SCATTERED SOUND FIELD SEPARATION
BASED ON THE EQUIVALENT SOURCE METHOD

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Scattered sound field separation is important for three-dimensional active scattered noise control. However, because the free-field Green's function fails to characterize the spherical scattering transfer function, the equivalent source strength of the solution is incorrect when the equivalent source method (ESM) is applied in near-field acoustic holography. A constrained scattered sound field separation method is proposed that overcomes the application limitations of the ESM. In this paper, a rigid sphere is considered. By constraining the direct sound pressure on the measurement surface and extending the equation to be solved, a more accurate equivalent source strength is obtained. The correctness of the method is shown with a theoretical study, and the feasibility and effectiveness are examined with a numerical simulation.

Keywords: Active noise control; Sound field separation; Equivalent sources model

1. Introduction

Scattered sound separation is the key to active noise control, which can be applied to reduce scattered noise and make objects invisible to detection in a 3D sound field. In 1980, Weinreich and Anold [1] proposed a method for measuring the sound field of a musical instrument by a double-layer spherical measurement. By using the measured spherical harmonic expression of the sound field on the
measurement surface and the independence of the outgoing and incident waves, the scattered sound can be separated from the sound field. Then, Frisk [2] combined near-field acoustic holography (NAH) with Weinreich's method to calculate the underwater reflection coefficient by a space Fourier transform (SFT). Cheng [3] also used an STF to achieve separation of the incident and scattered sound in cylindrical coordinates, and Yu et al. [4] achieved separation of noise sources. However, the STF can only separate a sound source with a particular shape, due to the winding error introduced by the Fourier transform.

The equivalent source method (ESM) was proposed by Koopmann [5], also known as the wave superposition method, and has been widely applied in NAH and wave separation [6, 7]. Bi [8] et al. proposed a sound field separation method employing a double-layer measurement and a single-layer measurement based on the ESM. Simulation and experimental results showed that the error of the separation method was less than 10%, and the method overcame the problems of the SFT. Jacobsen et al. [9] used the ESM to separate the radiated sound field of a plate structure under disturbed conditions, and the separation results were consistent with theory, verifying the effectiveness of the separation algorithm. However, the ESM has never been applied to the scattered sound separation of a rigid sphere.

Considering a rigid sphere, the ESM is applied to the separation of the scattered sound of the rigid sphere in a 3D sound field. Since the free-field Green's function cannot characterize the scattering transfer function, the equivalent source strength of the solution is incorrect. In this paper, by constraining the direct sound pressure at the measurement surface and extending the equation to be solved, a more accurate equivalent source strength is obtained. The performance of the proposed method and the separation results is tested in a simulation.

2. Constrained scattered sound field separation

Figure 1: Geometry of the measurement system and measurement surfaces

Figure 1 depicts the coordinate system of the method and the configurations of the source, rigid sphere, equivalent source surfaces and measurement surfaces. The origin is located at the center of a rigid sphere with a radius of \( r_a \). An arbitrary position in 3D space is described by \( \mathbf{r} = (r, \theta, \varphi) \), where \( r \) is the distance between the origin and the position, and \( \theta, \varphi \) are the elevation angle and azimuth angle, respectively. The equivalent source positions and the reconstruction positions are \( \mathbf{r}_{es} = (r_{es}, \theta_{es}, \varphi_{es}) \) and \( \mathbf{r}_{rec} = (r_{rec}, \theta_{rec}, \varphi_{rec}) \), respectively. In addition, the source is located at \((0, 0, r_0)\).

On account of the rigid sphere, the total sound field consists of the incident sound field and the scattered sound field. In a spherical coordinate system, a spherical harmonic decomposition is applied. Therefore, the total sound field can be expressed as a set of linear combinations of standard orthogonal bases. The incident sound field due to the point source can be described as [10]
where \( a(k) \) is the amplitude of the point source, \( j_n(\cdot) \) is the spherical Bessel function, \( h_n(\cdot) \) is the spherical Hankel function of the first kind, \( Y^m_n(\cdot, \theta, \phi) \) is a spherical harmonic, and \( r_0 \) is the distance between the origin and the point source. The total sound field is given by

\[
p(k, r, \theta, \phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} 4\pi i a(k) b_n(k, r, r_0) h_n(k r_0) Y^m_n(r_0, \theta_0, \phi_0) Y^m_n(r, \theta, \phi).
\]  

(2)

where \( j_n'(\cdot) \) and \( h_n'(\cdot) \) represent derivatives of the first kind, respectively.

As depicted in Figure 1, the pressure on the measurement surfaces is the total sound field. Suppose there are \( M \) measurement points on the measurement surfaces and \( N \) equivalent sources on the equivalent source surfaces. The transfer matrices from the \( N \) equivalent sources to the \( M \) measurement points are \( G_{11}, G_{12}, G_{21} \) and \( G_{22} \). Meanwhile, the total pressure on measurement surface 1 is

\[
p_1 = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} 4\pi i a(k) \left( j_n(k r_1) - \frac{j_n'(k r_0)}{h_n(k r_0)} h_n(k r_1) \right) h_n(k r_0) Y^m_n(r_0, \theta_0, \phi_0) Y^m_n(r_1, \theta_1, \phi_1).
\]  

(4)

and the incident pressure is

\[
p_{21} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} 4\pi i a(k) j_n(k r_1) h_n(k r_0) Y^m_n(r_0, \theta_0, \phi_0) Y^m_n(r_1, \theta_1, \phi_1).
\]  

(5)

Additionally, the total pressure and the incident pressure on measurement surface 2 are

\[
p_2 = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} 4\pi i a(k) j_n(k r_2) h_n(k r_0) Y^m_n(r_0, \theta_0, \phi_0) Y^m_n(r_2, \theta_2, \phi_2).
\]  

(6)

\[
p_{22} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} 4\pi i a(k) j_n(k r_2) h_n(k r_0) Y^m_n(r_0, \theta_0, \phi_0) Y^m_n(r_2, \theta_2, \phi_2).
\]  

(7)

respectively.

The framework of the ESM is to estimate the equivalent source strength according to the measured pressure and then employ a new Green’s function to reconstruct the sound field. Sound field separation can also be achieved by the ESM, and the expression is shown as [11]

\[
\begin{bmatrix}
p_1 \\
p_2
\end{bmatrix} =
\begin{bmatrix}
G_{11} & G_{21} \\
G_{12} & G_{22}
\end{bmatrix}
\begin{bmatrix}
Q_1 \\
Q_2
\end{bmatrix}.
\]

(8)

where \( p_1 \) and \( p_2 \) are the total pressure on the measurement surfaces, and \( Q_1 \) and \( Q_2 \) are the equivalent source strengths. \( G \) represents the free-field Green’s function, and the subscripts denote the equivalent source surface 1 or 2 and the measurement surface 1 or 2. However, since the free-field Green’s function cannot characterize the spherical scattering transfer function, the equivalent source strength of the solution is incorrect, and the scattered sound field cannot be separated.

Therefore, this paper proposes a method that constrains the direct sound pressure on the measurement surface, extending the separation equation to obtain a more accurate equivalent source strength. Then the equation in (8) becomes

\[
\begin{bmatrix}
p_1 \\
p_2
\end{bmatrix} =
\begin{bmatrix}
G_{11} & G_{21} \\
G_{12} & G_{22}
\end{bmatrix}
\begin{bmatrix}
Q_1 \\
Q_2
\end{bmatrix}.
\]

(9)

(9)

However, the number of equivalent sources is more than the number of measurement points; therefore, equation (9) is underdetermined. To overcome this shortfall, Tikhonov regularization is applied for the
minimization: \( \min \{ \| P - GQ \|_2^2 + \lambda^2 \| Q \|_2^2 \} \). Then, the optimal expression of the equivalent source strength is shown as
\[
Q = (G^H G + \lambda^2 I)^{-1} G^H P.
\]
where \((\cdot)^H\) denotes the conjugate transpose, \(\lambda\) is the regularization parameter, and \(I\) is the identity matrix.

3. Simulations

In this section, several simulations are performed to assess the efficiency of the scattered sound field separation method. The parameters are as follows: the radius of the rigid sphere is 0.5 m, and the frequency of the point source is taken from 100 to 1000 and 3000 Hz. The double-side array is 1 m \(\times\) 1 m with uniform distributed speakers, and the array spacing is 0.1 m. The origin is located at the center of the rigid sphere, as shown in Figure 1. Meanwhile, the center of the source, the center of the array and the center of the rigid sphere are on the same horizontal line. In the simulations below, the source is located at (0,0,0.7), and the distances between the measurement surfaces and the origin are 0.61 m and 0.63 m, respectively. The construction surface is measurement surface 1. The signal-to-noise ratio (SNR) is 40 dB, and the truncated length is designed according to [12]. In addition, the radius of the rigid sphere is much larger than the size of the point source so that diffraction from the edge is ignored.

To examine the efficiency of the proposed method, three frequencies are chosen in the simulation below: 300 Hz, 1000 Hz and 3000 Hz. The two subplots in the figure represent the true scattered sound and the reconstructed result. The total error is defined as
\[
\text{Error} = \frac{\| P_{\text{true}} - P_{\text{rec}} \|_2}{\| P_{\text{true}} \|_2} \times 100\%.
\]
where \(P_{\text{true}}\) and \(P_{\text{rec}}\) are the true scattered sound and the reconstructed sound, respectively.

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Figure 2: The simulation results of the proposed method with Tikhonov regularization. The colorbar corresponds to the sound pressure level (SPL), scaled to dB; (a) is the result for 300 Hz; (b) is the result for 1000 Hz; and (c) is the result for 3000 Hz.

Figure 2 shows that the reconstruction maps of the method are close to the true maps when the source frequency is 300 Hz, 1000 Hz and 3000 Hz, and the total error rates are 2.48%, 1.68% and 6.51%, respectively. Due to the use of Tikhonov regularization, the equivalent source strength is inaccurate at a high frequency, which results in larger error rates. In addition, the source is close to the rigid sphere, so the SPL is much larger at the center of the maps. The small squares in each map represent the position. If the number of reconstruction points is increased, the map will be smoother.

Figure 3 shows the error rate of the proposed method; the error is less than 5%. However, the error at 100 Hz is larger than that at other frequencies. The scattered sound at 100 Hz is small, and the SNR is small, which produces this phenomenon. The simulation results prove that the proposed method is feasible and effective at frequencies below 1000 Hz. In addition, at a high frequency, the proposed method can also achieve good agreement with the theory, and the error is less than 10% at 3000 Hz.

Figure 3: The error of the proposed method from 100 Hz to 1000 Hz.

4. Conclusion

In this paper, a constrained scattered sound field separation method is proposed. By constraining the direct sound pressure on the measurement surface and extending the separation equation, a more accurate equivalent source strength is obtained. The simulation results show that the separation method achieves
good agreement with theory below 1000 Hz, and the error is less than 5%. In addition, the proposed method also performs well at a higher frequency. However, to acquire a better error at high frequencies, Tikhonov regularization can be replaced by other algorithms, such as the steepest decent method and the weighted iterative method.

The scattered sound separation method proposed in this paper can effectively solve the problem of low-band scattered sound separation, which will enable active noise reduction of scattered sound from a rigid sphere and make objects invisible to detection. These results will play an important role in underwater active target detection and antisubmarine techniques.

REFERENCES