OPERATIONAL TRANSFER PATH ANALYSIS OF A DOMESTIC REFRIGERATOR

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Despite of the fact that the product design process of advanced mechanical systems have been developed substantially, the demand for noise and vibration diminution has become even more significant. Concordantly, Transfer Path Analysis (TPA) can be applied for the determination of important vibration and/or noise source(s), and crucial vibro-acoustic energy transfer path(s) in a system to develop the vibro-acoustic design. The technique used in this study is the Operational Transfer Path Analysis (OTPA) utilizing Crosstalk Cancellation (CTC) and Singular Value Decomposition (SVD).

In the scope of this study, OTPA is applied to a domestic refrigerator to identify and order the critical vibration transmission path(s) in the system. The source-transfer path-receiver model, sensor positions, and operational working conditions for the study are expressed in detail. Selection of correct transfer paths and receiver locations without changing the vibration source and operational conditions is investigated. In this sense, two different experimental studies are conducted on the refrigerator. The operational conditions are obtained by changing the compressor rotational speed and measurements at reference (input) and response (output) points are made for individual operational conditions. The OTPA is then implemented utilizing the obtained data sets and the contribution of vibro-acoustic energy transmission via individual transfer paths of the refrigerator are acquired and evaluated.

**Keywords:** Operational Transfer Path Analysis, Singular Value Decomposition, Vibration Signal Analysis, Domestic Refrigerator, Variable Speed Compressor

1. **Introduction**

Operational Transfer Path Analysis (OTPA) utilizes a signal processing methodology that acquires the linearized Transfer Functions (TFs) between a group of selected input and output quantities from a measurement [1, 2]. The TFs are linearly independent in regard to each other, the relation between the reference (input) and response (output) quantities is defined in this way [2]. The determined TFs are utilized in the OTPA method to ascertain the vibro-acoustic energy propagation and resultant contribution of a source in the response signal [1, 2]. The numerical computations conducted in the OTPA could be adversely affected from ill-conditioning, hence the Singular Value Decomposition (SVD) method is implemented to avoid this problem [1 – 3].

The primary purpose of the OTPA is to evaluate and rank different contributions of vibration and/or noise at the specific receiver position by classifying transmission paths from various structure-borne and/or airborne vibration and/or noise sources. A variety of studies are being conducted utilizing the
OTPA method in the field of Noise, Vibration and Harshness (NVH). De Klerk et al. [1] examined the analysis of a classic car Wartburg 311 with the OTPA by performing the real condition measurements (e.g., run-up, full load, partial load, coastdown) to identify critical vibration paths and source contributions to the interior noise of the driver’s compartment. De Klerk and Ossipov [2] discussed the theory of the OTPA enhanced with the SVD method, detailed the resemblances and dissimilarities between the Multi Input-Multi Output (MIMO) and OTPA techniques, and applied the OTPA method on a vehicle to investigate the tire noise propagation. Roozen et al. [3] searched the utilization of transmissibility approach on the OTPA improved with the SVD technique that was carried out on small gearbox test setup to specify the number of physical transmission paths. Tutu et al. [4] applied the OTPA based on Least Square Algorithm (LSA) and SVD techniques to a front-loading drum-type washing machine to determine and rank the contributions of vibration transmission paths via individual springs and dampers of the washing machine. Toome [5] presented the theoretical background of the OTPA method and introduced the practical considerations to choose sensor locations during the OTPA setup and post-processing by conducting a study on an idling road tractor.

The main goal of this study is to evaluate the practicality of the OTPA method in the case of a domestic refrigerator application. For this reason, an OTPA code based on the SVD technique is developed. OTPA is then applied to domestic refrigerator to determine and rank the contributions of critical vibration transfer paths to the response location. The content of the manuscript is as follows. The theoretical background is given and summarized in the next section. The third section details the application of this method to a domestic refrigerator. Ultimately, in the fourth and last section, the paper is finalized with a short summary of the fundamental results.

2. Theoretical background

The purpose of the OTPA is to designate the contributions of different sources, propagated over several vibration and/or noise transfer paths, to the receivers under operational conditions [2, 4, 6]. The sources or excitations of the mechanical system are described as the input Degrees of Freedom (DOFs) and the receivers are described as the output DOFs for the OTPA [1 – 4]. The essential objective of the OTPA is to obtain linear relations between reference (input) and response (output) DOFs by measuring the source/excitation and receiver/response signals of the system simultaneously under operational conditions [1 – 6]. An arbitrary linear(ized) system model described by a set of input and output DOFs is stated as:

\[ H(j\omega)x(j\omega) = y(j\omega) \]  

(1)

where \( x(j\omega) \) and \( y(j\omega) \) are the input and output vectors, respectively, \( H(j\omega) \) is the TF matrix connecting \( x(j\omega) \) and \( y(j\omega) \), and \( (j\omega) \) states the dependency on frequency. The OTPA technique tries to find all elements of the TF matrix from measured reference and response data. To investigate this approximation, the transpose of Eq. (1) is received and it is rewritten as:

\[
\begin{bmatrix}
    x^{(1)}, & \ldots, & x^{(m)} \\
    \vdots & \ddots & \vdots \\
    H_{m1} & \ldots & H_{mn}
\end{bmatrix}
\begin{bmatrix}
    H_{11} & \ldots & H_{1n} \\
    \vdots & \ddots & \vdots \\
    H_{m1} & \ldots & H_{mn}
\end{bmatrix}
\begin{bmatrix}
    y^{(1)}, & \ldots, & y^{(n)}
\end{bmatrix}
\]

(2)

where \( m \) and \( n \) define the number of input and output DOFs, respectively. Here, the dependency on frequency \( (j\omega) \) is omitted from Eq. (2) for the clarity. It is vital to state that receiving the transpose does not permit to find the TF matrix elements. For this reason, even though Eq. (2) has to be calculated at each frequency, a set of synchronized linearly independent operational measurement blocks is needed for the identification of the TF matrix elements. Since the sources/excitations of the mechanical system alter progressively during the measurement, the set of selected input and output quantities do not have the same content in general. It is assumed that the relation between the reference and response DOFs is
linear(ized) and remains constant during the entire measurement period, thus Eq. (2) should be valid for each individual measurement block. Therefore, the operational measurements are augmented \( r \) times, for which synchronous frequency spectrum for each input and output channel is calculated. This causes to the augmentation of Eq. (2) for whole measurement blocks \( r \) as:

\[
\begin{bmatrix}
X_1^{(1)} & \ldots & X_1^{(m)} \\
\vdots & \ddots & \vdots \\
X_r^{(1)} & \ldots & X_r^{(m)} \\
\end{bmatrix}
\begin{bmatrix}
H_{11} & \ldots & H_{1n} \\
\vdots & \ddots & \vdots \\
H_{m1} & \ldots & H_{mn} \\
\end{bmatrix}
= 
\begin{bmatrix}
y_1^{(1)} & \ldots & y_1^{(n)} \\
\vdots & \ddots & \vdots \\
y_r^{(1)} & \ldots & y_r^{(n)} \\
\end{bmatrix}
\]  

(3)

Here it is supposed in the conducted experiment that the number of operating conditions (i.e., measurement blocks) is higher than the total amount of input DOFs (i.e., \( r > m \)). This assumption provides a least-squares optimization problem solution for Eq. (3) which can be demonstrated in more neat form as:

\[ XH = Y \]  

(4)

Coherency between \( m \) measured input (reference) signals can be originated from crosstalk between the input DOFs [2, 6]. The calculation required to be carried out for each frequency line of the FFT spectrum [2]. Eq. (4) is pre-multiplied by \( X^T \) to solve the equation which provides the explicit determination of the TF matrix \( H \) as:

\[ H = (X^T X)^{-1}X^T Y = X^+ Y \]  

where matrix \( X^+ \) is the pseudo-inverse of matrix \( X \), which is defined as:

\[ X^+ = (X^T X)^{-1}X^T \]  

(6)

If reference signals are highly coherent in combination with measurement noise, the explicit determination of the TF matrix \( H \) by Eq. (5) can result in inaccurate estimations [2]. Additionally, since the number of rows is higher than the number of columns in the input matrix \( X \) (i.e., \( r > m \)), the pseudo-inverse calculation yields to an overdetermined least-squares problem. For this reason, a general and more accurate solution can be provided by SVD [2, 6, 7]. Thereby, the input matrix \( X \) is decomposed by a SVD as:

\[ X = U \Sigma V^T \]  

(7)

where \( U \) is an \( r \times r \) unitary column-orthogonal matrix, \( V^T \) is an \( m \times m \) unitary matrix that defines the conjugate transpose of an \( m \times m \) unitary column-orthogonal matrix \( V \). \( \Sigma \) is \( r \times m \) diagonal matrix with nonnegative numbers along the diagonal, representing singular values of the matrix \( X \). At this stage, very small singular values, generally because of noisy data, are taken away during this operation [1, 2, 6] and the pseudo-inverse \( X^+ \) via SVD is computed as:

\[ X^+ = V \Sigma^{-1} U^T \]  

(8)

Consequently, Eq. (8) is implemented to Eq. (5) to determine the linearized TF matrix \( \tilde{H} \) as:

\[ \tilde{H} = V \Sigma^{-1} U^T Y \]  

(9)

The term \( \tilde{H} \) in Eq. (9) specifies the synthesized TF matrix. The synthesized output (response) matrix \( \tilde{Y} \) is calculated utilizing measured input (reference) matrix \( X \) and estimated TF matrix \( \tilde{H} \) as:

\[ \tilde{Y} = X \tilde{H} \]  

(10)

As can be noticed from Eq. (10), the total response (output) at the receiver position is found by totting up the output contributions from individual inputs (sources) [6, 8]. The real measured total response \( Y \) is compared with the overall response synthesis \( \tilde{Y} \) (i.e., the total synthesis contributions) to illustrate the efficacy of the OTPA computations [1, 2, 6, 8].
3. Application of OTPA to a domestic refrigerator

One of the most usually utilized large home appliances is household refrigerators (i.e., domestic refrigerators) which are generally placed in the kitchens. Nathad et al. [9] stated that the refrigerators have the moving components (e.g., a pump or a compressor) to transmit the required work and heat input towards generating refrigeration. Compressor is considered as the dominant noise and vibration source in the domestic refrigerators. Marshall [10] represented an approach to diminish the compressor noise while regarding system interactions and classified the vibro-acoustic transmission paths as structurally coupled, acoustically coupled and sound waves radiated. Structural transmission paths are indicated as the structural energy flow from the compressor mounting feet into the base pan, the flow of energy along the refrigerant tubes on the discharge and suction sides, and the path along electrical cables.

In this study, the OTPA is applied to a domestic refrigerator to identify and rank the vibration transmission paths. For this reason, two different experimental studies are conducted. The domestic refrigerator is a bottom-free type refrigerator. The compressor which is a variable speed compressor is regarded as the main vibration source. The compressor is physically connected to the base pan of domestic refrigerator via grommets and to the body of domestic refrigerator via refrigerant tubes. The base pan is also physically linked to the body of domestic refrigerator. The compressor vibrations are structurally transmitted to the body of domestic refrigerator through the grommets and the refrigerant tubes on the discharge and suction sides. Thus, the vibration transmission paths are chosen as the grommets that are placed between the base plate and the compressor in order to provide vibration isolation and the refrigerant tubes on the discharge and suction sides. Here, it can be stated that the vibration transmission paths are assumed as the inputs for the application of OTPA in the scope of this study. The receiver location is selected as the side body of the refrigerator. The structural and noise transmission paths along electrical cables, the structure-borne and air-borne noise effects are neglected.

For the both experiments:

- The inputs are measured at the start of the selected vibration transmission paths.
- The necessary operational conditions for the application of OTPA are provided by changing the rotational speed of the compressor,
- The refrigerator is tested at different rotational speeds, from 1200 rpm to 3000 rpm, with a step of 200 rpm,
- A signal analyzer with 12-channel is utilized, permitting the measurements of 12 signals simultaneously.
- Measured signals are obtained as phase-assigned spectrums in 0-6400 Hz frequency range.
- The verified MATLAB code [4] is used to process the measurement results using the OTPA procedure to obtain the required data for the identification of critical transfer paths.

On the other hand, the main difference between two experimental studies is the selection of receiver position. In the first experimental study, the receivers are defined as connection points between the condenser pipe-refrigerator body and the base plate-refrigerator body. In the second experimental study, the receivers are defined on the right side of the refrigerator body as a result of the first experimental study. The details and results of experimental studies are explained in the next two subsections.

3.1 First experimental study details and results

In the first experimental study, the input signal measurements are made at the start of refrigerant pipes, and as close as possible to the front grommets located at left and ride sides. As defined above, the receivers are defined as connection points between the condenser pipe-refrigerator body and the base plate-refrigerator body. It means that 5 input and 6 output are selected in total.
The singular values obtained by decomposing the input data with the SVD method are shown in Fig. 1. Since some singular values show inbound character, the selected references could be dependent with each other per the result of SVD computation. For a representative comparison, the responses measured as acceleration on the both sides of the refrigerator body at 1600 rpm are compared in Fig. 2. As a result of the conducted OTPA application with the defined source-transfer path-receiver model, it may not be possible to distinguish among the selected inputs. It is observed that the measured acceleration values on the top and bottom connection points between the condenser pipe and the refrigerator body are very close to each other as shown in Fig. 2(a) and Fig. 2(b), hence the measurements can be made in such a way that one side of the body is output by assuming the equal vibration transmission from compressor to the body through the pipes on the both sides. On the other hand, it is noticed that the response of right side is higher than the response of left side with respect to measured acceleration values on the connection point between base plate and refrigerator body as shown in Fig. 2(c), thus the right side of the refrigerator body should be selected as the output for the OTPA application on the domestic refrigerator.

![Singular Values](image1)

**Figure 1:** Singular values for the first experimental study.

![Comparison of measured responses as acceleration](image2)

**Figure 2:** Comparison of the measured responses as acceleration: (a) the responses of right and left sides are very close to each other per measured acceleration values on the bottom connection point between condenser pipe and refrigerator body, (b) the responses of right and left sides are very close to each other per measured acceleration values on the top connection point between condenser pipe and refrigerator body, (c) the response of right side is higher than the response of left side per measured acceleration values on the connection point between base plate and refrigerator body.
3.2 Second experimental study details and results

In the second experimental study, the input signal measurements are made at the start of refrigerant pipes as in the first experimental study, but this time as close as possible to the front and back grommets located at ride side. As defined above, the receivers are defined as connection points between the condenser pipe-refrigerator body and the base plate-refrigerator body. It means that 4 input and 7 output are selected in total.

After simultaneous operational measurements on the selected input and output locations, TFs are estimated using the SVD method. The singular values obtained by decomposing the input data with the SVD method are shown in Fig. 3. Since the computed singular values show out of bound character, it can be stated that the chosen references are linearly independent per the result of SVD computation. Then, to validate the TF estimation, some comparisons of measured and calculated (synthesized) responses by implementing LSA and SVD approaches, shown in Fig. 4, are made for one of the response location. It is stated that there is a good accordance between actual measured and synthesized output signals.

![Figure 3: Singular values for the second experimental study.](image)

![Figure 4: Comparison of measured (blue) and calculated (red for LSA method and green for SVD technique) responses in dB scale (a) 1600 rpm and (b) 3000 rpm.](image)
After the TF estimation and verification process, the vibration transfer paths are determined by the contribution analysis where the overall response is divided into individual path contributions. An example contribution analysis at an operational condition (e.g., 3000 rpm) is illustrated in Fig. 5. As a result of the contribution analysis, the refrigerant pipes are the dominant vibration transfer paths in 0-100 Hz frequency range as shown in Fig. 5(a) while the compressor grommets are the dominant vibration paths in 4700-5100 Hz frequency range as shown in Fig. 5(b).

![Figure 5: Result of contribution analysis performed for the operation condition of 3000 rpm (a) in 0-100 frequency range, (b) in 4700-5100 frequency range.](image)

### 4. Conclusions

In this study, it is purposed to apply the OTPA to a domestic refrigerator for identifying and ranking the critical vibration transfer paths. In the domestic refrigerator application, the main excitation source is considered as the compressor. The sources to the vibration transfer paths are assumed as input variables, thus transmissibility approach is utilized for the operational TF determination. The operational conditions are varied by altering the rotational speed of the compressor. Different contribution analyses are conducted in order to decide and rank the significant transmission paths along the compressor grommets and refrigerant pipes of the domestic refrigerator. For this reason, two different experimental studies are carried out on the domestic refrigerator. In the case of vibration transmission from compressor to refrigerator side body, it is found that the refrigerant pipes (i.e., discharge and suction tubes) are more dominant on lower frequencies (e.g., 0-100 Hz frequency range) while the compressor grommets are more dominant on higher frequencies (e.g., 4700-5100 Hz).

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### REFERENCES


