CONCENTRATION AND ATTENUATION OF FLEXURAL VIBRATIONS IN TAPERED CYLINDRICAL BEAMS AND WEDGE-LIKE RECTANGULAR BEAMS WITH POWER-LAW THICKNESS

Pengyun Zeng¹, Ling Zheng¹*, Yinong Li¹, Jie Deng¹, Shuhong Xiang², Tingfei Yan², Yao Wu²
1. The State Key Laboratory of Mechanical Transmission, College of Automotive Engineering, Chongqing University, Chongqing 400044, P.R. China
2. Beijing Institute of Spacecraft Environment Engineering, Beijing 100029, P.R. China
email: zling@cqu.edu.cn

The Acoustic Black Hole (ABH) effect for vibration control and sound absorption can be realized in tailored structures with a power-law-profiled thickness variation as the wave velocity will gradually decrease to zero in an ideal scenario. This means that no reflection occurs since the bending wave are not able to arrive at the edge of ideal ABH structures. One merit of the ABH effect is that vibration energy can be completely captured at the tip of the wedge structures. This feature makes ABH structures have a great potential in the aspects of vibration reduction and energy harvesting. In this paper, in order to describe the energy focalization effect properly, the semi-analytical method will be used to established the dynamical model as it can describe the system in terms of vibration energy. The mean square velocity (msv) of some measuring points has been calculated to analyze the difference of the energy concentration effect between the tapered rod and the wedge-shaped beam. Numerical results reveal that both of them own similar effect on energy trapping. However, the energy density at the tip of the rod is obviously larger than that at the edge of the wedge beam, which denotes that adhering less viscoelastic materials on the edge of tapered rods can achieve similar energy concentration effect compared with wedge-shaped beams. Comparisons of modal loss factors have further verified this trait of tapered rod, and it therefore allows ABH rods to perform better in vibration attenuation compared with wedge-like beams.

Keywords: Acoustic black hole; Tapered beam; Energy concentration; Damping effect

1. Introduction

In the past two decades, many researchers have investigated the acoustic black hole effect in traditional wedge-like structures. Based on the geometrical acoustic method, Krylov V V had testified that flexural wave will be completely captured at the edge of wedge beams with no truncations [1, 2].
After that, ABH effects have received extensive attentions and researchers analysed how to use it to realize vibration control and energy harvesting [3, 4]. Compared with some active vibration reduction method [5, 6], an astounding merit of ABH beams is their steady performance in vibration attenuation and external environment has little influence on it. However, obtaining an ideal ABH structure is not practical due to manufacturing limitations. Fortunately, some researches revealed that a very thin viscoelastic materials layer can effectively compensate the adverse effects caused by the truncation [7, 8]. Some researchers had investigated the impact of constrained damping layers on ordinary plates [9-12], and Jie Deng applied the same materials in one dimensional wedge beams and observed better vibration damping effect [4].

Although the geometrical acoustic method was first proposed to investigate the flexural wave propagating in ABH structures, it is not suitable to solve the vibration problem when damping materials has been applied to improve the performance of ABH structures [1,4]. In 2016, Tang L adopted the semi-analytical method to solve this problem and verified its convergence and accuracy by comparing the results with FEM as well as experimental results [13, 14]. The semi-analytical method based on Lagrange principle can be used to build the dynamic model of ABH structures in finite size and to deal with arbitrary boundary conditions flexibly. Besides, it is convenient for researchers using this method to tackle the coupling problem between the basic ABH structure and damping layers [13]. Another merit associated with this method is that it describes vibration systems from the perspective of energy, and this character makes it convenient to analyse the energy concentration effect of ABH structures.

However, most researchers mainly focus on the energy collection effect and the vibration reduction effect of wedge-shaped beams, and few of them investigate the ABH effect of tapered cylindrical beams or further analyse the difference effect of the two structures [2, 8]. The main difference of the two structures is illustrated in Figure 1 and Figure 2. In this paper, improved semi-analytical method is used to establish the dynamic models of wedge-like beams and tapered beams, and Morlet wavelet is considered as basic shape function to fit the displacement field of ABH beams. Besides, its convergence and accuracy has been verified in our previous work [15]. Moreover, comparisons of energy concentration effect as well as vibration reduction effect reveal that less viscoelastic materials adhered at the tip of tapered rods can achieve even better effect than wedge-shaped beams.

2. The dynamic modelling based on semi-analytical method

As shown in Figure 1 and Figure 2, the two physical models consist of a uniform portion with constant thickness $h_b$ from $x_1$ to $x_2$ and a tailored part with power law profile described as $h(x) = 2 \varepsilon x^n$ from $x_0$ to $x_1$, when $x_0=0$, the wedge beam become an ideal ABH structure. However, in consideration of the practical condition, the truncation distance is set to $x_0$. To mitigate the adverse effects caused by truncation thickness, a thin damping layer with invariable thickness $h_v$ is adhered at the edge from $x_{v1}$ to $x_{v2}$. Both of the wedge-like beam and the tapered cylindrical beam are excited by a harmonic force $f(t)$, and the amplitude of the force is set to 1N. The one side of the two ABH structures is free and the other side is elastically supported by a rotational spring $k_2$ and a translational spring $k_1$. Adjusting the stiffness of the two springs, varieties of boundary conditions can be obtained. In order to tackle the coupling problem between the ABH beams and the damping layer, a complex form of stiffness is considered as $E=E(1+i\eta)$, here $\eta$ refers to the damping loss factor of both the basic beams and the viscoelastic materials.
According to the Euler-Bernoulli theory, the rotary inertia as well as the shear deformation terms are neglected at all times, and therefore the displacement field for the two ABH structures can be expressed as:

\[
(x, y) = \left[ -y \frac{\partial w}{\partial x}, w(x, t) \right].
\]  

(1)

The basic Lagrange equation is used to determine the deflection \( w(x, t) \), it is defined as follows.

\[
\frac{d}{dt} \left[ \frac{\partial L}{\partial \dot{q}(t)} \right] - \left[ \frac{\partial L}{\partial q(t)} \right] = 0 .
\]  

(2)

Here \( \mathbf{q}(t) \) is a vector referring to a set of general coordination related with time, and \( L \) represents the Lagrangian which is defined as

\[
L = E_k - E_s + W .
\]  

(3)

The three terms of the Lagrangian are described clearly as follows.

\[
\begin{align*}
E_k &= \frac{1}{2} \int \rho (\frac{\partial w}{\partial t})^2 dV \\
E_s &= \frac{1}{2} \int EI(x) \left( \frac{\partial^2 w}{\partial x^2} \right)^2 dx + E_{\text{edge}} \\
E_{\text{edge}} &= \frac{1}{2} k_1 w(x_1, t)^2 + \frac{1}{2} k_2 \left( \frac{\partial w(x_2, t)}{\partial x} \right)^2 \\
W &= F(t) \cdot w(x_1, t)
\end{align*}
\]  

(4)

Here \( E_{\text{edge}} \) denotes the strain energy of the two springs. \( E_k \) and \( E_s \) respectively represent the kinetic energy and the strain energy of the whole ABH structure and damping layers.

Some necessary algebraic operations need to be completed to determine the deflection \( w(x, t) \) shown in Eq.(4). In terms of variable separation method, \( w(x, t) \) can be assumed as follows.
Substituting Eq.(3)-(5) to (2), the dynamic equation of the vibration system can be obtained.

\[ \mathbf{Mq}(t) + \mathbf{Kq}(t) = \mathbf{f}(t). \]  

(6)

Where \( \mathbf{M} \) and \( \mathbf{K} \) represent the system’s mass matrix and stiffness matrix respectively, and \( \mathbf{q}(t) \) and \( \mathbf{f}(t) \) are, respectively, the dynamic response and the general force. Mass matrix and stiffness matrix can be described in details as

\[
\begin{align*}
\mathbf{M} &= \mathbf{M}_{\text{uni}} + \mathbf{M}_{\text{abh}} + \mathbf{M}_{\text{damping}} \\
&= \int_{x_{1}}^{x_{2}} \pi r^{2} \rho \mathbf{p}\mathbf{p}^{T} dx + \int_{x_{1}}^{x_{2}} \pi \left( \varepsilon x^{2} \right)^{2} \rho \mathbf{p}\mathbf{p}^{T} dx + \int_{x_{1}}^{x_{2}} \pi \left[ \left( \varepsilon x^{2} + h_{j} \right)^{2} - \left( \varepsilon x^{2} \right)^{2} \right] \rho \mathbf{p}\mathbf{p}^{T} dx \\
\mathbf{K} &= \mathbf{K}_{\text{uni}} + \mathbf{K}_{\text{abh}} + \mathbf{K}_{\text{damping}} + \mathbf{K}_{\text{edge}} \\
&= \int_{x_{1}}^{x_{2}} \pi \frac{d^{4}}{dx^{4}} \left( \frac{d^{2}\mathbf{p}}{dx^{2}} \right)^{2} \pi r^{2} dx + \int_{x_{1}}^{x_{2}} \pi \left( \varepsilon x^{2} \right)^{4} \left( \frac{d^{2}\mathbf{p}}{dx^{2}} \right)^{2} \left( \frac{d^{2}\mathbf{p}}{dx^{2}} \right)^{2} \pi \left( \varepsilon x^{2} \right)^{2} dx \\
&+ \int_{x_{1}}^{x_{2}} \pi \left[ \left( \varepsilon x^{2} + h_{j} \right)^{4} - \left( \varepsilon x^{2} \right)^{4} \right] \left( \frac{d^{2}\mathbf{p}}{dx^{2}} \right)^{2} \left( \frac{d^{2}\mathbf{p}}{dx^{2}} \right)^{2} \pi \left[ \left( \varepsilon x^{2} + h_{j} \right)^{2} - \left( \varepsilon x^{2} \right)^{2} \right] dx + \mathbf{K}_{\text{edge}}
\end{align*}
\]

This physical model can also be used to describe the wedge-shaped beam shown in Figure 1 through changing the cross section area \( A(x) \) and the inertia moment \( I(x) \). The response vector and the force vector can be assumed as

\[
\mathbf{q}(t) = \mathbf{Ae}^{i\omega t}, \mathbf{f}(t) = \mathbf{Fe}^{i\omega t}
\]

(8)

\[
\begin{align*}
\mathbf{A} &= [q_{1}, q_{2}, q_{3}, \ldots q_{n}]^{T} \\
\mathbf{F} &= \mathbf{Fe}^{i\omega t} \\
\end{align*}
\]

The damping effect of both the two ABH structures and damping layers can be taken into account according to the complex Young modulus \( E \), i.e. \( E = E \cdot (1 + i\eta) \), and therefore the natural frequencies can be expressed as

\[ \omega^{2} = \omega_{i}^{2} \left( 1 + i\eta \right) \]  

(9)

Here \( \eta \) is the modal loss factor.

In order to solve the deflection \( w(x, t) \), finding a set of proper shape function to fit the displacement field is the semi-analytical method’s priority. After some trails, we find that Morlet wavelet is suitable to be used to describe the rapidly varied wavenumber and wave length at the edge of ABH structures.

The fundamental function of Morlet wavelet is defined as

\[
p(x) = \pi \frac{1}{4} \cos(n \pi x) e^{-\frac{x^{2}}{2}}, (n = 1, 2, \ldots, 5)
\]

(10)

In this paper \( n \) is set to 1 to obtain the highest accuracy. By translating and squeezing the basic function, a series of shape functions can be obtained.

\[
p_{j,k}(x) = 2^{\frac{j}{2}} p(2^{j}x - k) \]

(11)

Where \( j \) and \( k \) are the scaling and translating coefficient respectively. Generally, the higher the value of scaling coefficient, the higher accuracy can be obtained, while the computational cost also increases. The two coefficients can be determined through the following equation.

\[
k = [\text{ceil}(x_{0}2^{j} - 4/2^{j}), \text{floor}(x_{2}2^{j} + 4/2^{j})], (j = 0, 1, 2, \ldots)
\]

(12)
3. Analysis and Discussion

All geometrical parameters and material parameters for numerical simulations are illustrated in Table 1. To match the practical condition, a tiny truncation is made at $x_0=0.05m$.

<table>
<thead>
<tr>
<th>Tapered cylindrical beam</th>
<th>Wedge-like beam</th>
<th>Damping layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material $\rho=7800 \text{ kg/m}^3$</td>
<td>$\rho=7800 \text{ kg/m}^3$</td>
<td>$\rho=950 \text{ kg/m}^3$</td>
</tr>
<tr>
<td>Geometry $x_2=0.5m$</td>
<td>Geometry $x_2=0.5m$</td>
<td>Geometry $x_{v1}=x_0$</td>
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<tr>
<td>$E=210 \text{ GPa}$</td>
<td>$E=210 \text{ GPa}$</td>
<td>$E=5 \text{ GPa}$</td>
</tr>
<tr>
<td>$\eta=0.005$</td>
<td>$\eta=0.005$</td>
<td>$\eta=0.3$</td>
</tr>
<tr>
<td>$d_b=0.01m$</td>
<td>$b=0.01m$</td>
<td>$\varepsilon=0.08$</td>
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<td>$\varepsilon=0.08$</td>
<td>$\varepsilon=0.08$</td>
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3.1 Energy concentration effect of ABH beams without damping layers

This part is designed to compare energy concentration effect of the tapered beam and the wedge-shaped beam. A harmonic exciting force with constant amplitude $f_0=1 \text{ N}$ is applied to both of the two beams at $x_f=0.45m$. Firstly, two measuring points are chosen for comparing the difference of energy concentration density between the two structures. One of the measuring point is located at the tip ($x=0.05m$) of the beam, while the other point is chosen at the uniform part ($x=0.4m$) of the beam. As shown in Figure 3, velocity responses of the two points are calculated. The mean square velocity (msv) is defined as $E_v = 20 \log (v / (2v_{ref}))$, and the reference velocity is set to 1m/s. Figure 3(a) shows the velocity responses of a completely uniform cylindrical beam, and the peaks of velocity response at $x=0.05m$ are almost equal to that at $x=0.4m$. By contrast, in Figure 3(b) the mean square velocity at the tip of the tapered rod is much larger than that at the uniform part of the tapered rod. This means that tapered beams are capable of concentrating vibration energy at the ABH portion. The results shown in Figure 4 are similar to that in Figure 3, which denote that the wedge-shaped beam are also able to aggregate energy in the wedge part. In order to analyse investigate the difference of energy concentration effect of the two ABH structures, mean square velocity ratio is defined as

$$10 \log \left[ \left( \frac{v_{abh}^2}{2} \right) / \left( \frac{v_{uni}^2}{2} \right) \right].$$

Figure 5 reveals that the $msv$ ratio of the tapered beam is obviously larger that of the wedge-like beam from 500-5000 Hz, and this implies the energy density at the tip of the tapered beam is greater.
3.2 Vibration reduction effect of ABH beams with damping layers

To further compare the difference of ABH effects of the tapered beam and the wedge-shaped beam, a thin damping layer is adhered at the edge of the two structures from 0.05m to 0.15m. According to Table 1, the damping loss factor $\eta$ of the basic beam is 0.005 and of the viscoelastic materials is 0.3. In Figure 6(a), when the thickness of damping layer is 0.005m, the modal loss factors of tapered rod saw a rapid increase to about 0.035 at the fourth resonant frequency and maintain this level until at the twenty
ninth natural frequency. The trend of the wedge-shaped beam is similar to that of the tapered rod, while its modal loss factors remain stable at 0.02. When the thickness of the damping layer increases to 0.001m, as illustrated in Figure 6(b), the modal loss factors of the two structures also become greater than that shown in Figure 6(a) but the modal loss factor of the tapered beam are still larger than the wedge-like beam. Overall, Figure 6 verifies that less viscoelastic materials applied in tapered beams can achieve similar (even better) vibration damping effect than wedge-like beams.

![Figure 6: Modal loss factor of the tapered cylindrical beam and the wedge-like rectangular beam (a) $h_v=0.0005$m; (b) $h_v=0.001$m.](image)

4. Conclusions

In conclusion, the semi-analytical method is adopted to investigate the mean square velocity on different measuring points along the cylindrical beams as well as the wedge-like beams with the tailed ABH structure. Numerical results reveal that both of two ABH beams can concentrate energy at the tip, while the energy density of the tapered beams is much greater than that of the wedge-shaped beams. Moreover, the damping layers are applied at the edge of the beams and the modal loss factor of each natural frequency is calculated to reduce the vibration of the two ABH beams. The results demonstrate that less damping materials applied to tapered beams can obtain better effect on vibration attenuation than the wedge-shaped beams.

This work provides an effective technique to analyse energy concentration and vibration reduction on the tapered beams. Future research work will be addressed to the practical application of one-dimensional ABH structures.

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