DYNAMIC ANALYSIS OF RUB IN HIGH-SPEED ROTOR-STATOR CONTACT

Zhang Yi and Su Li
Institute of Physical and Chemical Engineering of Nuclear Industry, Hedong, Tianjin, China
email: challenger1@126.com

This research includes the model based on high-speed rotor, and used multi-body dynamics to investigate the dynamic response of rotor-stator system during rub-contact condition caused by initial periodic excitation. How the physical parameters of limit device influence the capability of the limit device is investigated in this paper. The analytical model consists of a sub-critical rotor, bearings, a damper and a limit device. Radial clearance, impact stiffness and friction forces due to rub are viewed in this model. A set of non-linear differential equations is obtained, and numerical solution is calculated using Runge-Kutta method. This research indicates that the impact between flywheel rotor and limit device can generates low frequency procession. Friction force is the major factor affecting the stability of rotor rubbing with stator. Higher friction coefficient and higher impact stiffness produce more possibility of full-annular rub and instability. Increasing the impact stiffness raises the maximum impact forces rotor bear which may cause destruction of rotor. The stability of rotor can be enhanced by increasing radial clearance between and limit device. This, however, increases the maximum vibration amplitude during the impact. A critical friction coefficient curve due to stiffness change is derived. A stable area can be figured when taking both impact stiffness and friction coefficient into consideration.

Keywords: rotor dynamics, rotary machine, rub-impact, limit device, nonlinear dynamics

1. Introduction

Rotating machinery is commonly used nowadays for a wide range of energy conversion applications. High-speed flywheel can be used in energy storage. The idea of using stops to limit the excitive motion has existed for several decades. Light partial rub can be useful to absorb energy and decrease vibration. But it can also lead to serious damage when partial rub grows to full-annular rub. The collision and friction forces may rise to high level and will bring disastrous accident of machinery. The dynamics of rotor-stator contact have been investigated extensively in the past by many researchers. Muszynska reported a comprehensive survey on rub-related phenomenon[1]. F F. Ehrich[2] calculated a two-degree model to simulate one plate rotor contacting stator. Based on numerical method, he studied high order sub-hamonic response and chaotic vibration of high speed rotor rub system. Same conclusion has been reported by many researchers[3-5]. Investigation on these phenomena revealed that the rotating system showed a rich class of nonlinear dynamic behaviors include sub-and hyper-synchronous vibration, quasi-periodic responses and chaotic motions. Most studies focused on Jeffcott type rotors. Dai and X.Zhang has studied a kind of limit device which can absorb vibrations for flywheel[6]. They modeled the device as a spring placed near one side of two plate rotor and investigated the partial and
full annular rub. With the development of the finite element method (FEM), researchers were able to establish dynamic models of more complex rotor system employing FEM. Lumiao Chen considered the flexible deformation of a rotor-stator system involving a cylindrical shell [8].

We concern about the safety of the rotor under sudden transient external perturbation such as earthquake. So this research focus on how the design parameters of limit device influence the effect of limitation and protection. The remainder of this paper is organized as follows: Sec.2 describes the theoretical model of the rotor-stator system. Sec.3 presents numerical analysis results and analyse the effect of radial clearance, impact stiffness and friction coefficient changing. Sec.4 presents some brief conclusions.

<table>
<thead>
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<th>Nomenclature</th>
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<tr>
<td>$\omega_0$</td>
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<tr>
<td>$\delta$</td>
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2. Analytic model

The basic dynamic model of a flywheel rotor-bearing system is shown in Fig1. We set the rotor rotating anticlockwise in following discussion. The top bearing is a permanent magnetic bearing. The bottom bearing is a needle bearing with damping support. There is an inner limit device at the top. When the amplitude of rotor vibration is larger than radial clearance, collision and friction occurs between rotor and limit device. The stiffness of the limit device is higher than that of the top bearing. When the rotor revolve at high speed, disturbances can cause the amplitude rise. The collision and rub between the rotor’s inner wall and the limit device prevent contacts at other positions. The disturbances are applied on the bearing basis and we take harmonic excitation as an example for calculation.

When deriving the dynamic model for the flywheel rotor, two assignments should be formed to establish an analytic model. First, the flywheel itself is a sub-critical rotor and can be considered rigid. Second, the time of rub-impact is so short that the contact between rotor and stator is elastic impact and friction can be regard as a simple Coulomb model. The center of the stator is assumed as the origin of the co-ordinates. Displacements of the rotor’s upper journal’s center are $(x_1, y_1)$. Displacements of the rotor’s bottom plane center are $(x_2, y_2)$. $(x_3, y_3)$ represent the displacements of damping body.
2.1 Contact model

The limit device is designed to reduce radial contact force and friction force to protect the rotor. The well-known Hertz theory is widely used to establish the relations between contact force $f'$ and deformation $\delta$. As for the situation which a circular ring rotor and a cylinder stator contacting with each other, some research[9] proposes the contact force $f'$ can be determined by deformation as follows:

$$f' = k_d \delta + \alpha \delta^2 \quad (1)$$

We use the drop hammer test to measure the radial contact force. When the deformation of limit device is less than 0.4mm, the relationship between contact force and deformation is linear. The stiffness $k_d$ is between $2.2 \times 10^5$ N/m and $6.1 \times 10^5$ N/m. The stiffness increase sharply as the deformation grown to more than 0.4mm. But in this research we only investigate the linear situation. The impact force is given by:

$$f'' = k_4 (r_i - r_0) \quad (2)$$

Where $r_i = \sqrt{x_i^2 + y_i^2}$ is the radial displacement, $r_0$ is the initial clearance between the rotor and stator. According to Coulomb model the friction force can be expressed as:

$$f' = \mu k_4 (r_i - r_0) \quad (3)$$

Rub and impact forces can be written in the x-y coordinate as follows:

\[
\begin{align*}
    f_x^{k_4} &= -k_4 (r_i - r_0) \frac{x_i}{r_i} - k_4 \mu (r_i - r_0) \frac{y_i}{r_i} \\
    f_y^{k_4} &= -k_4 (r_i - r_0) \frac{y_i}{r_i} + k_4 \mu (r_i - r_0) \frac{x_i}{r_i}
\end{align*}
\]  

(4)
2.2 The governing equations of motion

According to the contact model and rotor dynamics theory, the motion equations of this rotor-bearing system can be written as:

$$\begin{bmatrix} M \end{bmatrix} \ddot{X} + [C] \dot{X} + [K] \{X\} = F_c(t) + F_k$$  \hspace{1cm} (5)

Where \( \{X\} = \{x_1, x_2, x_3, y_1, y_2, y_3\}^T \), \( M \) is mass matrix, \( C \) is damping matrix, \( K \) is stiffness matrix and can be written as:

$$M = \begin{bmatrix}
J / L & M / J & 0 & 0 & 0 & 0 \\
- J / L & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \hspace{1cm} (6)$$

$$C = \begin{bmatrix}
0 & 0 & 0 & -J_\rho \omega / L^2 & J_\rho \omega / L^2 & 0 \\
0 & 0 & 0 & J_\rho \omega / L^2 & -J_\rho \omega / L^2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & c_3 & 0 & 0 \\
- J_\rho \omega / L^2 & J_\rho \omega / L^2 & 0 & 0 & 0 & 0 \\
J_\rho \omega / L^2 & - J_\rho \omega / L^2 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \hspace{1cm} (7)$$

$$K = \begin{bmatrix}
k_1 & 0 & 0 & 0 & 0 & 0 \\
0 & k_2 & -k_2 & 0 & 0 & 0 \\
0 & -k_2 & k_3 + k_3 & 0 & 0 & 0 \\
0 & 0 & 0 & k_3 & 0 & 0 \\
0 & 0 & 0 & -k_2 & k_2 + k_3 & 0 \\
\end{bmatrix} \hspace{1cm} (8)$$

$$F_c(t) + F_k = \begin{bmatrix}
m_e \epsilon \omega^2 \cos \omega t + A \cos \omega t \\
m_e \epsilon \omega^2 \cos \omega t \\
0 \\
m_e \epsilon \omega^2 \sin \omega t + A \sin \omega t \\
m_e \epsilon \omega^2 \sin \omega t \\
0 \\
\end{bmatrix} \begin{bmatrix}
-k_4 (r_i - r_i) \frac{\dot{x}_1}{r_i} - k_4 \mu (r_i - r_0) \frac{\dot{x}_1}{r_i} \\
0 \\
-k_4 (r_i - r_0) \frac{\dot{y}_1}{r_i} + k_4 \mu (r_i - r_0) \frac{\dot{y}_1}{r_i} \\
0 \\
0 \\
0 \\
\end{bmatrix} \hspace{1cm} (9)$$

Set \( r_j = x_j + iy_j \), we can get the equations in complex terms:

$$[\overline{M}] \{\ddot{r}\} + (i \omega [\overline{C}] - \omega^2 [\overline{K}]) \{\dot{r}\} + [\overline{K}] \{r\} = \begin{bmatrix}
m_e \epsilon \omega^2 e^{i \omega t} + A e^{i \omega t} \\
m_e \epsilon \omega^2 e^{i \omega t} \\
0 \\
m_e \epsilon \omega^2 e^{i \omega t} \\
0 \\
0 \\
\end{bmatrix} \begin{bmatrix}
-k_4 (r_i - r_0) (1 + \mu i) \frac{\dot{x}_1}{r_i} \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix} \hspace{1cm} (10)$$
3. Numerical analysis results

The numerical solution of Eq.(5) is performed by using fourth-order Runge-Kutta method. Although the system has six degrees of freedom, the vibration at the top is much larger than at the bottom. Thus, the analysis concentrates on the degrees of freedom at the top. Modal analysis show that the system has three modal frequency. The first-order frequency is 3.45 Hz. Therefore, the excitation frequency of 3.45 Hz is used in the following calculation. The clearance between the rotor and stator is 0.3 to 0.9 mm, much larger than the amplitude of synchronous unbalance response. The simulated model parameters of the rotor bearing system is given in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
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<tbody>
<tr>
<td>$m$ (g)</td>
<td>2000</td>
</tr>
<tr>
<td>$m_1, m_2$ (g $\times$ m)</td>
<td>0.3*0.07</td>
</tr>
<tr>
<td>$m_3$ (g)</td>
<td>17</td>
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<tr>
<td>$J_1$ (kg$^2$m$^2$)</td>
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<tr>
<td>$J_2$ (kg$^2$m$^2$)</td>
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<tr>
<td>$k_2$ (N/m)</td>
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<td>$L$ (m)</td>
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<tr>
<td>$c_1$ (Ns/m)</td>
<td>20</td>
</tr>
</tbody>
</table>

3.1 Influence of rotational speed

Rotational speed is one key parameters that affect the dynamic behavior of a rotor system. Fig2 to Fig4 show the motion orbit of the top of the rotor at various rotation speed. In these case $\mu=0.1$ and $k_4=2 \times 10^5$. According to calculation results, when the rotation frequency is below 152 Hz, the solution is unstable. In this situation, rub excites the second order modal vibration of the system. When the rotation frequency is above 152 Hz, full-annular rub happens and expresses periodic characteristics.

The function of limit device is to reset the rotor. So we set the excitation last 2.5s, and study whether the rotor returns to backwards after excitation. Fig5 show the motion after excitation disappears. It can be figure out that the rotor and stator would separate quickly. After separation, A vibration at 3.4 Hz will exist for several seconds. Situations which excitation frequency equals to 10 Hz, 100 Hz, 500 Hz, are also calculated. Same low-frequency vibration are also derived. It seems to be the inherent frequency of the system.

![Figure 2](attachment:image.png)
3.2 Influence of the friction coefficient and impact stiffness

When the rubbing phenomenon occurs, friction conditions and impact stiffness are also key parameters that affect the dynamics responses of a system. Fig6 to show the influence of the friction coefficient with $k = 2 \times 10^5 \text{ N/m}$ and rotation frequency = 1500Hz. Still, excitation lasts 2.5s, and the motion of rotor after excitation is investigated. It illustrates that higher friction coefficient makes the duration of contact between rotor and stator longer and produce more possibility of full-annular rub and instability with other parameters holding constant. When $\mu$ reach to 0.4, full-annular rubbing continues even though excitation stops. The friction force transforms the rotating energy into unwanted lateral vibration energy and amplitude is not convergent.

Fig7 shows the influence of impact stiffness with $\mu = 0.38$ and rotation frequency = 1500Hz. We can also figure out that higher impact stiffness makes the duration of contact between rotor and stator longer and produce more possibility of full-annular rub and instability. Because the displacement in full-
annular rub is quite same, the high stiffness lead to high radial contact force which may cause destruction of rotor.

![Figure 6](image6.png)

**Figure 6** The motion of rotor after excitation disappear with different friction coefficient

![Figure 7](image7.png)

**Figure 7** The motion of rotor after excitation disappear with different impact stiffness

<table>
<thead>
<tr>
<th>Stiffness(N/m)</th>
<th>5*10⁴</th>
<th>1*10⁵</th>
<th>10⁶</th>
</tr>
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<tbody>
<tr>
<td>Maximum radial amplitude(mm)</td>
<td>1.052</td>
<td>0.996</td>
<td>0.936</td>
</tr>
<tr>
<td>Maximum force(N)</td>
<td>7.60</td>
<td>9.60</td>
<td>35.77</td>
</tr>
</tbody>
</table>

Table 2 Maximum radial contact force during rub with different contact stiffness (μ = 0.1)

When taking both two parameters into consideration, a critical curve which is shown in Fig8 can be derived. The area below the curve is the stable area, the area above the area is the unstable area. Friction and stiffness are determined by using different material and different designs. To make sure the rotor would not fall into continuous rubbing with limit device, the contact parameters should be kept in the stable area.

![Figure 8](image8.png)

**Figure 8** Critical curve and stable area of friction coefficient and impact stiffness
4. Conclusions

1. A theoretical model is established using the Coulomb friction model to simulate the rub and collision between rotor and limit device. Rub can excite low-frequency response of this system which decay slowly.

2. Higher friction coefficient and higher impact stiffness produce more possibility of full-annular rub and instability. There is a stable area when taking both impact stiffness and friction coefficient into consideration. We can choose proper material and design to ensure the parameters keep in the steady area to avoid full-annular rub after external perturbation. Limit device is effective only in this area otherwise it will have opposite effect.

REFERENCES


