REPRODUCIBILITY OF MODAL PARAMETERS USING DIFFERENT EXCITATION TYPES

Eric Hensel and Björn Knöfel
Fraunhofer Institute for Machine Tools and Forming Technology, Dresden, Germany

Experimental acquisition techniques of modal parameters are well-known topics in the dynamic characterization process of steel structures. The objective is to determine the structure’s natural vibration properties, which are eigenfrequencies, corresponding mode shapes and damping. Mostly, the experimentally obtained modal parameters are used to verify numerical models, which can be utilized e.g. for overall system vibration simulations. The modal parameters usually were taken from single measurements and thus, few information about the parameter variances are available. In experimental modal analysis, the choice of the boundary condition (e.g. free or grounded support) as well as the corresponding excitation type (e.g. impulse hammer or shaker) may impact the variance of the obtained modal data. The present article focuses on the differences in reproducibility of modal parameters using an impact hammer and a modal shaker, where measurements are carried out under free/free boundary conditions. The test object is a gear wheel, which first eigenfrequency occurs in a frequency range around 4.6 kHz. During the experimental modal analysis, the frequency response functions are obtained up to 20 kHz. Due to the high frequency range, a 3D Scanning Laser Doppler Vibrometer is used to measure the vibrations velocities in all configurations. In a first test setup, statistical variations of a single gear wheel’s modal parameters are analyzed and special attention will be payed on the reproducibility of the excitation force using an automated impulse hammer. In a second step, repeated measurements of modal parameters of one gear wheel are presented followed by a comparison of measurements on three identical gear wheels of one lot. Finally, repeating measurements are carried out using a modal shaker for system excitation and the corresponding modal parameters are compared to the results obtained using an impact hammer.

Keywords: Experimental Modal Analysis, Statistical Variance

1. Introduction

The dynamic behavior of a structure can be characterized by its natural vibration properties respectively by its modal parameters which are represented by natural frequencies (eigenfrequencies), vibration mode shapes and damping factors (cf. [1]). In general, there are two different ways to obtain the modal parameters of an object. Numerical modal analyses are often used within the design phase of a product. The corresponding influencing parameters, such as discretization or modeling approaches, are well-known and in most cases, a sensitivity analysis can clarify the interaction between input parameters and simulation results.
The other way to obtain a structure’s natural vibration characteristics bases on measurements. An experimental modal analysis (EMA) can be carried out using different test setups regarding excitation, support as well as the technique of measuring the system’s response to the corresponding excitation which all can affect the natural vibration properties of the investigated object. Like all measurements, EMAs are liable to variations as well. For example, these variations may occur due to inaccuracies during the assembly of the test setup or due to environmental influences. The following investigations deal with variations of EMAs and the corresponding impacts on the obtained modal parameters. Within the present paper, the evaluation of differences in mode shapes as scalable eigenvectors are dropped. For the underlying test structure which is described below, all mode shapes are nearly identically. Hence, the usage of the modal assurance criterion for example is not expedient. The measured frequency response function (FRF) between excitation and response point will be used instead. However, the two remaining modal parameters, natural frequency and damping factor, are content of the following investigation.

At first, the differences of modal parameters due to the chosen test setup are presented. The excitation of the system is analyzed in detail to ensure that there are no significant influences while exciting the corresponding system. The results of the impact excitation variance are followed by investigations on the differences of modal parameters of a single test structure for repeating assembly and disassembly of the test setup. Subsequently, the variances of three geometrical identically objects are compared. The last section of the paper consists of the comparison variances in modal parameters obtained by shaker and impact hammer excitation.

Numerical and experimental modal analysis methods are well-known for decades and thus, ambitious research work has been carried out which probably will fill specialized libraries if they were arranged at the same place. In case of the current paper, selected research works dealing with the statistical deviations of modal parameters are presented in a concentrated manner. Ndambi et al. compared different modal analysis techniques of concrete structures in 2000 [2]. Their work contains the comparison of modal parameters obtained by impact hammer as well as shaker excitations with different waveforms (sine and random). The results demonstrate that there are differences in the determined damping ratios especially for the comparison of impact hammer and shaker measurements up to 500 Hz. Additionally, slight differences can be found for the eigenfrequencies. In 2006, Pape and Adhikari acquired modal data of 100 nominally identical beam structures within a frequency range up to 1 kHz [3]. The system excitation has been carried out by impacts generated by an electromagnetic shaker equipped with a steel tip. The comparison includes FRFs from the excitation location to three response degree of freedom as well as the corresponding natural frequencies. Especially for some modes within their considered frequency range, conspicuous differences can be found. Finally, the work of Baqersad et al. will be listed at this point [4]. For frequencies up to 70 Hz, the modal parameters of a rotor blade system have been calculated for three different measurement strategies (multiple input multiple output, single input single output and impact excitation). The comparison denotes a good correlation for the investigated methods in terms of frequency and damping.

The current paper differs in some points from the described research works done in the past and listed above. Essentially, the frequency range is extended up to approximately 22 kHz and the investigations focus on the deviations which can occur in modal testing.

2. Test Setup

The EMA test setup can be seen in fig. 1 (right side). A gear wheel is mounted under free boundary conditions (on soft absorbing material, type CARUSO-ISO-BOND®) while an automated impact hammer (MAUL-THEET GmbH, type: vImpact-60) excites the gear wheel in the transversal z direction. This configuration enables a high reproducibility of the injected force impulse and the force vector. On the left
Figure 1: EMA test setup with impact hammer excitation and scan point mesh (SLDV).

side of the figure the scan point mesh is demonstrated which is utilized to measure the surface vibration velocities of the wheel with a 3D Scanning Laser Doppler Vibrometry (SLDV) (Polytec GmbH, type: PSV-400-3D). The focus of the following investigations is on the first 6 out of plane mode shapes which occur in a frequency range between 4.6 kHz and 21.7 kHz.

3. Evaluation of Excitation Spectrum

First aspect of the statistical observations is to look deeper into the reproducibility of the excitation force at one gear wheel. Fig. 2 points out the time history of 3 out of 143 force impulses which were taken on one gear wheel with an interval time of 1.5 s between two consecutive hits. Fig. 3 illustrates the force distribution at force maxima in a boxplot with an interquartile distance of factor 1.5. Notwithstanding the 6 points below the 79 N whisker, the deviations within the 143 hits while exciting the gear wheel are insignificant since 95.8 % of the hits (137 out of 143) are in a range which scatter at maximum 1.6 % (1.3 N) around the mean value of the average force (79.8 N).

In the frequency range, the spread of different force spectra are presented in fig. 4. In this aspect, three identical gear wheels where taken which were each measured 13 times. After each of the in total 39
measurements, the current gear wheel has been replaced by one of the two remaining gear wheels. The mean force spectrum is plotted with a red line while the area between $F_{\text{min}}$ and $F_{\text{max}}$ varies from 0.5 dB at 15 kHz to 1.4 dB at 25 kHz. The scatter of the force spectra are due to small differences in the alignment between impulse hammer and test object, which have been rearranged for each of the 39 measurements. Since the frequency range under investigation is assumed to have a linear system behavior, meaning that the response of the system is proportional to its excitation, the deviations shown are not thought to be crucial for the estimation of modal parameters of different gear wheels.

4. Deviation of Modal Parameters of one Gear Wheel

Considering one and the same gear wheel, modal parameters may vary during a series of $N = 19$ repeated repositionings of the wheel on the soft support an the corresponding alignments from the impact hammer. The varying modal parameters which were examined, are the eigenfrequency $f_i$ and the loss factor $\eta_i$, whereby the amplitude of the averaged transfer admittance $Y_i$ (unit dB, $Y_0 = 1 \cdot 10^{-6}$ N) is considered as well but not meant to be a modal parameter in an narrower sense. Within the following descriptions, the amplitude of the averaged transfer admittance is abbreviated with amplitude of admittance. The explicit indication of a transfer admittance will be omitted. Eigenfrequencies and loss factors have been derived from curve fittings of each of the 19 single measurements. The averaged eigenfrequencies of the first $I = 6$ out of plane modes are listed in tab. 1.

<table>
<thead>
<tr>
<th>mode $i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency $f$ / kHz</td>
<td>4.61</td>
<td>8.26</td>
<td>10.38</td>
<td>13.16</td>
<td>16.18</td>
<td>21.74</td>
</tr>
</tbody>
</table>

At first, the average values of eigenfrequency, loss factor and amplitude of admittance were calculated for each of the modes according to eq. 1.

$$\bar{x}_i = \frac{1}{N} \sum_{n=1}^{N} x_i^n \quad i = 1 \ldots 6$$  \hspace{1cm} (1)
Using eq. 2

\[ X = \{ x_1^1 - \bar{x}_1, \ldots, x_1^N - \bar{x}_1, x_2^1 - \bar{x}_2, \ldots, x_2^N - \bar{x}_2, \ldots, x_I^1 - \bar{x}_I, \ldots, x_I^N - \bar{x}_I \} , \quad (2) \]

a total sample number is achieved by \( N \cdot I = 114 \), where the variances of all out of plane modes were regarded instead of the 6 single modes. Tab. 2 lists the averages \( \bar{x} \) and the standard deviation \( \sigma \) of each of the 3 considered quantities. A test for normal distribution has been undertaken with the Kolmogorov-Smirnov test using the software Cornerstone. Fig. 5 depicts the corresponding histograms of eigenfrequencies, loss factors and amplitudes of admittance. The two latter quantities are normally distributed, while the eigenfrequencies show a light positive skew. Tab. 2 also highlights at a confidence level of 95% the p-values, which are measures, how close the measurements fit to a normal distribution. With a \( p \) value of less than 5% the measured values do not fit to a normal distribution, what is the case at the eigenfrequency distribution.

The eigenfrequencies can be characterized with its span width \( R_{f,i} \), for each of the 6 out of plane modes which were calculated as difference of the maximum and minimum value for each mode according to eq. 3

\[ R_{f,i} = f_{i,\text{max}} - f_{i,\text{min}} . \quad (3) \]

The maximum span width within the measured eigenfrequencies is 2.2 Hz, which is less than 0.05% deviation compared to the first eigenfrequency at 4.6 kHz. For this reason, the deviation of the eigenfrequency distribution from a normal distribution can be neglected.

<table>
<thead>
<tr>
<th>parameter</th>
<th>( \bar{x} )</th>
<th>( \sigma )</th>
<th>( 2\sigma )</th>
<th>( p ) value</th>
<th>norm. distr.?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f / \text{Hz} )</td>
<td>(-1.17 \cdot 10^{-5})</td>
<td>0.44</td>
<td>0.89</td>
<td>( \leq 0.01 )</td>
<td>no</td>
</tr>
<tr>
<td>( \eta / - )</td>
<td>(-4.41 \cdot 10^{-11})</td>
<td>3.27 ( \cdot 10^{-6})</td>
<td>6.55 ( \cdot 10^{-6})</td>
<td>0.1 \ldots 0.25</td>
<td>yes</td>
</tr>
<tr>
<td>( Y' / \text{dB} )</td>
<td>(2.05 \cdot 10^{-7})</td>
<td>0.54</td>
<td>1.09</td>
<td>( &gt; 0.25 )</td>
<td>yes</td>
</tr>
</tbody>
</table>

Figure 5: Histograms of eigenfrequencies \( f \), loss factor \( \eta \) and admittance amplitude \( Y' \).
Leaving the distributions and focusing on the single modes, fig. 6 depicts the loss factors for the 6 out of plane modes for gear wheel 1 with an error bar interval of $\pm 2\sigma$ which is induced by the repeated assembly and disassembly of the measurement setup. Finally, fig. 7 demonstrates the development of the calculated average loss factors in dependence of the number of conducted measurements for the considered 6 out of plane modes. The graphs reveal a convergence behavior of the average loss factors whereby each average curve never deviates more than 5% from its final average of 19 measurements.

5. Comparison of Modal Parameters of three Gear Wheels

Knowing the statistical distribution of one gear wheel raises the question of the deviations among a series of identical gear wheels. The procedure of section 4, has been repeated with two further gear wheels, so in total $13 \cdot 3 = 39$ eigenfrequencies, loss factors and amplitudes of admittance are available due to the repeated assemblies and disassemblies of the gear wheels in the test setup. In fig. 8 the loss factors for the three gear wheels are plotted. Again, the error bars indicate a range of $\pm 2\sigma$, where, according to the normal distribution, 95.4% of all measured values are located. Fig. 8 underlines the frequency
dependence of the loss factor which has been seen in fig. 6 before and which started from $3 \cdot 10^{-4}$ at 4.61 kHz to $1 \cdot 10^{-4}$ at 21.74 kHz. Furthermore, the three curves demonstrate only small variations of the loss factor among the three identical gear wheels.

6. Comparison of Shaker and Impact Hammer Excitation

As stated in the introduction section 1, the results obtained by impact hammer excitation are compared to modal parameters determined from shaker measurements. The test setup for the shaker measurements is identically to the impact hammer test setup (cf. section 2 and fig. 1). The only difference is the type of excitation. In contrast to the test setup with the impact hammer, a shaker has been applied at the same position which is denoted as the driving point in fig. 1. The shaker used within the measurements was a piezoelectric high frequency actuator (dynamics mechanics, type: dm2) with an operational frequency range from $\approx 3.5$ kHz to $> 25$ kHz which has been coupled directly to the structure. This direct coupling has been chosen to prevent possible stinger modes within the high frequency range of interest. The force has been obtained using a force transducer (PCB Piezotronics, Inc., type: 208C01). For the shaker input, a random signal waveform has been chosen. In analogy to the measurements using the impact hammer excitation (cf. section 4), the shaker tests has been repeated 13 times. To evaluate the influence of the shaker’s connection to the test object, the coupling has been disassembled subsequent to each single measurement.

At first, the standard deviations of the loss factors obtained by hammer and shaker excitation are compared. Fig. 9 denotes the results for both methods for the first 5 out of plane modes. Due to a limited shaker excitation to 20 kHz, the last out of plane mode at 21.74 kHz is omitted). It is obvious, that the deviations of the shaker measurements are considerably larger compared to the deviations obtained by impact hammer testing. The same differences can be recognized by comparing the amplitude of the averaged transfer admittances which are displayed in fig. 10. The deviations increase with raising mode numbers respectively with raising frequency. The third out of plane mode of the investigated gear wheel can be found at a frequency of about 10.4 kHz (cf. tab. 1) which means in the present analyses, the shaker influence on modal parameters increases significantly above frequencies of 10 kHz. Within the present paper, the reasons can not be analyzed in detail. A possible influencing parameter is the coupling method between shaker and test structure which has been realized using a two component adhesive which varies slightly during the different attachments to each single setup. Furthermore, the alignment of the shaker with respect to the structure’s surface plays a crucial role concerning the reproducibility of repeating
measurements. Even small misalignments of the shaker can result in a variation of the force vector which may lead to different results in modal testing. Finally it has to be noted, that for single measurements a split of the symmetrical modes of the gear wheel has been observed.

7. Conclusion

The present paper quantifies the deviations of the modal parameters eigenfrequencies and loss factors for a given test structure of a gear wheel. It has been shown, that the determination of modal parameters is highly reproducible using an impact hammer excitation. The reproducibility is acceptable still at higher frequencies up to 21.74 kHz. The standard deviations of the determined eigenfrequencies are comparable low which is plausible due to the fact that the system is not detuned by sensors or an excitation source. Furthermore, the results demonstrate small deviations in loss factors for the different measurements. In most modal testing scenarios, the excitation is realized by impact hammers or shakers. Due to this fact, the final section contains a comparison of modal parameters obtained by impact hammer and shaker excitation. The comparison quantifies the influence of the shaker coupling for the investigated structure. Within these investigations, modal testing using a shaker excitation leads to differences and significant higher deviations in comparison to impact hammer measurements. Some reasons for these deviations may be seen in the shaker attachment to the structure which need further investigations in future. Finally it has to be noted, that the presented investigation can not readily be generalized to arbitrary test setups. It should rather provide indicative values for the deviations of modal parameters for two well-established testing methods.

REFERENCES


