ACTIVE VIBRATION CONTROL USING A STROKE LIMITED INERTIAL ACTUATOR

Mattia Dal Borgo, Maryam Ghandchi Tehrani and Stephen John Elliott
Institute of Sound and Vibration Research, University of Southampton, Highfield, Southampton, UK
email: M.Dal-Borgo@soton.ac.uk

Inertial actuators can be used with velocity feedback controllers to reduce structural vibration, however, their finite stroke length can affect the behaviour and stability of the control system. Stroke saturation not only limits the amount of force available from the actuator but also causes the proof mass to suddenly decelerate, causing impulse-like excitations that are transmitted to the structure and may result in damage. The shocks produced by these impacts are in phase with the velocity of the structure, leading to a reduction of the overall damping of the system, which eventually destabilises the system. This paper examines the implementation of a nonlinear feedback controller to avoid collisions of the proof mass with the actuator’s end-stops, thus preventing this instability. A nonlinear feedback control strategy is then presented, which actively increases the internal damping of the actuator only when the proof mass approaches the end-stops. The nonlinear control strategy is investigated both theoretically and experimentally for the control of a cantilever beam, and a comparison in terms of stability is made when both control loops or only the velocity feedback loop are present. Finally, a virtual sensing approach based on an extended Kalman filter algorithm is discussed for the real-time estimation of the proof mass states that are used to calculate the feedback signal of the nonlinear control law. It is shown that larger velocity feedback gains can be used without the system becoming unstable when the nonlinear feedback loop is adopted.

Keywords: nonlinear feedback control, state estimation

1. Introduction

Velocity feedback controllers (VFCs) are a widely used active solution that can increase the effective damping of a structure, reducing its level of resonant vibration [1]. A VFC typically consists of an electromagnetic inertial, or proof mass, actuator attached to a structure, a collocated vibration sensor and a controller, which feeds back the velocity of the structure to the actuator.

The internal dynamics of the inertial actuator is known to affect the stability and performance of the VFC, which then becomes only conditionally stable [2], so that there exists a maximum velocity feedback gain, above which the control system becomes unstable [3]. Previous research has established that higher feedback gains can be used if the inertial actuator has a low natural frequency and a well damped resonance [3].

On the other hand, this increases the proof mass static displacement and its response to low frequency excitations. In practical applications, however, the displacement that the proof mass can reach is limited by the stroke length between the end-stops of the actuator [4]. For very low frequency motions or high
input currents, the proof mass can saturate in stroke as it hits the end-stops imparting large shocks to the structure, which may be damaged.

Moreover, it has been observed both theoretically and experimentally that this nonlinear dynamic behaviour is also undesirable in terms of the stability of the closed-loop control system, because it can reduce the stability margin of the velocity feedback loop, and in fact, enhance the level of vibration [5, 6]. The instability is due to the forces imparted during the collisions between the proof mass and end-stops being in-phase with the velocity of the structure, hence reducing the overall damping of the system. This motivates for having accurate models of proof mass actuators and in particular of the nonlinearities that may affect the dynamics of these devices at low frequency or large excitation signals [7].

This paper presents a theoretical and experimental study of a nonlinear feedback control (NLFC) strategy to prevent stroke saturation of inertial actuators. The proposed control strategy acts as a second loop alongside the linear VFC and implements a simple nonlinear control law using the proof mass relative velocity signal. The implementation of the control law requires real-time knowledge of the velocity and displacement of the proof mass, which often cannot be measured directly with a physical sensor. Thus, a virtual sensing approach is investigated based on an extended Kalman (EKF) filter algorithm [8].

This paper is organised in three main sections. Section 2 presents the theoretical analysis of the nonlinear feedback control law. Section 3 investigates the experimental implementation of the NLFC on a control unit attached to the free end of a cantilever beam. Section 4 introduces a state estimation approach for the internal states of the proof mass actuator. The conclusions are summarised in Section 5.

2. Nonlinear feedback control: theoretical analysis

This section presents a nonlinear strategy that accounts for the nonlinear dynamics of the stroke limited inertial actuator, enhancing the stability of the control system whilst maintaining the vibration attenuation performance provided by the VFC. A theoretical analysis of a stroke limited inertial actuator attached to a single degree of freedom (SDoF) structure, driven by either a VFC or a combination of the VFC and NLFC feedback loops is presented. The mathematical model of the nonlinear actuator, the structure and the feedback control system is first derived in a state space form. Then, the stability of the closed-loop VFC system is compared with the stability of the closed-loop VFC+NLFC system.

2.1 Mathematical model

Figure 1 shows a lumped parameter model of a SDoF system connected to a stroke limited inertial actuator, where the SDoF may represent the first mode of a real structure. The proof mass and structural mass displacements are denoted as \( x_p \) and \( x_s \), respectively. Also, \( x_r = x_p - x_s \) represents the deflection of the proof mass from its resting position. The proof mass \( m_p \) is connected to the structural mass \( m_s \) via a damping coefficient \( c_p \) and a nonlinear stiffness that models the physics of stroke saturation [7], which is given by,

\[
\kappa(x_r) = \begin{cases} 
        k_p & |x_r| < x_0 \\
        k_p + k_c \left( 1 - \frac{x_0}{|x_r|} \right) & |x_r| \geq x_0
\end{cases}
\]

where \( k_p \) is the underlying linear suspension stiffness, \( k_c \) is the end-stops stiffness and \( x_0 \) is the stroke length. The electrical winding of the actuator can be modelled as a series inductor \( L_e \) and resistor \( R_e \) and the voltage across the coil terminals is denoted as \( e_a \). The structural mass is connected to the ground via the stiffness and damping parameters \( k_s \) and \( c_s \), respectively. The structure is subject to the external, or primary, force \( f_s \) and the control, or secondary, force \( f_{a,s} = -\phi \dot{q} \) that is generated by the actuator transducer, where \( \phi \) is the transduction coefficient. The VFC loop is assumed to be implemented by scaling the structure’s velocity \( \dot{x}_s \) by a gain \( h_s \) and feeding this back to the actuator as the input current.
signal $i_a$. A NLFC loop is added alongside the VFC loop in Fig. 1, and is defined as a nonlinear function of the relative proof mass velocity and displacement $\psi(x_r, \dot{x}_r)$.

![Image of schematic model](image)

Figure 1: Lumped parameter model of the structure, nonlinear inertial actuator, velocity feedback controller (VFC) and nonlinear feedback controller (NLFC).

The equations of motion of the system in can be expressed in a state space form as follows,

$$\begin{align*}
\dot{x} &= Ax + Be \, f_e + Ba \, f_a, \\
y &= Cx,
\end{align*}$$

(2)

where $x$ is the state vector, $A(x)$ is the state dependent system matrix defined as,

$$A(x) = \begin{bmatrix} 0 & I \\ -m^{-1}k(x) & -m^{-1}c \end{bmatrix},$$

(3)

where $0$ and $I$ are the 2-by-2 null matrix and identity matrix, respectively. The mass matrix is denoted by $m$, the damping matrix is denoted by $c$ and the nonlinear stiffness matrix is defined as,

$$k(x) = \begin{bmatrix} k_s + \kappa(x) & -\kappa(x) \\ -\kappa(x) & \kappa(x) \end{bmatrix},$$

(4)

where $\kappa(x)$ is given by Eq. (1). The input vectors of the primary and secondary excitations are $B_e$ and $B_a$, respectively. The output vector is named $y = [x_s \quad \dot{x}_s]$ and the output matrix is denoted by $C$. The feedback control system, as shown in the schematic of Fig 1 is made by closing the loop around the output vector, thus the input current that drives the actuator becomes,

$$i_a = g_a \, H \, y,$$

(5)

where $g_a$ is the amplifier gain, which is assumed to be unity throughout the thesis and $H$ is the matrix of control gains. For a single VFC loop on the structure’s velocity, the matrix of feedback gains can be written as,

$$H_{vfc} = [h_s \quad 0],$$

(6)

hence, substituting Eq. (6) into Eq. (5) the driving current of the inertial actuator becomes,

$$i_{a,vfc} = g_a \, H_{vfc} \, y = h_s \dot{x}_s.$$  

(7)
For a single NLFC loop on the relative proof mass velocity, instead, the matrix of feedback gains can be rewritten as,

$$ H_{nlfc} = \eta_r(x_r) - \eta_r(x_r), \quad (8) $$

where

$$ \eta_r(x_r) = \frac{n_r}{(x_0 - |x_r|)^{2p} + b}, \quad (9) $$

is a nonlinear gain that depends on the proof mass relative position, \( n_r \) is the feedback gain of the nonlinear controller, \( b \) is a limitation parameter and \( p \) is an exponent parameter. For the simulations the parameters of Eq. (9) has been set to \( p = 1, b \) and \( n_r \) have been calculated to give a maximum damping of 10 N/ms and a minimum damping of 0.1 N/ms. Substituting Eq. (8) into Eq. (5) the driving current of the inertial actuator becomes,

$$ i_{a,nlfc} = g_a H_{nlfc} y = -\eta_r(x_r) x_r = -\psi(x_r, x_r). \quad (10) $$

For a double feedback loop, hence, combining the VFC loop with the NLFC loop, the matrix of feedback gains becomes,

$$ H_{vfc+nlfc} = H_{nlfc} + H_{vfc}. \quad (11) $$

The closed-loop control system block diagram of the state space Eq. (2) is illustrated in Fig. 2, where the driving current is given by the combination of both the VFC and NLFC loops.

Figure 2: Block diagram of the closed loop control system with both the VFC loop and the NLFC loop.

Considering a general matrix of feedback gains \( H \), the control force applied to the structure by the closed-loop system can be written as,

$$ f_{a,s} = -\phi_a = -g_a HCx, \quad (12) $$

where Eqs. (2,5) have been used. The state equation can thus be rewritten as,

$$ \dot{x} = A_0(x)x + B_e f_e, \quad (13) $$

where the closed-loop state dependent system matrix \( A_0(x) \) is derived as follows,

$$ A_0(x) = [A(x) - g_a \phi B_a HC]. \quad (14) $$

The parameters used in the numerical analysis of the system shown in Fig. 1 are provided in Table 1.
Table 1: Table of model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$m_s$</th>
<th>$k_s$</th>
<th>$c_s$</th>
<th>$m_p$</th>
<th>$k_p$</th>
<th>$c_p$</th>
<th>$x_0$</th>
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<td>N/ms$^{-1}$</td>
<td>mm</td>
<td>N/A</td>
<td>A/ms$^{-1}$</td>
</tr>
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2.2 Stability analysis

The stability of the closed-loop nonlinear control system is dependent on both the relative proof mass displacement and the velocity feedback gains. This dependency can be analysed through the Lyapunov exponents of the nonlinear system [9, 10]. This technique allows to investigate the behaviour of the system around an initial condition $\tilde{x}$ [10]. The local stability is analysed by evaluating the eigenvalues of the Jacobian matrix of the nonlinear state space equation calculated at the initial condition $\tilde{x}$.

![Figure 3: Real part of the maximum eigenvalue of the closed-loop matrix $A_{0,vfc}(\tilde{x})$ without NLFC for several proof mass displacements $x_r$ and feedback gain $h_s$. The stroke limit is indicated with the red dashed line. The magenta solid line delimits the stable/unstable region.](image)

If the eigenvalues of the Jacobian matrix have negative real part, then the nonlinear system is locally stable, if, instead, the eigenvalues of the Jacobian matrix have positive real part, then the nonlinear system is locally unstable [10]. The Jacobian matrix of the nonlinear state space equation is given by the closed-loop system matrix of Eq. (14). The stability of the closed-loop VFC system can be studied by substituting Eq. (6) into Eq. (14) and calculating the eigenvalues of the matrix for several proof mass relative displacements. If the real part of the largest eigenvalue is lower than zero, the system is stable. The real part of the maximum eigenvalue of $A_{0,vfc}$ for several proof mass displacements around the stroke limit region is displayed in Fig. 3, where the stroke limit is represented with the red dashed line. Figure 3 shows that the nonlinear system is stable for displacements within the stroke limits and feedback gains below $h_{s,max}$, as expected. Also, the system becomes unstable for $h_s > h_{s,max}$ regardless of the relative displacement. The delimiter of instability $\Re \{ \lambda_{max} [A_{0,vfc}(x_r,h_s)] \} = 0$ shows that for a particular feedback gain $h_s < h_{s,max}$ there exists a relative displacement $|x_r| > x_0$ above which the system becomes unstable, and the higher is the feedback gain, the lower is the relative displacement required to make the system unstable. The stability of the closed-loop system using the combination of VFC and NLFC can also be analysed through the Lyapunov exponents as shown for the single VFC loop by substituting Eq. (11) into Eq. (14) and calculating the eigenvalues of the Jacobian matrix for several proof mass relative displacements. The real part of the maximum eigenvalue of $A_{0,vfc+nlfc}$ for several proof mass displacements around the stroke limit region is displayed in Fig. 4. A comparison between Fig. 4 and Fig. 3 shows that the NLFC enlarges the stability region of the system in terms of relative proof mass displacements. In fact, the delimiter of instability $\Re \{ \lambda_{max} [A_{0,vfc+nlfc}(x_r,h_s)] \} = 0$ for the VFC+NLFC loop is shifted to larger relative displacements with respect to the one of single VFC loop.
3. Nonlinear feedback control: experimental implementation

This section examines the experimental implementation of the VFC and NLFC loops for the vibration reduction of the first mode of a cantilever beam using the stroke limited inertial actuator presented in [7].

A schematic of the experimental set-up is shown in Fig 5, which consists of an aluminium cantilever beam and a control unit attached at the nodal point of the second mode of the beam. The control unit comprises the inertial actuator, a collocated accelerometer and a force sensor. The external excitation is given by an instrumented hammer on the free end of the beam. The acceleration signal measured by the accelerometer is high-pass filtered and integrated to obtain the velocity. The velocity of the structure is then amplified by a fixed VFC gain. The signal measured by the force cell, instead, is combined with the inertia of the total mass attached to the control point and is used to calculate the relative acceleration of the proof mass. The proof mass states are then calculated by high-pass filtering and integrating the acceleration signal. The NLFC input signal to the actuator is then calculated in real-time using Eq. (9).

The results of the experimental implementation are shown in Figs. 6(a,b). Figure 6(a) shows a comparison between the uncontrolled scenario (red dashed line), the VFC case using \( h_s = 42\% h_s^{\max} \) (black dash-dotted lines) and the VFC+NLFC case also with \( h_s = 42\% h_s^{\max} \) (blue solid lines), where \( h_s^{\max} \) is the maximum gain calculated with the Nyquist criterion neglecting the nonlinearity. Figure 6(a) shows the spectrum of the excitation force and the response at the control point. It can be seen that implementing the VFC with that level of excitation and feedback gain leads to instability. If the NLFC is also implemented under the same conditions, the system remains stable. A parametric study has been carried out to
evaluate the stability of VFC and VFC+NLFC for increasing levels of excitation and increasing values feedback gain. The results are presented in Fig. 6(b), where the red circles are the experimental data points in which the VFC is stable, and the black asterisks are the experimental data points in which the VFC plus NLFC is stable.

Figure 6: (a) Spectrum of the excitation and time response. Red dashed line for the uncontrolled scenario, black dash-dotted line for VFC only, with $h_t = 42\% h_{t,\text{max}}$, blue solid line for NLFC switched on; (b) Comparison of the stability range: red circles is where the VFC is stable, black asterisks is where VFC+NLFC is stable.

4. Proof mass state estimation

Often the proof mass cannot be directly instrumented. Hence, its velocity can be estimated using a virtual sensor based on the available measurements and the mathematical model of the actuator. This section, presents an extended Kalman filter (EKF) algorithm [8] that calculates the optimal estimate of the proof mass velocity using the nonlinear model of the actuator and the voltage, current and base acceleration signals available from the measurements.

Figure 7: Time histories of the proof mass relative velocity estimated using the force gauge (solid black line) and with an extended Kalman filter algorithm (dash-dotted red line) when experiencing limit cycle oscillations.

The dynamic equations of the nonlinear inertial actuator in Fig. 1 can be written as,

\[
\begin{align*}
  m_p \ddot{x}_r &+ c_p \dot{x}_r + \kappa(x_r) \dot{x}_r = \phi a_m - m_p \ddot{x}_s, \\
  e_a &= \phi \dot{x}_r + R_e i_a
\end{align*}
\]

which can be rewritten in a state space form as,

\[
\begin{align*}
  \dot{x} &= Ax + Bu + W \\
  y &= Cx + Du + V
\end{align*}
\]
where the state vector is \( \mathbf{x} = [x_r \ x_r']^T \) and the input vector is \( \mathbf{u} = [\dot{x}_a \ \dot{x}_s]^T \). \( \mathbf{W} \) is the covariance matrix for the process noise, whereas \( \mathbf{V} \) is the covariance matrix for the measurement noise.

The EKF algorithm linearises the system at each time step, it predicts the state values using the inputs to the system, and then it updates the state estimation using the output measurement value. The EKF approach presented in this section is compared to the case where the proof mass velocity was calculated from the force gauge measurement. Figure 7 shows the response of the system when the actuator is subject to a limit cycle oscillation. In this scenario, the time history of the proof mass velocity given by the EKF algorithm matches well with the one measured with the force gauge except for a small time delay at the beginning of the measurements.

5. Conclusions

This paper presented a NLFC that aims to prevent stroke saturation instability and its implementation using a virtual sensor is discussed. Firstly, the mathematical model of a nonlinear inertial actuator connected to a single degree of freedom structure has been derived, where the nonlinearity has been modelled as a piecewise linear stiffness, consistently with the study in [7].

The proposed NLFC actively increases the internal damping of the actuator as the proof mass approaches the end stops. The stability of the nonlinear system with either a single VFC loop or a combination of VFC and NLFC has been studied using the Lyapunov exponents approach. It is shown that the NLFC is able to increase the safe operating region of the actuator with respect to the single VFC loop.

Secondly, the experimental implementation of the NLFC on a control unit attached to a cantilever beam has been introduced. Finally, it has been shown that an EKF algorithm based on the nonlinear model of the inertial actuator can accurately estimate the proof mass velocity. Future work may be related to the experimental implementation of such virtual sensor algorithm.

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