ACOUSTIC METAMATERIAL SOUND-INSULATOR BY HELMHOLTZ RESONATORS COMBINED WITH THIN MEMBRANES

Takashi Yamamoto
Kogakuin University, Faculty of Mechanical Engineering, Tokyo, Japan
e-mail: takashi_yamamoto@cc.kogakuin.ac.jp

Hideki Furusawa, Hiroshi Sakaguchi, Toshihiro Nomura
Ibiden Co., Ltd., Gifu, Japan

An acoustic metamaterial sound-insulator by Helmholtz resonators combined with thin membranes is developed to realize extraordinary sound transmission loss (STL). Small Helmholtz resonators are embedded periodically within the sub-wavelength throughout the insulation plate that covers an elastic plate with an intermediate air layer to compose a double-wall system. The throats of the Helmholtz resonator open at the intermediate air layer, and the bottom faces of the back cavities are composed of thin membranes. This system has three resonances; One is a structural resonance of a mass and spring system where the insulation plate works as a mass and the intermediate air layer as a spring. Second one is an acoustic resonance of the Helmholtz resonator. The last one is an elastic resonance of the membrane at the bottom surface of the back cavity. This system exhibits extraordinary high STL compared with a conventional double-wall insulation system in the frequency range where these three resonances are excited. This frequency range can be controlled by designing the geometry of the Helmholtz resonators, the intermediate air layer, and the size and the stiffness of the thin membrane. The theoretical analysis gives the expression of the equivalent properties of the intermediate air layer and the insulation plate. To demonstrate the acoustic performance of the developed system, an elastic steel plate of 1 mm is covered with a plastic plate of 20 mm thick where the resonators that has a resonance around 1.6 kHz. Air layer of 5 mm between these two plates gives resonant transmission loss around 800 Hz, and the membrane of 0.1 mm thickness gives an elastic resonance around 850 Hz or 2.2 kHz depending its elasticity.

Keywords: acoustic metamaterial, plate, Helmholtz resonator, thin membranes, sound transmission loss

1. Introduction

In electrically powered vehicles, noises that have been musked by engine noise, such as road noise, pattern noise and wind noise, are emerged especially in the low and mid frequency range up to around 2.0 kHz. Sound-proof structures such as sound-absorbing material and sound-insulation material are conventionally applied. However, these structures usually require the additional weight. Thus, we study a new structure to achieve simultaneously high sound transmission loss (STL) and the weight reduction by utilizing a concept of an acoustic metamaterial.

The concept of a metamaterial was introduced by Veselago [1] to allow the manipulation of electromagnetic wave propagation. Metamaterials are artificial engineered materials those usually comprise periodic structures with periods shorter than the wavelength to be manipulated. Split-ring
resonators, for example, are periodically aligned as unit cell structures. Inspired by the analogy between the governing equations of the electromagnetic and acoustic waves, the concept of using metamaterial to manipulate acoustic waves has been extended. Liu et al. first reported an acoustic metamaterial that includes periodically arranged lead balls coated with silicone rubber to enhance STL [3]. Lewińska et al. proposed a locally resonant acoustic metamaterial with inclusions coated with viscoelastic material to manipulate band gaps [4]. Jimenez et al. enhanced sound absorption in the low frequency range with Helmholtz resonators aligned in a thin panel at intervals less than the wavelength [5]. Sui et al. realized high STL below 1 kHz using membranes attached on a honeycomb [6]. Langfeldt et al. proposed a membrane acoustic metamaterial with a ring mass and perforations that improve transmission loss at low frequencies [7]. Varanasi et al. presented a planar cellular metamaterial that increased STL at low frequencies range relative to a homogeneous solid panel of equal area-mass density [8]. Xiao et al. and Wang et al. developed metamaterial thin plates comprising multiple sub-wavelength arrays of spring-mass resonators [9] [10] [11]. Oudich et al. extended a general approach for computing the STL of a thick plate embedded with a square periodic array of low-frequency spring-mass resonators [12].

The one of the authors of this proceeding presented an acoustic metamaterial insulator embedded periodically with Helmholtz resonators in sub-wavelength periods at ICSV 25. However, the insulator has drawbacks due to the coupled effect between the resonator and the intermediate air in the resonant transmission frequency range.

In this study, we propose a new acoustic metamaterial insulator that is embedded periodically with Helmholtz resonators in sub-wavelength periods backed by thin membranes to enhance STL at the resonant transmission frequency range. Analytical solutions for the STL of the proposed AMI are derived based on the effective medium theory. A linear harmonic regime is assumed for vibration and sound herein, and time dependence $e^{j\omega t}$ is omitted, where $j$ is the imaginary unit and $\omega$ is the angular frequency.

2. Transmission loss by effective medium theory

![Figure 1: Sound insulation by double wall embedded with Helmholtz resonator and membrane.](image)

A conventional double wall insulation system that has two impermeable plates with intermediate air layer generally degrades sound transmission loss at the resonant frequency where the plates and the intermediate air work as a mass and a spring, respectively.
In our previous proceeding, an acoustic metamaterial insulator in which Helmholtz resonators are embedded in both side of the insulator was proposed.

This acoustic metamaterial can improve sound transmission loss, however, the insulator can be thick since a large back cavity of Helmholtz resonator is required to realize relatively low Helmholtz resonance frequency.

As shown in Fig[1] in this study, an acoustic metamaterial insulator where a thin membrane is applied as an elastic vibrating wall of the back cavity is proposed and verified through several numerical examples.

Sound transmission loss of a double wall system that consists from an elastic plate covered by the proposed acoustic metamaterial insulator is derived using the effective medium theory where the displacements of the plate and the insulator are assumed to be uniform throughout the unit cell, and acoustic pressure in the back cavity is also assumed to be uniform. This assumption can be valid when the periodicity is much smaller that the acoustic wave length. In the model presented here, Helmholtz resonators are assumed to be embedded periodically within one fourth of the acoustic wave length.

When $x_{s2}$ and $x_{r2}$ are the displacements of the insulator and the air in the throat, respectively, and $v_{s2}$ and $v_{r2}$ are the velocities of the insulator and the air in the throat, respectively, equations of motions for the insulator and the membrane are respectively written as follows,

$$m_{s2}\ddot{x}_{s2} = (S_0 - S_{r1})p_a(L, y) + (S_{r1} - S_{r2})p_{r1} - (S_0 - S_{m1})p_t - \frac{k_{m1}}{j\omega}(v_{s2} - v_{r2}),$$

$$m_{n2}\ddot{x}_{r2} = p_{r1}S_{m1} - p_tS_{m1} - \frac{k_{m1}}{j\omega}(v_{r2} - v_{s2}),$$

where $S_0$, $S_{r1}$, and $S_{r2}$ are the area of the unit cell, the throat section, and the thin membrane, respectively, and $S_{m1}$ is equal to $S_{r2}$. $m_{s2}$ and $m_{n2}$ are mass of the insulator and the thin membrane, $p_t$ and $p_{r1}$ are sound pressure of the incident and the transmitted plane acoustic waves, respectively. $p_a$ is sound pressure in the intermediate air, and $p_{r1}$ is sound pressure in the back cavity of Helmholtz resonator. $k_{m1}$ is an equivalent spring coefficient for the first resonant frequency of the thin membrane.

Divided by $S_0$ and $S_{m1}$, these equations of motions are rewritten as,

$$z_{s1}v_{s1} = p_t + p_{r1} - p_a(0, y),$$

$$z_{s2}v_{s2} = (1 - \beta_1)p_a(L, y) + (\beta_1 - \gamma_1)p_{r1} - (1 - \gamma_1)p_t - \gamma_1 z_{m1}(v_{s2} - v_{r2}),$$

$$z_{n1}v_{r1} = p_a(L, y) - p_{r1},$$

$$z_{n2}v_{r2} = p_{r1} - p_t - z_{m1}(v_{r2} - v_{s2}),$$

where $\beta_1 = \frac{S_{r1}}{S_0}, \gamma_1 = \frac{S_{r2}}{S_0}, \frac{k_{m1}}{S_{j\omega}} = z_{m1},$ and $\frac{k_{m1}}{S_{j\omega}} = \frac{S_{m1}}{S_0}$. Now $z_{s1}$, $z_{s2}$, $z_{r1}$, and $z_{r2}$ are mechanical impedances of the plate, the insulator, the air in the throat, and the thin membrane, respectively. These impedances can be expressed as follows,

$$z_{s1} = \rho_s h_{s1} j\omega + D_1 (1 + j\eta) \frac{1}{j\omega},$$

$$z_{s2} = \rho_s (h_{s2} - \beta_1 h_{r1} - h_{r1}) j\omega + D_2' (1 + j\eta) \frac{1}{j\omega},$$

$$z_{n1} = (\rho_1 c L_{r1} + 2\rho_0 \delta r_1) j\omega + \alpha_1 \frac{1}{2} \sqrt{2\mu_0 \rho_0 \omega},$$

$$z_{n2} = \rho_m h_{m} j\omega,$$

where $D_1$ and $D_2'$ is the equivalent bending stiffness of the plate and the acoustic metamaterial insulator embedded periodically with resonators, respectively. $\rho_{s1}$, $\rho_{s2}$, $\rho_0$, $\rho_m$ are mass densities of the plate, the insulator, air, and the thin membrane, respectively. $h_{s1}$, $h_{s2}$, and $h_{m}$ are the thicknesses of...
the plate, the insulator, and the thin membrane, respectively. \( \eta \) and \( \eta_2 \) are loss factors of the plate and the insulator, respectively. \( k_y \) is \( y \) component of acoustic wave number. The other parameters are defined as \( h_{v1} = \frac{V_{r1}}{S_0} \), \( k_{r1} = \frac{K_0}{h_{r1}} \), and \( k_a = \frac{K_0}{L_a} \) where \( K_0 \) is bulk modulus of air.

When the radius of the throat is narrow, viscous loss in the throat should be considered. \( \rho_{1c}^e \) is the equivalent complex density, \( r_1 \) is the radius of the throat, and \( \mu_0 = 1.84 \times 10^{-5} \) Ns/m\(^2\) is the viscosity of air. \( \alpha_1 \) and \( \delta_1 \) are the coefficients for end corrections of the resistive and reactive terms, respectively, and are given by Allard and Atalla \[13\] as

\[
\alpha_1 = 4 + 2\frac{L_1}{r_1}, \quad \delta_1 = 0.48\sqrt{\frac{\pi}{1 - 1.14\sqrt{\beta_1}}},
\]

which is valid for \( \beta_1 < 0.4 \). The equivalent complex density can be expressed with a function \( F \) that is defined in accordance with the shape of the flow-channel section \[13\] \[14\] given as

\[
\rho_{1c}^e = \frac{\rho_0}{F(\mu_0/\rho_0, r_1)}.
\]

For a circular shape, \( F \) is given by

\[
F(\eta, r_1) = 1 - \frac{2}{sr_1} \frac{J_1(sr_1)}{J_0(sr_1)}, \quad s = \left( -\frac{j\omega}{\eta} \right)^{1/2},
\]

where \( r_1 \) is the radius of the throat and \( J_i \) is the \( i \)-th order Bessel function. \( F \) can be derived from the analytical solution of the linearized Navier-Stokes equation for compressive viscous fluid \[14\]. The real and imaginary parts of \( 1/F(\eta, r_1) \) in Eq. \( (13) \) represent the inertia effect and viscous loss that arise in the sound propagation through a narrow cylindrical flow channel, respectively.

Volume velocity of air is assumed to be continuous at the boundaries between air and the plate or the insulator, the following equations can be deduced as,

\[
\begin{align*}
&v_i \cos \theta - v_r \cos \theta = v_{s1} = v_{ax}(0, y), \\
&v_{ax}(L, y) = (1 - \beta_1)v_{s2} + \beta_1v_{r1}, \\
&(1 - \gamma_1)v_{s2} + \gamma_1v_{r2} = v_t \cos \theta.
\end{align*}
\]

Here \( \theta \) is the angle in which the incident acoustic plane wave is coming, \( v_{s1} \) and \( v_{r1} \) are the respective velocities of the plate and the air in the throat. \( v_{ax} \) is \( x \) component of particle velocity of air in the intermediate air gap. Substituting the expressions of the mechanical impedances and the boundary conditions into the equations of motions, the equations to be solved are derived as follows,

\[
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34} \\
a_{41} & a_{42} & a_{43} & a_{44}
\end{bmatrix}
\begin{bmatrix}
v_{s1} \\
v_{s2} \\
v_{r1} \\
v_{r2}
\end{bmatrix} =
\begin{bmatrix}
2p_t \sinh(jk_x L_a) \\
0 \\
0 \\
0
\end{bmatrix},
\]

(17)
where
\[
\begin{align*}
  a_{11} &= (z_{s1} + z_{\theta}) \sinh(jk_x L_a) + z_{\theta} \cosh(jk_x L_a) \\
  a_{12} &= -(1 - \beta_1) z_{\theta} \\
  a_{13} &= -\beta_1 z_{\theta} \\
  a_{14} &= 0 \\
  a_{21} &= a_{12} \\
  a_{22} &= [z_{s2} + (\beta_1 - \gamma_1) z_{r1} - (\beta_1 - \gamma_1) z_{r2} + (1 - \gamma_1)^2 z_{\theta} + \gamma_1 z_{m1}] \sinh(jk_x L_a) \\
  &\quad + (1 - \beta_1)^2 z_{\theta} \cosh(jk_x L_a) \\
  a_{23} &= -(\beta_1 - \gamma_1) z_{r1} \sinh(jk_x L_a) + (1 - \beta_1) \beta_1 z_{\theta} \cosh(jk_x L_a) \\
  a_{24} &= [(\beta_1 - \gamma_1) z_{r2} + (1 - \gamma_1) \gamma_1 z_{\theta} - \gamma_1 z_{m1}] \sinh(jk_x L_a) \\
  a_{31} &= a_{13} \\
  a_{32} &= a_{23} \\
  a_{33} &= \beta_1 (z_{n1} + z_{r1}) \sinh(jk_x L_a) + \beta_1^2 z_{\theta} \cosh(jk_x L_a) \\
  a_{34} &= -\beta_1 z_{r2} \sinh(jk_x L_a) \\
  a_{41} &= a_{14} \\
  a_{42} &= a_{24} \\
  a_{43} &= a_{34} \\
  a_{44} &= \gamma_1 (z_{n2} + z_{r2} + z_{\theta} \gamma_1 + z_{m1}) \sinh(jk_x L_a)
\end{align*}
\]

Solutions can be obtained by setting \( p_i = 1 \), and sound pressure in the transmitted field \( p_t \) can be calculated by \( p_t(\theta) = z_{\theta} v_{s2} \). Finally, sound transmission loss \( TL_1 \) when the plane acoustic wave is incident with the angle \( \theta \) is given by,
\[
TL_1(\theta) = 10 \log_{10} \left| \frac{1}{p_t(\theta)} \right|^2.
\]

### 3. Demonstrations

Steel plate of 0.8 mm thickness is covered by the insulator made from expanded polypropylene (EPP) of 20 mm thickness with the intermediate air of 5 mm thickness. Young’s modulus, Poisson’s ratio, mass density, and loss factor of steel are 210 GPa, 0.29, 7860 kg/m\(^3\), and 0.001, respectively. Young’s modulus, Poisson’s ratio, mass density, and loss factor of EPP are 170 MPa, 0.35, 30 kg/m\(^3\), and 0.05, respectively. Helmholtz resonators are periodically embedded in the insulator with 15 mm periodicity. The radius and length of the throat of the Helmholtz resonator are 1.5 mm and 10 mm, respectively, and the radius and height of back cavity are 5 mm and 10 mm, respectively. The uncoupled Helmholtz resonant frequency \( f_r \) is calculated as 1.5 kHz.

Circular thin membrane is assumed to be applied without any initial tension, and all edges are clamped at the wall of Helmholtz resonator. The first resonance frequency of the thin membrane \( f_m \) of 0.1 mm thickness is now designed as 850 Hz. Poisson’s ratio \( \nu_m \), mass density \( \rho_m \), and loss factor \( \eta_m \) are 0.35, 1100 kg/m\(^3\), and 0.05, respectively. When the radius of the membrane is set to 1.4 mm, Young’s modulus of the membrane \( E_m \) is calculated as 1.9 MPa by using the formula \([15]\) given by,
\[
 f_m = \frac{1}{2\pi} \frac{10.2158}{r_m^2} \sqrt{\frac{D_m}{\rho_m h_m}},
\]
where \( D_m = E_m h_m^3 / 12(1 - \nu_m^2) \) is the flexural rigidity of the circular membrane.
Figure 2 shows sound transmission loss when the acoustic plane wave is impinged at normal direction. Black line gives sound transmission loss for the conventional solid insulator without resonator and thin membrane, blue line for the insulator periodically embedded with Helmholtz resonators, and red lines for the proposed acoustic metamaterial insulator periodically embedded with Helmholtz resonators backed by the thin membranes. Sound transmission loss is improved around 800 Hz by the proposed acoustic metamaterial insulator.

The first resonance frequency of the thin membrane is then designed as 2.0 kHz. When the radius of the membrane is set to 1.2 mm, Young’s modulus of the membrane $E_m$ is calculated as 5.7 MPa.

As shown in Fig. 3, sound transmission loss is improved around 2.0 kHz by the proposed acoustic metamaterial insulator.

4. Conclusions

An acoustic metamaterial insulator embedded periodically with small resonators backed by thin membranes is proposed in this study. The double wall system utilizing the proposed acoustic metamaterial insulator with an intermediate air layer could achieve higher STL in low and mid frequency range compared with a conventional double wall system. Theoretical STL results validate the efficiency of the proposed acoustic metamaterial insulator. The embedded resonators work as a kind of a tuned mass damper to the mass-spring resonance of the double wall, however, it produce drawbacks due to the coupled effect between the resonators and the intermediate air. In this study thin membrane is proposed to attach at the back surface of the resonators. By designing the resonant frequency of the membrane, the drawback of STL can be enhanced.
Figure 3: Sound transmission loss for acoustic metamaterial insulator with membranes resonating at 2.0 kHz.
REFERENCES


