There is an interest in many situations to achieve a perfect acoustic cloak, which renders an object acoustically invisible. Although significant contributions have been made to realising passive acoustic cloaks, there is significant potential in the use of active control technologies for cloaking. However, this requires that an accurate estimate of the scattered pressure can be obtained in real-time to provide an error signal to the controller. This is non-trivial, since a standard pressure sensor would detect the contributions from both the incident and scattered fields. The measured pressure must, therefore, be decomposed into these two constituent parts, which has previously been achieved using a double layer of pressure sensors enclosing the scattering object. This paper proposes an alternative method of estimating the scattered component of the sound field, which does not use a double layer of sensors. The proposed virtual sensing method is based on an adaptation of the Remote Microphone Technique that has previously been used in active noise control applications. The proposed method filters the measured pressures using an optimally designed filter to estimate the scattered component of the sound field. The paper first formulates the proposed virtual sensing method for scattering detection and then presents an investigation into the accuracy of this estimation procedure using a series of measurements taken in an anechoic chamber. The effect of varying both the number of sources in the incident sound field, and the number of microphones used in the estimation is investigated. Finally, the practicability of designing an active control system using the estimated scattered field is discussed, and results from offline simulations of active control are presented.

Keywords: Active Control, Acoustic Scattering, Active Cloaking, Virtual Sensing, Signal Processing

1. Introduction

The interaction between an incident acoustic field and an object results in acoustic scattering, assuming the object is large relative to the acoustic wavelength. This scattered acoustic field is superimposed on the incident field, as shown in Fig. 1, at which point it becomes non-trivial to decompose the scattered and incident components into their constituent parts. The scattered pressure field measured at a number of discrete points can therefore be defined as

$$P_s = P_t - P_B,$$ (1)

where $P_s$ is the scattered acoustic pressure, $P_t$ is the total acoustic pressure and $P_B$ is the background acoustic pressure.
It has already been demonstrated computationally that the implementation of an active control system to reduce acoustic scattered pressure, effectively cloaking the scattering object to an outside observer, is highly effective if the scattered acoustic pressure is known \[1, 2\]. However, as shown in Fig. 1 a real-time measurement of the scattered acoustic pressure is non-trivial. Friot \textit{et al} \[3\] have demonstrated a potential solution to this scattered field detection problem, with the development of an active control system that uses two concentric rings of microphones surrounding the scattering object. The correlation between each ring is used to determine the incident signal from the scattered signal. This method was initially validated via simulations which demonstrated effective performance at lower frequencies, and was then applied to both a 1D experimental problem \[3\] and a 3D experimental problem \[4\]. In \[4\], reductions in the scattered field due to a parallelepiped scattering object of between 5dB and 20dB were achieved.

Although Friot \textit{et al} were able to demonstrate a cloaking capability, the system required a large number of microphones to accurately distinguish the incident and scattered pressure waves, which may not always be practical to implement. Han \textit{et al} \[5\] eliminate the need for the second ring of error sensors by modelling the transfer function between the scattered pressure and the total pressure with a computational model. They then use the simulated transfer function to estimate the scattered component that would be measured at the microphones, and this is then used as the disturbance signal for the active control system. This method was implemented in a laboratory setup using a spherical scatterer, and an 8dB attenuation in the scattered pressure was demonstrated.

This paper will present an alternative approach that does not rely on accurate modelling but instead uses the measured transfer responses between total pressure and scattered pressure to design a set of ‘observation filters’, in a similar way to the Remote Microphone Technique \[6, 7\]. This virtual sensing system would allow real-time estimation of acoustic scattered pressure based only on standard microphone signals and knowledge of the transfer responses of the scattering object, which could be obtained during a prior set-up phase.

The formulation of these optimal observation filters will first be discussed, before a normalised mean square estimation error is derived. A set of transfer responses has been measured for a cylindrical scattering object, which will be used in an off-line frequency domain implementation of the proposed virtual sensing strategy. Optimal frequency domain active control will then be implemented using the estimated scattered pressures as a disturbance signal, and the results will be discussed for varying arrangements of sources and sensors.
2. Formulation

An adaptation of the Remote Microphone Technique [8, 9] is presented, which attempts to estimate the scattered acoustic pressure at \(N_v\) virtual sensors, based on real-time measurements of total acoustic pressure at \(N_m\) positions. At a single frequency, an \(N_v \times N_m\) matrix of observation filters, \(O\), are defined such that the estimate of the scattered disturbance, \(\hat{d}_s\), is given as

\[
\hat{d}_s = Od_t,
\]

where \(d_t\) is a measurement of the total disturbance. The estimation error can therefore be defined as

\[
e = d_s - Od_t.
\]

The value of \(O\) which achieves the minimum estimation error can be calculated using a similar method to that given in [10], by optimising the cost function \(J\) as

\[
J = \text{trace} \left[ e e^H \right],
\]

\[
J = \text{trace} \left[ (d_s - Od_t) (d_s - Od_t)^H \right],
\]

\[
J = \text{trace} \left[ d_s d_s^H + Od_t d_t^H O^H - d_s d_t^H O^H - Od_t d_s^H \right].
\]

This can be minimised by differentiating the real and imaginary parts and setting the results to 0, as detailed in [11], leading to

\[
O_{opt} = d_s d_s^H (d_t d_t^H)^{-1}.
\]

The disturbance signals \(d_s\) and \(d_t\) can both be defined at each microphone position as the source strengths of each incoherent source making up the incident acoustic wave \(v\), multiplied by the transfer functions between each source and the scattered and total pressures respectively \(H_s\) and \(H_t\)

\[
d_s = H_s v \quad \text{and} \quad d_t = H_t v,
\]

such that Eq. 7 can be written as

\[
O_{opt} = E \left[ H_s v v^H H_t^H \right] E \left[ (H_t v v^H H_t^H)^{-1} \right],
\]

where \(E\) is the expectation operator. This is equivalent to

\[
O_{opt} = H_s S_{vv} H_t^H \left[ H_t S_{vv} H_t^H \right]^{-1},
\]

where \(S_{vv}\) is the power spectral density matrix of the input voltages. To ensure the observation filters are robust to small changes or errors in the plant response, the above process will now be repeated in a similar process to that carried out in [12], to allow for uncertainty in \(H_s\) and \(H_t\).

Assuming that the plant matrices in Eq. 8 have an uncertainty associated with them, they can be written as

\[
H_s = H_{s0} + \Delta H_s \quad \text{and} \quad H_t = H_{t0} + \Delta H_t,
\]

where the subscript \(0\) represents the nominal response and \(\Delta\) terms represent a measurement uncertainty. The estimation error can, therefore, be written as

\[
e = (H_{s0} + \Delta H_s) v - O (H_{t0} + \Delta H_t) v.
\]

Assuming that the measurement noise is random and uncorrelated with the true plant responses, the cross terms tend to 0 giving the optimal robust observation filters as

\[
O_{opt} = H_{s0} S_{vv} H_{t0}^H \left[ H_{t0} S_{vv} H_{t0}^H + \Delta H_t S_{vv} \Delta H_t^H \right]^{-1},
\]
which can be written as

$$O_{opt} = H_s S_{vv} H_t^H [H_t S_{vv} H_t^H + \epsilon I]^{-1},$$  \hspace{1cm} (14)$$

where $\epsilon I \approx \Delta H_t S_{vv} \Delta H_t^H$ is the level of uncertainty. It is interesting to note that this formulation is identical to the optimal observation filters including Tikhonov Regularisation [13], showing that the inclusion of regularisation is the optimal way to address uncertainty in the virtual sensing algorithm. This is consistent with the findings presented by Elliott et al [12].

The performance of the virtual sensing strategy can be quantified by the normalised mean square estimation error level at each sensor position, defined as [10]

$$L_e = 10 \log_{10} \left| \frac{S_{ee}}{S_{dd,sd}} \right|,$$  \hspace{1cm} (15)$$

where $S_{ee}$ and $S_{dd,sd}$ are the spectral density matrices of the error signals given in Eq. 3 and the scattered acoustic pressures respectively. By substituting Eqs. 3 and 8 into Eq. 15, the normalised mean square estimation error level can be written as

$$L_e = 10 \log_{10} \left| \frac{H_s S_{vv} H_t^H + OH_t S_{vv} H_t^H O^H - H_s S_{vv} H_t^H O^H - OH_t S_{vv} H_s^H}{H_s S_{vv} H_s^H} \right|$$  \hspace{1cm} (16)$$

This measure will be used in the following section to evaluate the performance of the scattering estimation.

The optimal filters calculated previously can then be used to estimate the acoustic scattered pressure, which becomes the disturbance signal for an active control system. At each frequency, a vector of optimal control signals can be calculated as [11]

$$f_{opt} = - \left( G^H G + \beta I \right)^{-1} G^H O d_t,$$  \hspace{1cm} (17)$$

where $G$ is the plant matrix between the control sources and the error sensors, $\beta$ is a regularisation parameter for the controller, and $I$ is the identity matrix. The error of the active control system, or the residual acoustic scattering after control, $e_s$ is therefore given by

$$e_s = d_s + G f_{opt},$$  \hspace{1cm} (18)$$

and the attenuation in the scattered field is calculated as

$$\text{attenuation} = -10 \log_{10} \left( \frac{\text{trace} [S_{ee,e_s}]}{\text{trace} [S_{dd,sd}]} \right),$$  \hspace{1cm} (19)$$

where $S_{ee,e_s}$ and $S_{dd,sd}$ are the spectral density matrices of the error signals given in Eq. 18 and the scattered acoustic pressures respectively. This measure of attenuation will be used in the following section to evaluate the performance of the active control system.

3. Frequency Domain Implementation

The performance of the proposed virtual sensing strategy will now be investigated using real-world data. Initially, the experimental procedure to measure the transfer responses will be discussed, before the transfer responses are used in an offline frequency domain implementation of the virtual sensing strategy which in turn becomes the disturbance signal for an optimal active control system.

3.1 Experimental Procedure

The acoustic scattered pressure from a cylindrical scattering object has been measured in an anechoic chamber as shown in Fig. 2. A circular array of 20 measurement microphones was placed around the cylinder, and a loudspeaker was positioned inside the microphone array and used to generate an incident acoustic wave.
The response to the incident field at all 20 microphone positions was measured with and without the presence of the cylinder, giving the total pressure field, $p_t$, and the background pressure field, $p_b$. Equation 1 was used to calculate the scattered pressure field, $p_s$. The loudspeaker was then moved, generating a different incident field, from which a different set of scattered pressures was calculated as detailed above. This process was repeated for 25 different loudspeaker positions. A further set of measurements was carried out between 10 structural actuators, mounted inside the cylinder, and all 20 microphones. This set of structural-acoustic transfer responses will be used to implement active control.

3.2 Frequency Domain Active Control of Predicted Acoustic Scattering

The measured transfer responses will now be combined with the virtual sensing formulation in Section 2 to design optimal observation filters, and then to use these observation filters to predict the acoustic scattered pressure which forms the disturbance signal for an optimal frequency domain active control system. For clarity, a subset of the large dataset discussed previously will be used, as shown in Fig. 3.

The scattered pressure at sensor 17 will be predicted, when the incident field is made up of all 7 of the sources shown in Fig. 3, from 5 measurement microphones (mic15-mic19), from 3 measurement...
microphones (mic16-mic18), and from a single measurement microphone (mic17). In each case, the normalised mean square estimation error level has been calculated for a variety of different regularisation values, and is shown in Fig. 4 plotted at 3 different frequencies, the middle of which corresponds to a resonance of the scattering object.

![Figure 4](image-url)

Figure 4: Normalised mean square estimation error of the proposed virtual sensing strategy when estimating the acoustic scattering at a single microphone. This has been predicted from measurements at increasing numbers of measurement microphones, and with varying amounts of regularisation, at three different frequencies. The legend is consistent across all three plots.

It can be seen from Fig. 4 that the performance of the virtual sensing method varies significantly at higher levels of regularisation, however remains reasonably constant below $1 \times 10^{-2}$. The effect of frequency on the estimation performance has also been investigated, and is shown in Fig. 5 with a similar sensor/source arrangement however now calculated with a constant regularisation value of $1 \times 10^{-2}$.

![Figure 5](image-url)

Figure 5: Normalised mean square estimation error of the proposed virtual sensing strategy when estimating the acoustic scattering at a single microphone. This has been predicted from measurements at increasing numbers of measurement microphones across a wide bandwidth, with a fixed regularisation value.

The predicted acoustic scattered pressure detailed above is then used as the disturbance signal for an active control system, given by Eqs. 17, 18 and 19. The incident field is made up of all 7 sources shown in Fig. 3 with the virtual sensing method being used to predict the acoustic scattered pressure at location 17 based on measurements from 3 different measurement microphone arrangements, with increasing levels of regularisation in the active control formulation, as shown in Fig. 6.
Due to the fact that the signal being minimised by the controller (predicted acoustic scattered pressure) is different to the true acoustic scattered pressure, there is an optimal regularisation range, as shown in Fig. 6. If the controller is over-regularised, the scattered pressure attenuation starts to decay as the matrix inversion becomes too inaccurate. If the controller is under-regularised, it attempts to perfectly control the scattered pressure estimate, at the expense of controlling the true acoustic scattered field. It can be seen from Fig. 6 that, regardless of the number of measurement microphones, this optimal range for the controller regularisation is between $\beta = 1 \times 10^{-15}$ and $\beta = 1 \times 10^{-25}$.

Based on the results presented in Fig. 6, a value of $\beta = 1 \times 10^{-20}$ has been selected for the regularisation in the active control formulation. With this value of regularisation, the attenuation in acoustic scattered pressure when detected using the proposed virtual sensing strategy is presented in Fig. 7.

It can be seen that significant reductions in acoustic scattered pressure are being predicted, even in the case where only a single measurement microphone is being used and, therefore, the estimation error is relatively high. All measurement microphone arrangements are able to achieve reductions in the
acoustic scattered pressure across the entire frequency range of interest.

4. Conclusions

A virtual sensing strategy has been proposed, based on the Remote Microphone Technique, to estimate scattered acoustic pressure from real-time total acoustic pressure measurements. Formulations for the optimal observation filters were derived, before a set of anechoic acoustic measurements of a cylindrical scattering object were carried out. These resulted in a set of transfer responses between 25 acoustic source positions, 20 total pressure sensors and 20 scattered pressure sensors, as well as between 10 structural source positions and 20 acoustic pressure sensors. The proposed virtual sensing strategy was then tested in an off-line frequency domain implementation using this set of measured transfer responses, which in turn formed the disturbance signal for an optimal frequency domain active control system.

It was found that for the non-trivial case where there are more independent sources in the incident field than there are measurement microphones, the attenuation of the active control system was limited by the number of microphones. However, even in the case where a single microphone was estimating the scattered pressure at a single location with an incident field made up of 7 incoherent sources, the active control system is able to achieve attenuation in acoustic scattering across the bandwidth of interest.

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