A NON-CONFORMING COMPONENT MODE SYNTHESIS METHOD FOR EFFICIENT VIBRO-ACOUSTIC ANALYSIS OF PASSENGER CARS

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The efficiency of vibro-acoustic analysis has been a major concern in dealing with large-scale vibro-acoustic systems such as a passenger car, owing to that its finite element model usually contains a huge amount of degrees of freedoms (DOFs) because of its geometrical complexity. To speed up the analyses while keeping desired computational accuracy, this paper develops a modified component mode synthesis method which emphasizes on the coupling techniques of non-conforming structural-acoustic interfaces. The non-conforming interfaces are introduced intentionally by considering that acoustic cavities are usually not necessary to be discretized as finely as structures to deliver equally accurate results. A radial basis function-based, master-slave interpolation formulation is developed to connect the physical field variables across the boundaries of substructures, while the left internal DOFs in each substructure are condensed to fewer modal variables. The computational performances of the proposed method are verified through a passenger car model.

Keywords: finite element method; non-conforming interface; vibro-acoustics

1. Introduction

Vibro-acoustic systems are involved in various engineering applications, e.g. passenger car compartment, airplane cabin, and submarine hull [1, 2]. A good understanding of the structural-acoustic interactions in such systems is an essential step in efforts to reduce the undesired vibration and noise. The finite element analysis is a commonly used method to assist and accelerate this process, which has been widely adopted by both scientific and industrial communities [3, 4]. In order to achieve accurate results, structures and acoustic cavities in a vibro-acoustic system should be finely discretized to a large number of finite elements, resulting in a large-scale finite element model with prohibitive degrees of freedom (DOF). Moreover, the finite element meshes have to be modified or refined along with the changing of underlying geometries, leading to a rather cumbersome solution procedure. Based on such a model, the vibro-acoustic simulations are computationally inefficient, hindering the overall progress of noise and vibration
reduction. Therefore, many efforts have been made to develop numerical methods for efficient analysis of large-scale vibro-acoustic systems.

This paper develops a non-conforming component mode synthesis (CMS) technique for vibro-acoustic analyses. The CMS technique is a classical method in structural dynamics, mainly used for model reduction and DOF condensation. Recently, it has been extended to two-field problems, such as vibro-acoustic analysis and contact analysis [5, 6]. In its applications in vibro-acoustic analysis, the physical DOFs inside the structure and acoustic cavity are respectively condensed to two sets of modal DOFs, while the boundary DOFs are kept unaltered. The structural and acoustical boundary DOFs are in a “node-to-node” correspondence, which facilitates both CMS sub-domain assembly and direct structural-acoustic coupling. However, it is usually not necessary to discretize an acoustic cavity as finely as a structure to deliver equally accurate results. Therefore, to further reduce the total DOF, the fine mesh of acoustic cavity could be replaced by a coarse one. To deal with the resultant non-conforming structural-acoustic interface, a radial basis function-based, master-slave interpolation formulation is developed.

2. Theoretical Formulations

Figure 1 Theoretical model of the vibro-acoustic system:
(a) internal and boundary sets; (b) non-conforming interface and the virtual layer.

The discretized governing equations of a vibro-acoustic system are written as

\[
\begin{bmatrix}
\mathbf{K}_s & -\mathbf{C}_{sf} \\
0 & \mathbf{K}_f
\end{bmatrix}
- \omega^2 \begin{bmatrix}
\mathbf{M}_s & 0 \\
\rho_f \mathbf{C}_{sf}^T & \mathbf{M}_f
\end{bmatrix}
\begin{bmatrix}
\mathbf{u} \\
\mathbf{p}
\end{bmatrix}
= \begin{bmatrix}
\mathbf{f}_s \\
\mathbf{f}_f
\end{bmatrix}
\]

(1)

in which \(\mathbf{K}_s\) and \(\mathbf{K}_f\) are stiffness matrices of the structure and acoustic cavity, respectively; \(\mathbf{M}_s\) and \(\mathbf{M}_f\) are mass matrices of the structure and acoustic cavity, respectively; \(\mathbf{C}\) is the structural-acoustic coupling matrix; \(\rho_f\) is the density of the fluid medium; \(\mathbf{f}_s\) and \(\mathbf{f}_f\) are external forces and sound sources applied to the structure and acoustic cavity, respectively; \(\omega\) is the angular frequency; \(\mathbf{u}\) and \(\mathbf{p}\) are structural displacement and sound pressure, respectively.

The DOFs of the structure and acoustic cavity are divided to two sets: internal set and boundary set., as shown in Fig. 1(a). In order not to break a direct structural-acoustic coupling, the structural DOFs and acoustical DOFs that lie on the coupling interface are chosen to be the boundary sets. Hence, Eq. (1) can be re-written as
Note that we assume no external forces or sound sources are applied to the boundary sets.

According to the fixed-interface component mode synthesis technique, physical variables in the internal sets are condensed to modal variables while those in the boundary sets remain physical. This process is written as

$$\begin{bmatrix}
u_i \\ \nu_b \\ p_i \\ p_b\end{bmatrix} = \begin{bmatrix} \Phi_i^s & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & \Phi_i^f & \Phi_i^c \\ 0 & 0 & 0 & I\end{bmatrix} \begin{bmatrix} \xi_i \\ \eta_i \\ \xi_b \\ \eta_b\end{bmatrix} = \mathbf{T} \begin{bmatrix} \xi_i \\ \eta_i \\ \xi_b \\ \eta_b\end{bmatrix}$$

in which \(\mathbf{I}\) is identity matrix; \(\Phi_i^s\) and \(\Phi_i^f\) are the fixed-interface modal bases, which are obtained by solving the following eigenvalue problems:

$$\mathbf{K}_i^s \Phi_i^s = \Lambda_i^s \mathbf{M}_i^s \Phi_i^s$$

$$\mathbf{K}_i^f \Phi_i^f = \Lambda_i^f \mathbf{M}_i^f \Phi_i^f$$

\(\Phi_i^c\) and \(\Phi_i^f\) are the constraint modal bases, which are expressed as

$$\Phi_i^c = - (\mathbf{K}_i^s)^{-1} \mathbf{K}_i^b$$

$$\Phi_i^f = - (\mathbf{K}_i^f)^{-1} \mathbf{K}_i^b$$

Substituting Eq. (3) to Eq. (2) and pre-multiplying the derived equations by \(\mathbf{T}^T\) yields:

$$\begin{bmatrix} \bar{K}_{ii}^s & 0 & 0 & 0 \\ 0 & \bar{K}_{bb}^b & 0 & -C_{uf} \\ 0 & 0 & \bar{K}_{ii}^f & 0 \\ 0 & 0 & 0 & \bar{K}_{bb}^b\end{bmatrix} - \omega^2 \begin{bmatrix} \bar{M}_{ii}^s & \bar{M}_{ib}^s & \bar{M}_{bb}^s & \bar{M}_{ib}^s \\ \bar{M}_{bi}^s & \bar{M}_{bb}^s & \bar{M}_{bb}^b & \bar{M}_{ib}^b \\ 0 & 0 & \bar{M}_{ii}^f & \bar{M}_{ib}^f \\ 0 & 0 & \rho_0 C_{uf}^c & \bar{M}_{ib}^f \end{bmatrix} \begin{bmatrix} \xi_i \\ \eta_i \\ \xi_b \\ \eta_b\end{bmatrix} = \begin{bmatrix} (\Phi_i^s)^T f_i^s \\ (\Phi_i^c)^T f_i^c \\ (\Phi_i^f)^T f_i^f \end{bmatrix}$$

The submatrices in Eq. (6) are defined as

$$\bar{K}_{ii}^s = \Lambda_i^s \bar{M}_{ii}^s, \quad \bar{K}_{bb}^b = \mathbf{K}_i^b \Phi_i^c + \bar{K}_{bb}^c$$

$$\bar{M}_{bb}^s = \mathbf{I}_{bb}^s, \quad \bar{M}_{bb}^f = (\Phi_i^c)^T (\bar{M}_{bb}^c \Phi_i^c + \bar{M}_{ib}^c \Phi_i^c), \quad \bar{M}_{bb}^c = (\bar{M}_{bb}^c)^T$$

$$\bar{M}_{bb}^o = (\Phi_i^c)^T (\bar{M}_{bb}^c \Phi_i^c + \bar{M}_{ib}^c \Phi_i^c) + (\Phi_i^c)^T (\bar{M}_{bb}^c \Phi_i^c + \bar{M}_{ib}^c \Phi_i^c)$$

Note that \(\diamond\) is “s” or “f” for the structure or acoustic cavity, respectively.

The structural-acoustic coupling matrix in above equations is given by assuming that the structural mesh and acoustical mesh are conforming across their coupling interface. That is, the structural nodes and acoustical nodes are in “one-to-one” correspondence. For the non-conforming case, as shown in Fig. 1(b), we insert a virtual layer \(p_v^c\) between the structural and acoustical boundaries. The mesh of this virtual layer is identical with the structural boundary mesh so that the structural-acoustic coupling matrix could be built straightforwardly as in the conforming case. The only task left is to connect the sound pressures on the virtual layer and the acoustical boundary mesh, which could be done by a master-slave formulation. Specifically, the acoustical boundary mesh is chosen as the master side and its sound pressure is written as
\[ p(x) = \sum_{i=1}^{N_x} c_i \phi(\| x - x_i^m \|) \]  

(8)

in which \( \phi(\| x - x_i^m \|) = e^{-\lambda \| x - x_i^m \|} \) is the radial basis function and \( \lambda \) is the shape parameter. Substituting nodal coordinates of the master side and slave side to Eq. (8) yields

\[ p^M = A^M c \]
\[ p^S = A^S c \]  

(9)

which immediately leads to

\[ p^M = A^M c \]
\[ p^S = A^S (A^M)^{-1} p^M = A_p^M \]  

(10)

where \( A^m \) and \( A^s \) are master-side and slave-side interpolation matrices, respectively.

Based on Eq. (10), the non-conforming structural-acoustic coupling in the finite element context can be expressed as

\[ \delta u^T_b C_{uf} P^f_v = \delta u^T_b C_{uf} P^S = \delta u^T_b C_{uf} A_p^M = \delta u^T_b \overline{C}_{uf} P^M \]  

(11)

Therefore, to implement the non-conforming CMS method, one only needs to replace the conforming structural-acoustic coupling matrix \( C_{uf} \) in Eq. (6) by its non-conforming counterpart \( \overline{C}_{uf} \) in Eq. (11).

### 3. Numerical Example and Discussions

![Figure 2](image_url)

Figure 2 The passenger car vibro-acoustic system: 
(a) dimensions of the structure and acoustic cavity; (b) coordinates of the response points.

A vibro-acoustic system that resembles a passenger car is designed to evaluate the computational performances of the proposed non-conforming CMS method. As shown in Fig. 2(a), the system consists of a thin panel backed by a compartment-like acoustic cavity, the dimensions of which are given along with the figure. The panel is made of aluminium with elastic modulus \( E = 71 \text{GPa} \), poisson ratio \( \nu = 0.3 \), and density \( \rho_s = 2700 \text{kg/m}^3 \). The cavity is filled with air with sound speed \( c = 340 \text{m/s} \) and density \( \rho_f = 1.225 \text{kg/m}^3 \). The panel is simply supported along its edges and a unit force is vertically applied at the point E. The structural displacement at point R1 and the sound pressure level at point R2 will be calculated in the frequency range 1–2000Hz, as shown in Fig. 2(b).

The panel is meshed by 306 MITC4 shell elements, resulting in 342 structural nodes and 2052 DOFs. The cavity is meshed by 7446 hexahedron elements, resulting in 8370 acoustical nodes and 8370 acoustical DOFs. The projection of the acoustical elements on the coupling interface has 250 elements, which is completely non-conforming with the structural elements. For both the structure and acoustic cavity,
all the fixed-interface modes with modal frequencies within 5000Hz are kept in the fixed-interface modal basis. To serve as references, the structural displacement and sound pressure are also calculated based on a conforming finite element model using direct frequency response analysis, rather modal response analysis. The results calculated by the proposed method are given in Fig. 3, along with the referential results. It can be seen the results are in good agreement with the referential results, which validates the accuracy of the proposed method.

Figure 3 Comparison between the results given by the proposed method and the referential model: (a) structural displacement at point R1; (b) sound pressure level at point R2.

The CPU time for carrying out the above simulation based on the reduced model and the full model are given in Table 1. It can be seen that the proposed method requires much less CPU time to complete the simulation, which validates its higher computational efficiency.

Table 1 CPU time required by the reduced model and the full model to complete the simulation

<table>
<thead>
<tr>
<th></th>
<th>Reduced model</th>
<th>Full model</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPU Time (in s)</td>
<td>270</td>
<td>3100</td>
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</table>

4. **Conclusions**

This paper develops a non-conforming component mode synthesis method for efficient simulation of large-scale vibro-acoustic systems. The non-conforming interfaces are introduced intentionally by considering that acoustic cavities are usually not necessary to be discretized as finely as structures to deliver equally accurate results. A radial basis function-based, master-slave interpolation formulation is developed to connect the physical field variables across the boundaries of substructures, while the left internal DOFs in each substructure are condensed to fewer modal variables. Numerical example shows that the proposed method is of better computational accuracy and higher computational efficiency.
REFERENCES


