A method to generate buzz-saw noise source is proposed to reduce the computational cost in modeling fan blades with random stagger angles. Considering a full annulus fan rotor with an arbitrary stagger angle on each blade, the multiple pure tone noise source is found to be able to be reconstructed from merely three steady simulations. The theoretical basis is, under the assumption of small stagger angles, the linear relation between the shock strength and the stagger angle. This method is applied on modified rotor of NASA rotor 67, pressure distribution on the reconstructed reference source plane is validated by simulation of the same modified rotor.

Keywords: buzz-saw noise, instant source generation, random stagger angles, mode decomposition

1. Introduction

In turbofan engines, a special noise component is generated when staggered fan blade is exposed to transonic or supersonic relative flow. This noise component is generally documented as "buzz-saw" or "multiple pure tone (MPT)" noise. Prediction of buzz-saw noise could be accomplished by the following three methods: 1) Propagation of N waves in quasi-two-dimensional models; 2) CFD/Computational Aero-Acoustics (CAA) hybrid method; 3) CAA method. Pre-known acoustic sources are obligatory for the first two methods.

Given a narrow annulus rotor or the shockwave is concentrated in the tip region, the first method is recommended. In this case, theoretical method proposed by Morfey and Fisher [1] is adequate to predict hard-wall propagations. Numerical method proposed by McAlpine et al. performs well for lined-duct predictions [2]. Initial source of these models are time signals. Stratford and Newby introduced an empirical formula to generate the initial shock waves based on statistics [3]. McAlpine et al. proposed an hybrid method of measurements and manipulation to generate random N waves [4].

On the other hand, if the leading-edge-shock structures violate the one-dimensional assumption, the first prediction strategy becomes less reliable. Moreover, in case of inlet distortion and other more complicated situations, the second or the third strategy should be adopted. However, using CAA method to model full-annulus rotor is still too expensive. As a result, CFD/CAA hybrid methods is left to tackle these situations. But it is found that none of the initial shock-generating methods used in one-dimensional simulations are appropriate for this case. For each random staggered rotor, full-annulus meshing and simulation are required.
Consider the mechanisms of buzz-saw noise generation, several factors are found to be responsible for the formation of buzz-saw noise. Among all the factors, blade stagger angle is proved to be the most important. Furthermore, shock strength near the rotor leading edge is found to be proportional to stagger angles \[3, 5\]. This linear relation provides a promising way to reconstruct pressure field on the source plane by no more than three steady simulations.

The goal of this work is to build a method to instantly generate initial buzz-saw noise source on the reference plane. The main idea of this method is described in section \[2\], validation of the assumptions is displayed in section \[3\], application of this method to NASA rotor 67 is located in section \[4\], finally, discussion and conclusions are made in section \[5\].

2. Methods

According to the linear relation mentioned above, it is assumed to be possible to reconstruct the pressure filed on the reference plane upstream of supersonic or transonic rotors by two steps. Firstly, three steady simulations of the element rotors are required to generate a database. Then, the pressure field of a rotor with arbitrary staggered blades could be generated instantly from the database.

2.1 Element Cases

In this section, three element cases are defined based on three blade channel elements.

- An ideal case is a steady simulation of the ideal rotor, see the left figure in Fig. 1.
- A first-type element case is a steady simulation of the rotor with one blade staggered a \(\delta \theta_0\) angle, see the middle figure in Fig. 1. A first-type element rotor is composed of one channel element of the first type (see section \[2.1.1\]) and several ideal channels.
- A second-type element case is a steady simulation of the rotor with positive and negative staggered blades, see the right figure in Fig. 1. A second-type element rotor is made up of two conjugated channel elements of the second type (see section \[2.1.2\]).

Figure 1: Schematic view of the rotor leading edge from the inlet. Ideal rotor (left); rotor with one blade modified by angle \(\delta \theta_0\) (middle); rotor with all blades rotated by a positive or negative angle \(0.5 \delta \theta_0\) (right).

2.1.1 First Type of Channel Element

The first type of channel element is defined as follows. Given a fan rotor, pick a pair of two adjacent channels, which contains a staggered blade with its left and right vicinal blades, see Fig. 2. This type of channel pair is termed as Channel Element I, which is assumed to be the element composing a leap-randomly staggered rotor, see Fig. 3. A leap-randomly staggered rotor has no more than one staggered blade in sequence, i.e., each staggered blade has at least one ideal blade before and after it.
2.1.2 Second Type of Channel Element

Another type of element of fan rotor is termed as Channel Element II, see Fig. 4, in which half of the blades in a sequence are rotated by $+0.5a^\circ$ and the other half of blades are rotated by $-0.5a^\circ$. Consequently, one blade channel is modified by angle $-a^\circ$ and the blade channel in the axisymmetric position is modified by angle $+a^\circ$. Unlike channel element of the first type, the positive and negative channel elements of the second type are conjugated in one rotor. Rotor with continuous staggered blades is assumed be able to be represented by a series of Channel Element II.

![Channel element of the first type.](image1)

![Blade pattern of a leap-randomly staggered rotor.](image2)

![Channel element of the second type.](image3)

2.2 Build the Database

A database is a set of normalized disturbances introduced by the presence of one element channel in the ideal-rotor background.

2.2.1 Database of Element Case of the First Type

On the presence of the staggered blade, see the middle figure in Fig. 1, pressure field of the shadowed channels are disturbed. According to the linear relation between stagger angle and shock strength, pressure distribution on the reference plane of rotor with the $0_{th}$ blade modified can be separated into a uniform part and a perturbation part, i.e.,

$$p_s(r, \theta, z, t) = \bar{p}_s(r, \theta, z, t) + p'_s(\theta, z, t, \delta \theta_0). \quad (1)$$

Assumptions:
The linear relation between stagger angle and shock strength preserves in the vicinal upstream of rotor where the pressure disturbance is low enough compared with the background flow.

The circumferential phase-shift angle of pressure distribution introduced by stagger angle shares the same linear relation with the first assumption.

The effects induced by different blades with arbitrary stagger angle have no non-linear interactions with each other. Thus, linear superposition of these effects is valid.

Following the first assumption, the perturbation part in Eq. (1) is solvable by assuming that the uniform part equals the pressure distribution yielded by the ideal rotor with an additional phase-shift angle $\delta \phi_0$. Thus, $\bar{p}_s(r, \theta, z, t) = p_i(r, \theta + \delta \phi_0, z, t)$. The pressure perturbation introduced by $\delta \theta_0$ stagger angle on the $0_{th}$ blade is then

$$p'_{s0}(r, \theta, z, t, \delta \theta_0) = p_s(r, \theta, z, t) - p_i(r, \theta + \delta \phi_0, z, t).$$

Recalling the linear relationship between the stagger angle and the shock strength and the second assumption, pressure perturbation and phase-shift angle induced by arbitrary stagger angle $\delta \theta$ is then written as

$$p'_{s} = \frac{p'_{s0}}{\delta \theta_0} \delta \theta, \quad \delta \phi = \frac{\delta \phi_0}{\delta \theta_0} \delta \theta. \quad (3)$$

In order to consider the radial distributions, Tyler and Sofrin’s modal theory [6] is adopted for detailed analysis. Based on acoustic modal theory, the in-duct pressure distribution on a $z$-constant reference plane is decomposed into modes as

$$p_s(r, \theta, t) = M \sum_{m=1}^{N} \sum_{n=0}^{N} A_{mn} F(J_m) e^{i(m \theta + \phi_{mn} - m \Omega t)}. \quad (4)$$

In which $F(J_m)$ is a function of Bessel functions expressing the radial distribution of the pressure field. $A_{mn}$ and $\phi_{mn}$ is the amplitude and phase angle of mode $(m, n)$. It is noticed that the plane and reflection waves are not involved in this equation. In case of buzz-saw noise, the circumferential mode $m$ is related to the rotor shaft frequency instead of the blade passing frequency. Substitute Eq. (4) to Eq. (2) to obtain

$$p_s(r, \theta, t) = \sum_{m=1}^{M} \sum_{n=0}^{N} F(J_m) \left[ A_{mn} e^{i(m \theta + \phi_{mn} + \delta \phi_{mn} - m \Omega t)} + A'_{mn} e^{i(m \theta + \phi'_{mn} - m \Omega t)} \right]. \quad (5)$$

In which, $A_{mn}$ and $\phi_{mn}$ are the mode amplitude and phase angle of the ideal rotor case. Equation (5) is the modal expression of the element case of the first type. Due to the linear property of Eq. (5), $\delta \phi_{mn}$ and $A'_{mn}$ share the same relation with Eq. (3). Thus, the amplitude and phase angle disturbances induced by an arbitrary stagger angle $\delta \theta$ on the $0_{th}$ blade are written as

$$A'_{mn} = \frac{A'_{mn}(\delta \theta_0)}{\delta \theta_0} \delta \theta, \quad \delta \phi_{mn} = \frac{\delta \phi_{mn}(\delta \theta_0)}{\delta \theta_0} \delta \theta. \quad (6)$$

In Eq. (6), $A'_{mn}(\delta \theta_0)$ and $\delta \phi_{mn}(\delta \theta_0)$ are achieved by subtracting the mode amplitude and phase angle of the element case by those of the ideal rotor case, see Fig. 5. As illustrated in the following sections, the phase angle of perturbation modes, $\phi'_{mn}$, is almost constant for stagger angle with the same sign. Thus, $\phi'_{mn}$ is obtained by

$$\phi'_{mn}(\delta \theta) = \phi'_{mn}(\delta \theta_0), \quad i f \ \delta \theta_0, \ \delta \theta > 0 \quad (7)$$

As a result of the previous discussions, the database is built by one steady simulation of the ideal rotor, two steady simulation of the element rotor with stagger angle $\pm \delta \theta_0$ and two times of mode decompositions. The detailed procedure is displayed in Fig. 5.
2.2.2 Database of Element Case of the Second Type

The main strategies in building the database of the second-type element case is the same with the first type. However, a further assumption is needed:

- The influential area of one expanded or shrunk channel is half-annulus of the rotor.

See the right figure in Fig. 1 if the influential area is limited to half-annulus, the pressure field on the reference plane can be split into two parts. The upper part represents disturbed flow introduced by a $\delta \theta_0$ expansion of one channel and the lower part represents the disturbed flow introduced by a $\delta \theta_0$ shrink of one channel. As a result, disturbances introduced by only one expanded channel or only one shrunk channel are obtained by manually setting disturbance on the other half-annulus be zero.

2.3 Reconstruct the Pressure Field

Once the database is built, pressure field of rotor is able to be generated instantly. Two types of rotors have slightly difference in reconstruction.

2.3.1 Leap-randomly Staggered Rotor

Rotor composed of channel elements of the first type is called leap-randomly staggered rotor, see Fig. 3. As discussed above, pressure field of rotor with only the $0_{th}$ blade staggered is expressed by Eq. (5).

In similar manner, if the $k_{th}$ blade in a rotor has a stagger angle $\delta \theta_k$, the superposition of all disturbances on the ideal background is expressed as

$$p_s(r, \theta, t) = \sum_{m=1}^{M} \sum_{n=1}^{N} F(J_m)(A + A').$$

(8)

In which, $A$ is the ideal background with phase drifting and $A'$ is the deviation introduced by blade stagger angles. These terms are detailed as

$$A = \bar{A}_{mn} e^{i[m\theta + \phi_{mn} + \sum_{k=1}^{B-1} \delta \phi_{mn}(\delta \theta_k) - m \Omega t]}$$

$$A' = \sum_{k=1}^{B-1} A'_{mn}(\delta \theta_k) e^{i[m\theta + \phi_{mn}(\delta \theta_k) + \frac{2k\pi}{B} - m \Omega t]}.$$
2.3.2 Full-randomly Staggered Rotor

In a full-randomly staggered rotor, the angle $\delta \theta_k$ is defined as the difference between adjacent blade stagger angles, i.e., $\delta \theta_k = \delta \theta_{sk} - \delta \theta_{sk-1}$. It is the angle variation of the $k_{th}$ channel. The pressure field is also expressed by Eq. (8) and Eq. (??), except for that the index $k$ represents the $k_{th}$ channel instead of the $k_{th}$ blade. Moreover, database of the element case of the second type should be used.

3. Validation of Assumptions

3.1 Geometry, Mesh and Flow Field Solver

NASA Rotor 67 [7] with 22 blades is selected for validation and further application. Based on this rotor, element rotors of the first type is created and tested to validate the assumptions mentioned above. In order to avoid unnecessary computational cost, half-annulus and periodic boundary conditions are used in the simulations. It is noticed that for the first type of element rotors, periodic boundary conditions actually introduce two channel elements of the first type in the equivalent full-annulus. Nevertheless, the idea of reconstruction is left unchanged. The geometry of half-annulus element rotor is displayed in Fig. 6, in which the only staggered blade is centered at the half-annulus rotor for the convenience of investigating the influential area of Channel Element I.

![Staggered Blade](image)

**Figure 6: Half-annulus of NASA Rotor 67 with stagger angle $\delta \theta_0$ on the 6th blade**

SST $k - \omega$ model is adopted in solver euranusTurbo to calculate the steady flow field of all rotors. It is proved that 1.71 million mesh per channel is enough to capture the steady shock structures. Overall performance of both the ideal rotor and the element rotors are close to the design point of NASA Rotor 67. The reference plane of buzz-saw noise source is located at axial position $z=-0.0203$m, which is about one chord length upstream of the leading edge at blade tip region. Figure 7 shows the static pressure distribution on the reference plane of one element rotor and the ideal rotor. Disturbance between the element rotor and ideal rotor is calculated by subtracting the pressure field of ideal rotor from the element rotors.

3.2 Validation of the First and the Second Assumptions

One ideal case and six element cases are investigated to validate the assumptions. The stagger angle varies from $-0.5^\circ$ to $+0.5^\circ$. Amplitude of the perturbation modes are calculated and normalized with respect to the stagger angle, in other words, it stands for the amplitude introduced by $1.0^\circ$ stagger angle. The left figure in Fig. 8 shows the normalized amplitude of mode $m$ by the range of [18, 28]. Each peak in the figure consists of six radial modes in a row, for instance the first peak labeled as (18, 1-6) displays mode (18, 1), (18, 2) up to (18, 6). The linear relation between the stagger angle and the shock strength...
is proved by the consistency of the normalized mode amplitude. All odd circumferential modes have an amplitude of zero due to the periodic boundary conditions. Persistence of phase angle both at the blade passing mode and engine order modes is proved by the middle and right figure in Fig. 8. Phase angle of the disturbances, $\phi'_{mn}$, is then proved to be constant.

Figure 8: Validation plots. Normalized amplitude of the perturbation modes (left); phase angle of perturbation modes $m=12$ (middle), phase angle of perturbation modes $m=22$ (right)

4. Application

Reconstruction method is imposed on leap-randomly staggered rotors based on modifications of NASA Rotor 67. Reconstruction of the pressure field on the reference plane of leap-randomly staggered blade is plotted in Fig. 9. Blade stagger angles of half-annulus of the rotor is displayed in the upper part of the sub-figures. Zeros in them are manually designed according to the limit of channel element of the first type, non-zero angles are generated by randomly picking numbers in the range of $[−0.5^{\circ}, +0.5^{\circ}]$. It is proved that the reconstruction method behaves well for most modes. This in turn proved the third assumption that non-linear interactions between introduced disturbances are negligible.

Attempts are also made to reconstruct the pressure field of full-randomly staggered rotors with the database generated by channel elements of the first type. However, large discrepancies are observed due to the physical difference between the two types of rotors. The reconstruction of fully-randomly staggered rotor with database of channel elements of the second type is to be revealed in future work.
5. Conclusion

A method of pressure-field generation is proposed in attempt to reduce the computational cost in generating buzz-saw noise source. The reconstruction procedure is started by building a database from one ideal rotor simulation and two element rotor simulations. Then, a new pressure field is able to be generated instantly from the database. Fan rotors composed of staggered blades are categorized into two types, leap-randomly staggered blade and full-randomly staggered rotor. Channel elements of the first type and of the second type are defined to build the database suitable for each rotor type. Validation is performed based on channel element of the first type. The assumptions are proved to be valid by the reconstruction procedure. Attempt is made to reconstruct pressure field of rotor with full-randomly staggered blades using channel element of the first type. However, the discrepancies announced the failure. Reconstruction with channel element of the second type is planed in future work.

REFERENCES


