WAVE-CONVERSION ACTIVE LINER BASED ON GENERALIZED SNELL-DESCARTES LAW

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The Cremer’s impedance theoretical framework has been used for years to optimally design acoustic liners for the aeronautic industry, such as honeycomb liners. This passive mean of sound absorption, which consists in rather thin (a few centimeters) honeycomb-shaped cavities glued between a perforated sheet and a rigid back-wall, has several downsides: an optimal acoustic absorption can only be achieved over a limited bandwidth, centered around one (eventually two) prescribed central frequency. Their performance at low frequencies is also limited by their size, owing to the “quarter-wavelength rule”. In this paper, we propose a fully tunable active liner based on electrodynamic resonators, employing loudspeaker membranes as sound absorbers, and an active control framework allowing modifying their acoustic impedance over a wide frequency band. An optimization of these active absorbers is achieved by considering the theoretical framework of the generalized Snell-Descartes law, which accounts for impedance gratings over an interface. Through this formalism, it is possible to devise a wave conversion strategy within an active acoustic liner in order to absorb multimodal sound propagation inside a rigid duct, improving the noise attenuation performance of the liner over an extended bandwidth.

Keywords: Active liner, meta-material, wave conversion.

1. Introduction

With the rise of the number of flights around the whole globe, airports and air traffic are becoming preoccupying sources of noise. One of the key actions to solve this problem is to reduce the noise emitted by the planes. Some studies have shown the importance of the noise radiated by the engines and transmitted to the air via the nacelles. Several solutions are already in place, such as liners. Such devices are limited by their resonant behaviour and the bulkiness needed to be effective at low frequencies.

Recent work have proposed to turn loudspeakers in low-frequency sound absorbers. By its versatility, its compactness with regards to the wavelength at which it can work, the loudspeaker can be used as one of the most efficient and thinnest sound absorbers, thus opening a field of study to propose solution for sound absorption in ducts to prevent propagation of emitted noise (such as in nacelles [1]). Loudspeakers can also be used to impose an acoustic impedance, offering a versatile tool to the acoustician in the form of an adjustable specific impedance to be placed anywhere.
Combining the ideas of the already existing liners with impedance control and active sound absorption, this work aims at finding new solutions to achieve better performance for transmission loss devices. The goal will be to use loudspeakers as unit cells of a novel meta-material presenting extraordinary properties in terms of absorption coefficient. The ability to tune such a system will be a strong point in the study, as noise nowadays is strongly situation-dependant, especially in the case of airplanes, where the engine’s regime strongly is directly correlated to the emitted sound, and quite variable by nature.

2. State of the art

2.1 Electrodynamic resonator

In this work, the main study is the interaction of a wave and an interface between the fluid where the wave propagates and the material, resulting in propagation changes. This interface can be modelled as a complex acoustic impedance:

\[ Z_{ac} = R_{ac} + jX_{ac} = \frac{p}{q}. \]  

The specific impedance over an area \( S \) is defined as \( Z_s = Z_{ac}S \) and can be compared with the characteristic specific impedance of the propagating medium : \( Z_c = \rho_0 c_0 \). When a wave impinges a surface with a given specific impedance \( Z_s \), we can define the reflection coefficient :

\[ R = \frac{Z_s - Z_c}{Z_s + Z_c}. \]  

In the case of an homogeneous impedance along an infinite interface and the propagation of plane waves, there exists a simple relation between the incident wave and the reflected one. In the case of a 2D propagation, the Snell-Descartes law dictates the angle \( \theta_r \) between the normal to the surface and the propagation direction of the reflected wave, given the angle \( \theta_i \) of the incident wave coming from P to Q (see Fig.1):

\[ \sin \theta_i + \sin \theta_r = 0. \]  

The fact that the impedance must be homogeneous is a very strong hypothesis and opens a vast field of study if this impedance should be inhomogeneous in a controlled manner, for example with the presence of resonators.

![Figure 1: Illustration of the Snell-Descartes law (left) and of the closed box loudspeaker (right)](image)

Various resonators are used in the acoustic domains, for example quarter-wavelength resonators or Helmholtz resonators. Interestingly, electrodynamic loudspeakers can also be used as acoustic resonators. From an acoustic point of view, it is a membrane of area \( S_d \) with a velocity \( v \), submitted to a force \( S_dP_f \) on the front and \( S_dP_r \) on the rear part of the membrane. When enclosed in a box, its behaviour follows
an equation linking its mechanical impedance $Z_m$, its force factor $Bl$, its effective area $S_d$, the pressure in front of the loudspeaker $p$, its velocity $v$ and the input current $i$ (see Fig.1):

$$Z_m \cdot v = S_d \cdot p - Bl \cdot i.$$  \hfill (4)

The impedance $Z_m$ is modeled by a single degree of freedom (DOF) oscillator: a mass $M_m$, a compliance $C_n$ and a resistance $R_m$. This is written in the form:

$$Z_m = R_m + j\omega \cdot M_m + \frac{1}{j\omega \cdot C_m}.$$ \hfill (5)

By imposing the ratio between $p$ and $i$, the ratio between $p$ and $v$ can be arbitrary imposed. This changes the specific impedance presented by the loudspeaker. The transducer, a single degree of freedom resonator (SDOF) can for example be tuned with an added passive circuit to an ideal acoustic resistance at a given frequency, thus presenting an ideal absorption [2].

In order to improve the behaviour of that kind of system, and especially its effective frequency bandwidth, Rivet et al. have shown in [3] a multiple degree of freedom (MDOF) approach to the control of loudspeakers. They presented a control strategy that enables the loudspeakers to present virtually any kind of acoustic impedance. To achieve a target specific impedance, the pressure $p$ in front of the loudspeaker is measured by a microphone, then processed through a controller and a command current $i$ is injected into the circuit of the loudspeaker, following the equation (4). The same equation can be re-written to have a target transfer function $\Phi = i/p$ dependant on the model of the loudspeaker’s mechanical impedance $Z_m$ and the target specific impedance $Z_s = p/v$ [4]:

$$\Phi = \frac{S_d}{Bl} \left(1 - \frac{Z_m}{S_d Z_s}\right).$$ \hfill (6)

This control function gives a target impedance over a large frequency band (provided the target impedance yields a stable control function, property that will be assumed here). This strategy requires to know the model of the loudspeaker ($Bl$, $S_d$ and $Z_m$). A strategy of model identification has been described in [4]. The specific impedance presented by the loudspeaker is then of the form:

$$Z_s = \frac{1}{\sum_{i=1}^{n} Z_{si}}, \quad \text{with} \quad Z_{si} = R_i + j\omega L_i + \frac{1}{j\omega C_i},$$ \hfill (7)

where $R_i$, $L_i$ and $C_i$ can be tuned independently.

### 2.2 Electroacoustic Meta-material

Originally inspired by electromagnetic meta-materials, the term of acoustic meta-material defines an engineered acoustic material presenting effective properties that cannot be presented by its basic components [5][6]. The arrangement of those components in a composite, or in other forms, leads to a tailorable material, specified by physical characteristics such as mass density or bulk modulus.

Composed of subwavelength cells, sometimes called meta-atoms, the material properties are based on the interaction between the acoustic field and the behavior of a single unit cell of a more or less periodic structure. This cell may often be a resonator of any type. Owing to such engineered structure, it is possible to spatially arrange the meta-atoms, so that they present a specified grating of acoustic properties (for example the phase of their individual reflection coefficient, see Eq. (3)) for wave reflection/transmission manipulations. Such gradient-based metasurfaces can then be designed to achieve helical wavefronts [7] or anomalous reflection [8]. These anomalous refraction properties can be derived from the Generalized
Snell-Descartes law. In two dimensions, this law links the angle $\theta_i$, the angle $\theta_r$ and the phase of the reflection coefficient of the surface $\psi(x)$:

$$\sin(\theta_i) + \sin(\theta_r) = -\frac{1}{k} \frac{\partial \psi}{\partial x}$$  \hspace{1cm}(8)

Most gradient-based meta-materials use passive meta-atoms, such as labyrinths [9] to change the path length of the wave, then its phase, Helmholtz resonators [10] of different sizes to have a phase gradient, quarter wavelength resonators or even hybrid resonators [6] to achieve various performances. Very few attempts were made to make an active meta-material. Some work has been done with flow [11], but almost none with loudspeakers.

Acoustic liners are intensively used in aeronautics. The goal of this surface treatment is to leave the main flow undisturbed, without any obstacle in the pipe, but absorb sound on the wall of the pipe, whether one dimensional propagation or multi-modal three dimensional propagation is considered. One first limitation of the traditional passive design comprised of an arrangement of quarter-wavelength resonators is that to treat very low frequencies, the liner has to be very thick. MDOF liners have been investigated, resulting in a wider bandwidth of absorption but this technique has a very strong disadvantage: multiple layers of honeycomb structures have to be stacked. To have efficient absorption, dimensions of the treatment are calculated with a method based on a classic paper by Cremer [12]. This gives a way to optimize the impedance presented by the treatment in order to maximize insertion loss. One of the problems is that the optimal acoustic resistance is very low, which yields highly resonant materials and a very thin effective bandwidth, even with MDOF liners. This method, although well used, is also not entirely valid at low frequencies [13]. Here, we propose an alternative to the Cremer principle to produce efficient transmission loss at low frequencies, with a reduced thickness thanks to active impedance control.

3. Non reflective surface

A metasurface being composed of discrete elements, the coordinate along the $x$-axis can be written $x = dm$ with $d$ the dimension of the element and $m$ the indices of the cell. Therefore, to apply the equation (8) it has to be re-written to derive the target reflection phase of every cell that will impose a relationship between impinging and reflected waves:

$$\psi_m = -kd_m (\sin(\theta_i) + \sin(\theta_r)).$$  \hspace{1cm}(9)

This defines a target phase function for the reflection coefficient, in the $[-2\pi, 0]$ interval, that has to be imposed on the whole surface. The reflection coefficient being directly linked to the specific impedance of the cell, a linear arrangement of loudspeakers is chosen to achieve the target phase gradient. Early work already published in [14] describes the method, which can be summarized as follow: every loudspeaker presents the same resonant behaviour with a slight frequency-shift in order to present different resonant frequencies. The phase presented by each loudspeaker is then slightly different from its neighbour, resulting in a phase gradient and wave redirection in a prescribed manner, with impedance control. One limitation of this approach comes from the analysis of the validity of the Snell-Descartes law: an existence condition for the reflection angle comes out of the equation. In fact, for a fixed phase gradient between two consecutive cells:

$$\sin(\theta_r) \leq 1 \Leftrightarrow \frac{\Delta \psi_m}{kd} - 1 \leq \sin \theta_i, \quad \text{for} \quad \theta_i \in [-90^\circ, 0^\circ]$$  \hspace{1cm}(10)
This equation means that, if the phase gradient is chosen adequately, there exists an incident angle over which the reflected angle is not defined. In this case, there is a conversion of the impinging airborne wave to a surface wave. Some more precise explanations can be found in the literature [15], but here, because of losses induced by the loudspeaker, we can consider that the engineered surface is very poorly reflective. This is illustrated by Fig.2.

Figure 2: Pressure field scattered by a reflective panel (left) and by the non-reflective condition (right), simulated at 250Hz with FEM.

4. Elementary MDOF cell

The goal of the design of this MDOF cell control is to achieve the phase-gradient designed to yield a non-reflective surface. Previous work done in [14] was based on a SDOF strategy for the impedance of the loudspeakers. It consists in shifting the resonant frequency of identical SDOF resonators such as to obtain a phase gradient of their individual reflection coefficient at a given work frequency. This gives a controlled phase reflection that follows the target gradient with some errors and several problems. The first one is that a SDOF impedance will yield a \([-2\pi, 0]\) phase between \(0\) and \(\infty\) thus leading to impractical frequency shifting to obtain very low or very high phase values in the target interval. Therefore, we investigate a MDOF to have a complete \([-2\pi, 0]\) phase in a small frequency interval. It is quite straightforward that a 2-DOF specific impedance will only yield phases between \(\pi/2\) and \(-\pi/2\), a 3-DOF impedance is therefore chosen.

The second constraint is the frequency dependence of the phase of the reflection coefficient. The previous paper has shown that for the frequency shifting to be optimal at the center frequency, the phase \(\phi\) of the reflection coefficient of the shifted resonator has to be of the form \(\phi = a \log f + \phi_0\) over the frequency band necessary to shift between 0 and \(2\pi\), which is not the case with a SDOF strategy. A perspicacious reader can argue that this is false if we want to have a constant behaviour over a large frequency band, given equation [9] which already depends on the frequency, but this form of phase is purposely chosen anyway. A target equation for the phase of the reflection coefficient, between the frequency \(f_0\) where this phase is equal to 0, and \(f_{-2\pi}\) where it’s equal to \(-2\pi\), can be written:

\[
\phi(f) = \frac{2\pi}{\log f_{-2\pi}} \log f - \frac{2\pi \log f_0}{\log f_{-2\pi}}, \quad \text{for} \quad f \in [f_0, f_{-2\pi}] \tag{11}
\]

This target is then extended to ensure the behaviour inside the frequency interval : we extend the equation to \([f_0/\sqrt{2}, f_{-2\pi} \cdot \sqrt{2}]\). This extended target is illustrated on fig.(3). A cost function is then defined as the absolute difference between the phase of the actual reflection and the target defined above.

As we saw that the reflection coefficient can be expressed as a function of the specific impedance, we can express the cost function as a function of the nine parameters of our 3-DOF specific impedance.
A matlab algorithm is used to optimize these nine parameters in order to obtain the target specific impedance that yields the correct phase-behavior of the reflection coefficient over the given frequency band. Constraints on the parameters are obtained with regards to the physical limits of the system. Those limits are not detailed here, where the model is supposed to be perfect and totally independent. One limit is still visible here: the acoustic resistances imposed to the loudspeaker cannot be negative, otherwise the transfer function will obviously present some unstable poles. The resulting phase of the optimization is shown on fig. (3). We can see that the phase behavior is very close to the defined target. On fig. (4) we can see a phase function obtained with equation 9 for a given central frequency, and the achieved reflection coefficient phase at each cell with the defined control strategy. We can see the quasi-exact correspondence between the two and therefore validate the method. This figure also shows the acoustic impedance which has to be imposed to every cell. Several other simulation results allow us to validate the method for a large frequency band.

Figure 3: Reflection coefficient optimization: initializations of the algorithm (orange), result (blue), target (black) and phase function using the whole phase interval (circles).

Figure 4: Left: phase function target (blue), and achieved via impedance control (black circles). Center: Target impedance for frequency shifted resonators yielding the desired phase gradient at the central frequency. Right: Experimental results of the same impedances.

On the right of fig. (4) we can see experimental results of the impedance control, showing very good adequacy with the target. One difficulty is to obtain a precise impedance control around the natural frequency of the loudspeaker, although it is not very visible in this result. Divergence between the model estimation of the loudspeaker and the real components are shown to be quite small but already large enough to cause some discrepancies in the control. Another factor of discrepancy can be the discretization method of the transfer function applied to control the loudspeaker. This calls for further investigation, but the results are good enough to be applied in a linear strategy.
5. **Liner simulation**

The previous parts of this article show that if a correct phase gradient is chosen, waves impinging the surface at a high angle with regards to the normal of the surface, close to grazing incidence, won’t be reflected. If this case is applied to a lining treatment in a duct, this should mean a strong transmission loss. To test that strategy, a COMSOL simulation is done, using a very short length of treatment compared to the wavelength (around 1/4 of the wavelength). Six loudspeakers are put in a row, in an infinite duct of a height smaller than half of the wave length, with only plane waves propagating. Results are promising and are shown in Fig.5.

![Figure 5: Propagation from left to right of a wave in a duct treated with a wave-conversion liner, at 509Hz.](image)

To assess the efficiency of this strategy, the same simulation is carried out with an homogeneous impedance (the same as the non-reflective surface without the frequency-shifting), a randomly frequency shifted set of resonators and the phase-gradient condition. The transmission loss is calculated for the three cases and illustrated on Fig.6. The difference between a random shift and a controlled gradient is not obvious on this simulation, due to a small number of loudspeakers, but the broadband effect of shifting resonators is clearly shown on these figures. More simulation will be conducted with more loudspeakers. These results are shown here because the setup corresponds to the experimental setup used to carry out experimental validation.

![Figure 6: Transmission loss (in dB) for an homogeneous impedance (left), a randomly frequency shifted set of resonators (center) and a phase-gradient condition (right).](image)

6. **Conclusion and perspectives**

This article shows a novel way of designing a liner, founded on the generalized Snell-Descartes law. Although widely used in the meta-material domain, this law is here used in a non-trivial way, by taking...
advantage of its limitations, not by simply redirecting waves. A strategy to obtain an elementary cell that can give the proper phase gradient is proposed here, also. This strategy is applied for a simulation of liner that shows promising results, although limited by the small number of resonators that are used.

Experimental validation is being carried out, but the main perspective is to prove this concept with a larger number of resonators. A comparison with traditional liners has to be carried out, especially focusing on the bandwidth of efficiency.

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