Tuned vibration absorber with liquid elastic chamber (TVA-LEC) is a novel device for structural vibration suppression. The TVA-LEC consists of a rigid cylindrical chamber covered with two rubber membranes to provide stiffness, and liquid inside to provide inertia. In this paper, we present an approach based on the piezoelectric effect to convert the TVA-LEC to an energy harvester, which can be used to supply power to other devices, such as IoT sensors. First, the design of this energy harvester is described. An array of cantilever beams is connected to the inner wall of the rigid cylindrical chamber. Then piezoelectric ceramic patches are attached to the beams to absorb the vibration induced by the liquid flow when the TVA-LEC vibrates. Next, an analytical model is developed to predict both the natural frequency and the voltage generated by this device. The nonlinear liquid impact term is linearized according to the principle that equal works are done within one complete cycle of vibration. Finally, experiments are carried out on an electrodynamic shaker to obtain the energy harvesting efficiency with different excitation frequency. The experimental results agree with the prediction.

Keywords: Piezoelectric energy harvesting, Tuned vibration absorber, Vibration semi-active control

1. Introduction

With increasing global energy consumption and pollution, energy issue has become a major concern [1]. Energy harvesting device is capable of collecting valueless energy from the environment and supplying it to low-power device such as IoT sensors, creating self-contained packages. Energy harvesting based on piezoelectric method has been validated to harvest vibration energy, and has received increasing attention from academia and industry [2]. The piezoelectric method, where vibration of a structure causes deformation of piezoelectric patch attached to it, through which electricity is generated [3], has several successful applications in harvesting of hydro-kinetic energy. However, many researches still apply unidirectional flows or alternating media flows, which require complex conditions to create [4, 5, 6].

Tuned vibration absorber with liquid elastic chamber (TVA-LEC) [7, 8] is a novel device to suppress vibrations from the environment. It is composed of a rigid cylindrical chamber sealed with two rubber membranes as spring, and the liquid enclosed as mass. This paper proposes TVA-LEC-based energy
harvesting by fabricating an array of cantilever beams with piezoelectric patches attached, onto the inner wall of the chamber. When the TVA-LEC vibrates, the beams are induced by the liquid flow, and the piezoelectric patches are bent, through which a new type of vibration energy harvester could be realized.

This paper is organized as follows: Section 2 details the proposed new energy harvesting scheme. The physical model is described and governing dynamic equation is obtained. An analytical model is established in Section 3 to predict the voltage generated by the device. In Section 4, TVA-LEC harvester is tested on an electrodynamic shaker to validate the energy harvesting efficiency. Finally, Section 5 draws the conclusion of the paper.

2. Mathematical modeling

2.1 Equivalent model of TVA-LEC harvester

The picture of TVA-LEC harvester used for this study are shown in Fig. 1(a). The rigid cylindrical chamber made of resin is covered with two elastic membranes for holding the liquid inside. An array of cantilever beams is connected to the inner wall of the rigid chamber. Piezoelectric patches are attached to the beams, which is shown in Fig. 1(b). Fig. 1(c) shows a schematic diagram of TVA-LEC harvester. The shape of the membranes is assumed to be a spherical cap. When the TVA-LEC harvester vibrates, the relative motion between the inner liquid and the rigid chamber produces periodic impact on the cantilever beam connected to chamber, driving the piezoelectric patches to deform and generate voltage. The mass of the membranes is neglected, while the liquid in the chamber is regarded incompressible.

Based on the assumptions above, the TVA-LEC harvester can be reduced to a SDOF system shown in Fig. 1(d). In the schematic model and the equivalent model, $x, x_p$ denote the displacement of the liquid mass centroid and the rigid chamber, respectively. The mass $m$ represents the liquid mass and can be directly measured. The stiffness $k$ is related to the design parameters of TVA-LEC harvester and its expression will be derived in Section 2.2. Finally, the damping coefficient $c$ consists of piezoelectric damping coefficient and mechanical damping coefficient, will be identified in testing.

![Figure 1: (a)(b) Photo of TVA-LEC harvester; (c) schematic model; (d) equivalent model](image-url)
2.2 Dynamic equation of TVA-LEC

Figure [1]c) is the geometry depiction of the TVA-LEC harvester, where \( r \) is the radius of the rigid chamber, \( b \) is the height of the chamber, \( t \) is the thickness of the membranes, \( L_1 \) and \( L_2 \) are length of the cantilever beams and piezoelectric patch respectively. The liquid impact on the beams can be neglected when studying the natural frequency of TVA-LEC, because area of the beams are limited and the natural frequency of the beams are much higher than that of TVA-LEC.

According to the former study [7, 8], the dynamic equation of TVA-LEC can be written as:

\[
m\Delta \ddot{x} + c\Delta \dot{x} + k\Delta x = 0 \tag{1}
\]

where the stiffness \( k \) can be written as:

\[
k = (F'(\theta_{ue}) + aF'(\theta_{le})) / (G'(\theta_{ue}) + aG'(\theta_{le})) \tag{2}
\]

\( \theta_{ue} \) and \( \theta_{le} \) represent the contact angles of upper and lower membranes when the force equilibrium is satisfied under gravity \( g \). Functions \( F \) and \( G \) are related to the contact angles and other geometric parameters of TVA-LEC harvester.

The natural frequency \( f \) is a vital parameter representing the vibration frequency to be suppressed. It can be calculated as:

\[
f = \sqrt{\frac{k}{m}} / 2\pi \tag{3}
\]

The frequency caused by gravity could be neglected since the stiffness non-linearity is relatively weak. Thus the expression in Eq. (3) can be simplified as:

\[
f = \sqrt{\frac{E_1 t}{2\pi^2 \rho r^3}} \varphi (\theta_e); \quad \theta_e \geq \theta_0 \tag{4}
\]

where \( \varphi \) is a dimensionless function:

\[
\varphi = \sqrt{\frac{(1 + 2\varepsilon E_2/E_1 + 3\varepsilon^2 E_3/E_1) (1 - \theta/\tan \theta) \sin \theta/\theta_0 + (\varepsilon + \varepsilon^2 E_2/E_1 + \varepsilon^3 E_3/E_1) \cos \theta}{I_1 + I_2 + I_3}}
\]

\[
I_1 = \frac{1}{2} \left( 1 + \left( \frac{h}{r} \right)^2 \right) \left( \frac{1 - \cos \theta}{\sin^2 \theta} \right) \left( \frac{3}{4} \left( \frac{2R}{r} - \frac{h}{r} \right)^2 / \left( \frac{3R}{r} - \frac{h}{r} \right) - 1/ \tan \theta + \frac{\lambda}{2} \right)
\]

\[
I_2 = \frac{1}{6} \frac{h}{r} \left( 3 + \left( \frac{h}{r} \right)^2 \right) / \sin^2 \theta
\]

\[
I_3 = -\frac{1}{8} \frac{h}{r} \left( 3 + \left( \frac{h}{r} \right)^2 \right) \left( \frac{2R}{r} - \frac{h}{r} \right) \left( \frac{4R}{r} + \frac{2R}{r} \cos \theta - \frac{h}{r} \right) / \left( \frac{3R}{r} - \frac{h}{r} \right)^2 \sin^2 \theta
\]

\( \theta_e \) is the equivalent contact angle and \( \theta_0 \) is the installation contact angle. \( \theta_e \) should be larger than \( \theta_0 \) to ensure that membranes are tensioned. \( \varepsilon, h/r, R/r \) are functions of \( \theta_e \) and \( \theta_0 \).

When the TVA-LEC harvester is connected to an electrodynamic shaker, the governing equation of the model can be written as:

\[
\begin{bmatrix}
m_p & 0 \\
0 & m
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_p \\
\dot{x}_p
\end{bmatrix}
+ \begin{bmatrix}
c & -c \\
-c & c
\end{bmatrix}
\begin{bmatrix}
\dot{x}_p \\
\dot{x}_p
\end{bmatrix}
+ \begin{bmatrix}
k & -k \\
-k & k
\end{bmatrix}
\begin{bmatrix}
x_p \\
x_p
\end{bmatrix}
= \begin{bmatrix}
F_e \\
0
\end{bmatrix} \tag{6}
\]

\( m_p \) is the total mass of the TVA-LEC shell. The transfer function of the acceleration of \( m_p \) to the external excitation force \( F_e \) could be expressed as:

\[
\frac{\ddot{x}_p}{F_e} = \frac{m_0 s^2 + cs + k}{m_p ms^2 + (m_p + m) (cs + k)} \tag{7}
\]
2.3 Governing equation of piezoelectric beams

Assuming that the cantilever beam and piezoelectric patch are classical Euler-Bernoulli beams shown in Fig.2, the vibration displacement of the composite beam is \( \omega(y,t) \). Consider only the section with piezoelectric patches.

Figure 2: (a)(b) Piezoelectric cantilever beam

Connected to the inner wall of the rigid chamber, the fixed end of the composite beam has a displacement \( x_p \). The governing equation of the beam can be written as following.

\[
\rho_b \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial y^4} + p(t) = -\rho_b \frac{\partial^2 x_p(t)}{\partial t^2} \quad y \in [0, L_2]
\]  

(8)

with boundary conditions

\[
\omega(0, t) = \frac{\partial \omega}{\partial y}(0, t) = 0
\]

\[
EI \frac{\partial^2 \omega}{\partial y^2}(L_2, t) = \left[ p(t) + \rho_1 b_1 h_1 \frac{\partial^2 x_p(t)}{\partial t^2} \right] \left( \frac{L_1 - L_2}{2} \right)^2
\]  

(9)

where \( p(t) \) is the liquid impact force on the beam. \( \rho_b \) and \( EI \) are the linear density and interface bending stiffness of the composite beam respectively, and could be denoted as:

\[
\rho_b = \rho_1 b_1 h_1 + \rho_2 b_2 h_2
\]  

(10)

\[
EI = \frac{1}{12} E_{p1} b_1 h_1^3 + \frac{1}{12} E_{p2} b_2 h_2^3
\]  

(11)

where \( \rho_1, b_1 \) and \( h_1 \) are the density, width, and thickness of the cantilever beam respectively. \( \rho_2, b_2 \) and \( h_2 \) are the density, width, and thickness of piezoelectric patch respectively. \( E_{p1} \) and \( E_{p2} \) are the elastic modulus of the cantilever beam and piezoelectric patch respectively.

3. Output voltage of TVA-LEC harvester

To evaluate TVA-LEC harvester as an energy harvester, it is crucial to derive and measure the energy harvesting efficiency under certain external excitation. The output voltage is an effective index for piezoelectric energy harvesting method. In this section, the output voltage under sinusoidal excitation is predicted based on the analytical model in Section 2.

3.1 Simplified expression

When the harvester is mounted on an electrodynamic shaker providing sinusoidal excitation, the acceleration of the rigid chamber is also in a single-frequency sinusoidal form:

\[
\ddot{x}_p = A \sin \Omega t
\]  

(12)
According to the governing equation Eq. (6), the liquid is in a forced vibration state under sinusoidal excitation. The displacement of the liquid mass centroid $x$ is related to $x_p$.

$$x = B \sin(\Omega t + \varphi_0) + x_p$$

$$B = \frac{A}{\sqrt{\left(\frac{k}{m} - \Omega^2\right)^2 + \left(\frac{c\Omega}{m}\right)^2}}, \quad \varphi_0 = -\pi - \arctan\frac{c\Omega}{k - \Omega^2 m}$$ \hspace{1cm} (13)

The liquid impact is associated with the relative velocity of the liquid flowing through the chamber $v$, which is dependent on the relative velocity of liquid mass center and rigid chamber. $H(x)$ is a Heaviside function.

$$v = 3r \left[1 + \left(\frac{1}{\sin \theta_{ue}} - \frac{1}{\tan \theta_{ue}}\right)^2\right] \frac{1 - \cos \theta_{ue}}{\sin^2 \theta_{ue}} \frac{\dot{x} - \dot{x}_p}{G'(\theta_{ue}) + aG''(\theta_{ie})}$$ \hspace{1cm} (14)

$$p(t) = \frac{1}{2} \rho v^2 [2H(\dot{x} - \dot{x}_p) - 1]$$ \hspace{1cm} (15)

The liquid impact is non-linear, making it difficult to obtain the analytical model. However, the vibration amplitude of the harvester is limited, so the non-linear term could be linearized according to the principle that equal works are done in one cycle of vibration. $P(t)$ is the equivalent liquid impact:

$$P(t) = P \sin(\Omega t + \varphi_0 + \pi/2) = P \sin(\Omega t + \varphi)$$

$$\int_{t_0}^{t_0 + \frac{2\pi}{\Omega}} P^2(\sin \Omega t)^2 dt = \int_{t_0}^{t_0 + \frac{2\pi}{\Omega}} p^2(t) dt \quad \text{for} \ t_0$$ \hspace{1cm} (16)

Replacing the liquid impact with the linear term, the governing equation of the composite beam Eq. (8) is shown as follows. The external excitation includes inertia term and liquid impact term, and there is a phase difference $\varphi$ between them.

$$\rho_b \frac{\partial^2 \omega}{\partial t^2} + EI \frac{\partial^4 \omega}{\partial y^4} = -\rho_b A \sin(\Omega t) - P \sin(\Omega t + \varphi) = -K \sin(\Omega t + \phi)$$

$$K = \sqrt{(P \cos \varphi + \rho_b A)^2 + (P \sin \varphi)^2}, \quad \cos \phi = \frac{P \cos \varphi + \rho_b A}{K}$$ \hspace{1cm} (17)

Obtain the solution using separation of variable method. The governing equation is transformed into ordinary differential equation about $y(y)$.

$$\omega(y, t) = Y(y)T(t) \quad T(t) = \sin(\Omega t + \phi)$$ \hspace{1cm} (18)

$$-\Omega^2 \rho_b Y + EI Y''' = -K$$ \hspace{1cm} (19)

$$Y(0) = Y'(0) = 0$$

$$Y''(L_2) = -\frac{3(L_1 - L_2)^2}{E_1 b_1 h_1^3} K_0 \cos(\phi_0 - \phi)$$

$$Y'''(L_2) = \frac{12(L_1 - L_2)}{E_1 b_1 h_1^3} K_0 \cos(\phi_0 - \phi)$$ \hspace{1cm} (20)

$$K_0 = \sqrt{(P \cos \varphi + \rho_b b_1 h_1 A)^2 + (P \sin \varphi)^2}, \quad \cos \phi_0 = \frac{P \cos \varphi + \rho_b b_1 h_1 A}{K}$$
The general solution for equivalent governing equation Eq. (17) can be given as below. Applying the boundary condition Eq. (20), the coefficient $C_1$, $C_2$, $C_3$ and $C_4$ can be solved.

$$ Y(y) = C_1 e^{ay} + C_2 e^{-ay} + C_3 \cos(ay) + C_4 \sin(ay) + \frac{K}{\Omega^2 \rho_b}, \quad a = \left( \frac{\Omega^2 \rho_b}{EI} \right)^{1/4} \quad (21) $$

The average bending strain of the piezoelectric patch $\varepsilon(y,t)$ can be obtained:

$$ \varepsilon(y,t) = -\frac{(h_1 + h_2)}{2} \frac{\partial^2 \omega}{\partial y^2} \quad (22) $$

According to the piezoelectric energy harvesting principle, the output voltage of the piezoelectric patch is determined as:

$$ V(y,t) = 4E_{p2} \varepsilon(y,t) \cdot (g_{31}/h_2) \quad (23) $$

where $g_{31}$ is the piezoelectric voltage coefficient. The output voltage is multiplied by four, because four piezoelectric patches are included in the array.

### 4. Experimental validation

To validate the theoretical results, the experiment is divided into two parts. The first part focuses on determining the natural frequency and damping coefficient in the theoretical model. The second part verifies the energy harvesting efficiency of TVA-LEC harvester.

#### 4.1 Natural frequency determination

The baseline parameters of TVA-LEC harvester used in the experiment is shown in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
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<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{m1}$</td>
<td>3.10e6 Pa</td>
<td>$\theta_0$</td>
<td>18.47 deg</td>
<td>$r$</td>
<td>0.37 m</td>
<td>$m$</td>
<td>0.246 kg</td>
</tr>
<tr>
<td>$E_{m2}$</td>
<td>-6.75e6 Pa</td>
<td>$g$</td>
<td>9.8 m/s²</td>
<td>$b$</td>
<td>0.40 m</td>
<td>$m_p$</td>
<td>0.266 kg</td>
</tr>
<tr>
<td>$E_{m3}$</td>
<td>9.44e6 Pa</td>
<td>Liquid</td>
<td>water</td>
<td>$t$</td>
<td>0.11 mm</td>
<td>$\rho$</td>
<td>1000 kg/m³</td>
</tr>
</tbody>
</table>

Figure 3: (a)(b) Experiemtn set-up; (c) Broad band test
The experimental setup and the apparatus is illustrated in Fig. 3(a). The TVA-LEC harvester is mounted on an electrodynamic shaker which could provide random excitation. An impedance head between the shaker and TVA-LEC harvester measures the force acting on TVA-LEC and acceleration of the TVA-LEC frame.

The equivalent coupling model is illustrated in Fig. 3(b), while the governing equation for this model is shown in Eq. (6). When the shaker provide broad band random excitation, the transmissibility curve for the transfer function Eq. (7) is presented in Fig. 3(c). The first anti-resonating frequency represents TVA-LEC harvester’s natural frequency, 12.4Hz. The experiment result shows two resonating frequencies on both side of the predicted resonating frequency, probably because of the influence of liquid and cantilever beam in resonance region, which is not modeled in this paper. The damping coefficient can be identified from the resonant peak which is calculated as 2.75 Ns/m.

4.2 Energy harvesting validation

To validate the output voltage, sinusoidal excitation is provided by the shaker to ensure the chamber moving in sinusoidal form and the output voltage is measured with an oscilloscope. The parameters for the piezoelectric beams are shown in Table 2.

<table>
<thead>
<tr>
<th>Parameters</th>
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<th>value</th>
<th>Parameters</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1$</td>
<td>0.027 m</td>
<td>$L_2$</td>
<td>0.020 m</td>
<td>$\rho_1$</td>
<td>1160 kg/m$^3$</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.012 m</td>
<td>$b_2$</td>
<td>0.010 m</td>
<td>$\rho_2$</td>
<td>7700 kg/m$^3$</td>
</tr>
<tr>
<td>$h_1$</td>
<td>0.001 m</td>
<td>$h_2$</td>
<td>0.001 m</td>
<td>$g_{31}$</td>
<td>-30e-3 Vm/N</td>
</tr>
<tr>
<td>$E_{p1}$</td>
<td>2.0e9 Pa</td>
<td>$E_{p2}$</td>
<td>7.5e10 Pa</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 4: (a)The relationship between the amplitude of TVA-LEC harvester’s output voltage and excitation frequency; (b)-(d) Output voltage with different excitation frequency.
Fig. 4(a) shows the amplitude of TVA-LEC harvester’s output voltage as the excitation frequency varies. As the frequency increases from 8Hz to 12.4Hz, the peak to peak value of output voltage increases from 105mV to 126mV, while value decreases to 104mV when the frequency increases to 16Hz. The amplitude of the output voltage shows a relatively sharp peak near the natural frequency, which agrees well with the theoretical prediction.

The waveform of the output voltage measured by the oscilloscope is shown in Fig. 4(b)-(d), with frequency of 12Hz, 12.4Hz and 14Hz respectively. When the difference between excitation frequency and resonance frequency is relatively large, the output voltage accords well with the predicted sinusoidal form, as shown in Fig. 4(b) and (d). However, due to the large displacement of liquid mass at resonance frequency, the non-linearity of liquid impact can not be neglected. As shown in the experiment, when the excitation frequency is close to resonance frequency, the waveform of output voltage becomes asymmetric and unstable (Fig. 4(c)). Therefore, further modeling and optimization considering the non-linearity of the system is required in the vicinity of resonance frequency.

5. Conclusion

This paper presents a Tuned vibration absorber and harvester with liquid elastic chamber (TVA-LEC harvester). It consists of a piezoelectric beams attached rigid cylindrical chamber covered with two rubber membranes and liquid inside. Analytical approach is adopted to model this device as a SDOF oscillator with an array of Eular-Bernoulli beams. Based on this analytical model, the natural frequency and output voltage of TVA-LEC harvester are obtained. Finally, an experiment on the electrodynamic shaker is conducted. The experimental results validate the analytical predictions. The voltage generated can reach the magnitude of 100mV.

Future work includes optimizing design parameters and investigating the nonlinear properties of the TVA-LEC harvester.

REFERENCES