached. The subharmonic vibration occurs at around $f = 15$ Hz.

Experiments were conducted using the same parameters of the dynamic absorber at points A, B, C and D in Fig. 8. Figures 11(a)-(d) show the frequency response curves when a dynamic absorber having the characteristics at points A, B, C and D, respectively, in Fig. 8 is attached. In Fig. 11(a), subharmonic
vibration occurs in two regions. The dashed red line represents the response of quasi-periodic vibration. In Figs. 11(b) and 11(c), small subharmonic vibrations occur at around \( f = 13.5 \) Hz and 17 Hz, respectively. The subharmonic vibrations are not suppressed perfectly, but the dynamic absorber reduces the amplitudes of the subharmonic vibrations. In Fig. 11(d), the subharmonic vibration is completely suppressed, as found in the numerical analyses.

![Figure 10: Frequency response curve.](image)

![Figure 11: Experimental results when the dynamic absorber is attached.](image)
6. Conclusions

This paper discusses fundamental design method of the optimum dynamic absorber to suppress subharmonic vibration in a torque converter with a piecewise-linear spring using a two-degree-of-freedom system numerically and experimentally. It was found that optimum values of the natural frequency and the damping ratio of the dynamic absorber can suppress the subharmonic vibration effectively. The natural frequency of the dynamic absorber should not be set close to the excitation frequency but rather the natural frequency of the system. This dynamic absorber characteristic completely suppresses the subharmonic vibration. The experimental results agreed with results of the numerical analyses.

In the actual torque converter, the centrifugal pendulum absorber which is tuned to the excitation frequency will not be enough to suppress the subharmonic vibrations. Even though the natural frequency is not included in the excitation frequency range, the increase of the modal damping by attaching the dynamic absorber contributes to suppress the subharmonic vibrations.

The study of the relationship between the nonlinear natural frequency and the subharmonic vibration will be the future work.

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