FUNDAMENTAL FREQUENCY OF COMPOSITE PLATE WITH STAIRCASE INTERNAL-THICKNESS-TAPER

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Considering the outstanding engineering properties, such as high strength/stiffness to weight ratios and the capability to be stiff at one location and flexible at another location as desired, internally-thickness-tapered composite plates are used in aerospace, mechanical and green power generation structures. Due to its distinct characteristics, such plates require comprehensive research to understand their dynamic response. In the present paper, the free vibration response of composite plates with staircase internal-thickness-taper configuration is considered considering clamped-free boundary condition. Since closed-form exact solution cannot be obtained for the resulting complex partial differential equation with variable coefficients in space and time coordinates, the Ritz method in conjunction with the Classical Laminated Plate Theory (CLPT) is used to obtain the system mass and stiffness matrices for out-of-plane bending vibration. In this approach, the stress and strain distributions in the laminated plate are determined in terms of mid-plane displacements and rotations corresponding to CLPT and as functions of taper angle and fiber orientation angle, and using these the kinetic and strain energies of the plate are calculated. Following the variational approach of the Rayleigh-Ritz method, the eigenvalue problem for the free vibration response is obtained, and the natural frequencies and mode shapes of the plate are determined by solving this eigenvalue problem. Numerical and symbolic computations have been performed using the software MATLAB. The influences of taper angle under thickness constraint and length constraint on the natural frequencies of the laminated composite plate are investigated for different stacking sequences of the laminate. Important design aspects are systematically brought out.

Keywords: Rayleigh-Ritz method, free vibration, composite plates

1. INTRODUCTION

Due to outstanding mechanical properties, composite materials are widely used in industry for years and they come in various shapes and structures depending on the requirements. As an instance, the tapered composite plates are popular in the aerospace industry and are used in manufacturing the structures such as rotor blades of helicopters and aircraft wings. Thickness reduction in tapered composites can be implemented by the termination of plies at different locations providing the tapered plate with customized stiffness property which is an absent capability in uniform laminates. The initial application of tapered laminated composites dates back to mid-1980s when commercial and military sectors demanded, elastically customizable components with higher weight to stiffness ratio [1].
In the course of the recent decades, few researchers have conducted analysis of such tapered structures. Recently, Seraj and Ganesan [2] investigated the dynamic instability of rotating doubly-tapered laminated composite beams under periodic rotational speeds. Liu and Ganesan [3] studied tapered composite plates for their dynamic instability. Ananda Babu et al [4] performed the dynamic characterization of thickness-tapered laminated composite plates using finite element analysis. Free vibration analysis of variable-stiffness laminated composite plates using Ritz method has not so far been conducted in existing literature and it is addressed in the present paper.

2. STRESS AND STRAIN TRANSFORMATIONS

The local coordinate system \(x''y''z''\) is considered for the \(k\)th layer with the \(x''\) axis directed along the fiber orientation and \(z''\) perpendicular to the surface of the layer as shown in Fig. 1. By the angle \(\alpha\), global coordinate system \(xyz\) is rotated counter-clockwise about the \(y\)-axis to establish the \(x'y'z'\) coordinate system and in turn, \(x'y'z'\) is rotated by angle \(\beta\), counter-clockwise, to correspond to the local coordinate system \(x''y''z''\).

![Diagram showing coordinate systems](image)

Figure 1: Ply orientation in the tapered laminated plate (left) and staircase taper configuration (right)

The stress-strain relationship in the global coordinate system is written for the \(k\)th layer.

\[
[\sigma]_{6\times1}^{[k]} = [C]_{6\times6}^{[k]}[\varepsilon]_{6\times1}^{[k]} \tag{1}
\]

Where \([C]_{6\times6}^{[k]}\), \([C']_{6\times6}^{[k]}\) and \([C'']_{6\times6}^{[k]}\) are stiffness matrices in \(xyz\), \(x'y'z'\), and \(x''y''z''\) coordinate systems respectively. The relation between stress and strain matrices, in \(x'y'z'\) and \(xyz\) coordinate systems, according to [6] is expressed:

\[
[\sigma'] = [T_{\alpha\alpha}][\sigma] \tag{2}
\]

\[
[\varepsilon'] = [T_{\varepsilon\alpha}][\varepsilon] \tag{3}
\]

Considering Eqs. (1), (2) and (3) for the coordinate systems depicted by Fig. 1, the relation between the stiffness matrices in local and global coordinate systems \(x''y''z''\) and \(xyz\) for the \(k\)th layer is as follows.

\[
[C]^{[k]} = [T_{\alpha\alpha}]^{-1}[T_{\alpha\beta}]^{-1}[C'']^{[k]}[T_{\varepsilon\beta}][T_{\varepsilon\alpha}] \tag{4}
\]
3. FORMULATION BASED ON CLPT

In order to apply the Ritz method and to determine the fundamental frequency, stiffness and mass matrices are obtained from the calculation of displacements, strains and stresses expressed based on Classical Laminated Plate Theory (CLPT). Considering the pure bending condition (displacement on midplane is zero in \( x \) and \( y \) directions), displacements and strains are as follows.

\[
\begin{align*}
\frac{\partial}{\partial x} &= 0 \\
\frac{\partial}{\partial y} &= 0 \\
\frac{\partial}{\partial z} &= 0
\end{align*}
\]

\[
\begin{align*}
u &= -\frac{\partial w_o}{\partial y}z \\
w &= w_o \\
\epsilon_x &= -\frac{\partial^2 w_o}{\partial x^2}z \\
\epsilon_y &= -\frac{\partial^2 w_o}{\partial y^2}z \\
\gamma_{xy} &= -2\frac{\partial^2 w_o}{\partial x \partial y}z
\end{align*}
\]

Where \( u, v \) and \( w \) are displacements in \( x, y \) and \( z \) directions, respectively. In order to facilitate further calculation of strain and kinetic energies, Eqs. (5) to (10) are written in form of multiplication of matrices using the joint matrix \([s]\). Equations (11) to (13) represents these matrices in a detailed form.

\[
\begin{align*}
[Z_u]_{3\times6} &= \begin{bmatrix} 0 & z & 0 & 0 & 0 & 0 \\
0 & 0 & z & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
[Z_\epsilon]_{3\times6} &= \begin{bmatrix} 0 & 0 & 0 & z & 0 & 0 \\
0 & 0 & 0 & 0 & z & 0 \\
0 & 0 & 0 & 0 & 0 & z \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
[s]_{6\times1} &= \begin{bmatrix} w_o \frac{\partial w_o}{\partial x} \frac{\partial w_o}{\partial y} -\frac{\partial^2 w_o}{\partial x^2} -\frac{\partial^2 w_o}{\partial y^2} -2\frac{\partial^2 w_o}{\partial x \partial y} \end{bmatrix}^T
\end{align*}
\]

Equations (5) to (10) are expressed in matrix using Eqs. (11) to (13).

\[
\begin{align*}
[u]_{3\times1} &= [Z_u]_{3\times6} [s]_{6\times1} \\
[\epsilon]_{3\times1} &= [Z_\epsilon]_{3\times6} [s]_{6\times1}
\end{align*}
\]

Where \([u]\) and \([\epsilon]\) are column matrices containing displacements and strains elements, respectively. The stresses are determined using the stress-strain relationship. The elements of the stiffness matrix are obtained using Eq. (4) and engineering constants of the material which have been given by Table 2.

Considering the CLPT assumptions, the reduced stiffness matrix in the stress-strain relationship is expressed.

\[
\begin{align*}
\begin{bmatrix} \sigma_x \\
\sigma_y \\
\tau_{xy} \end{bmatrix} &= \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\
Q_{12} & Q_{22} & Q_{26} \\
Q_{16} & Q_{26} & Q_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x \\
\epsilon_y \\
\gamma_{xy} \end{bmatrix}
\end{align*}
\]

Equation (16) is express in compact form as follows.

\[
\begin{align*}
[\sigma]_{3\times1} &= [Q]_{3\times3} [\epsilon]_{3\times1}
\end{align*}
\]

The resin is considered as an isotropic material and the corresponding stiffness matrix is independent of angles \( \alpha \) and \( \beta \). The reduced stiffness matrix based on CLPT can be obtained considering the CLPT assumptions.

Displacements, strains, and stresses presented by Eqs. (14), (15) and (17) are essential in energy calculation in the next section. Function \( w_o \) introduced by Eq. (7), is expressed in the form of series.
\[
W_o = \sum_{i=1}^{I} \sum_{j=1}^{I} A_{ij} X_i(x) Y_j(y)
\]  
(18)

Terms \(X_i\) and \(Y_j\) are admissible functions dependent on the boundary conditions. The stresses and strains are key components for strain energy \(U\) calculation, and the density of the material is the essential factor in kinetic energy \(T\) computation. The strain and kinetic energies are expressed according to [6] and written in matrix form as follows.

\[
U = \frac{1}{2} \iiint \sigma^t [\varepsilon] \, dV
\]  
(19)

\[
T = \frac{1}{2} \omega^2 \iiint \rho [u]^t [u] \, dV
\]  
(20)

Matrix form is superior in terms of computational efficiency. Hence, kinetic and strain energies are calculated at the same time in this form. In order to apply the Ritz method, the derivatives of the strain and kinetic energies with respect to parameters \(A_{mn}\) are denoted by \(\bar{U} = \partial U / \partial A_{mn}\) and \(\bar{T} = \partial T / \partial A_{mn}\) and calculated using Eqs. (19) and (20).

\[
\bar{U} = \frac{1}{2} \iiint ([Q] [\dot{\varepsilon}])^t [\varepsilon] + ([Q] [\varepsilon])^t [\dot{\varepsilon}] \, dV
\]  
(21)

\[
\bar{T} = \frac{1}{2} \omega^2 \iiint \rho ([\ddot{u}]^t [u] + [u]^t [\ddot{u}]) \, dV
\]  
(22)

Equations (21) and (22) are written in the following form.

\[
\bar{U} = \frac{1}{2} \iiint [\dot{\varepsilon}]^t [Q] [\varepsilon] + ([\dot{\varepsilon}]^t [Q] [\varepsilon])^t \, dV
\]  
(23)

\[
\bar{T} = \frac{1}{2} \omega^2 \iiint \rho ([\ddot{u}]^t [u] + ([\ddot{u}]^t [u])^t) \, dV
\]  
(24)

The terms \([\dot{\varepsilon}]^t [Q] [\varepsilon]\) and \([\ddot{u}]^t [u]\) on the right-hand side of the Eqs. (23) and (24) should be scalars as they follow scalar values of \(\bar{U}\) and \(\bar{T}\) on the left-hand side. In addition, this can also be realized from the size of the matrices on the right-hand side. Therefore, considering scalars \([\dot{\varepsilon}]^t [Q] [\varepsilon]\) and \([\ddot{u}]^t [u]\):

\[
[\dot{\varepsilon}]^t [Q] [\varepsilon] = ([\dot{\varepsilon}]^t [Q] [\varepsilon])^t
\]  
(25)

\[
[\ddot{u}]^t [u] = ([\ddot{u}]^t [u])^t
\]  
(26)

Equations. (25) and (26) are substituted in Eqs. (23) and (24).

\[
\bar{U} = \iiint [\dot{\varepsilon}]^t [Q] [\varepsilon] \, dV
\]  
(27)

\[
\bar{T} = \omega^2 \iiint \rho [\ddot{u}]^t [u] \, dV
\]  
(28)

Substituting Eqs. (15) and (14), in Eqs. (27) and (28), respectively, one can get:

\[
\bar{U} = \iiint [\dot{\varepsilon}]^t [Z_e] [Q] [Z_e] [s] \, dV
\]  
(29)

\[
\bar{T} = \omega^2 \iiint \rho [\ddot{u}]^t [Z_u] [Z_u] [s] \, dV
\]  
(30)

Equations (29) and (30) are written in the following form.

\[
\bar{U} = \int [\dot{\varepsilon}]^t \left( \int_{-\frac{h}{2}}^{\frac{h}{2}} [Z_e] [Q] [Z_e] \, dz \right) [s] \, dA
\]  
(31)
\[ \hat{T} = \iint [\mathbf{s}]^t \left( \omega^2 \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho [\mathbf{Z}_u]^t [\mathbf{Z}_u] \, dz \right) [\mathbf{s}] \, dA \]  

(32)

Integration through the laminate thickness, matrices containing functions of \( z \) are taken into account within the integral so that matrices \([\mathbf{Z}_e], [\mathbf{Z}_u]\) and \( Q_{ij} \) as well as scalar \( \rho \) participate in the integration.

\[ \hat{U} = \iint [\mathbf{s}]^t [\mathbf{Z}_e] [\mathbf{s}] \, dA \]  

(33)

\[ \hat{T} = \iint [\mathbf{s}]^t [\mathbf{Z}_u] [\mathbf{s}] \, dA \]  

(34)

\[ [\mathbf{Z}_e] = \int_{-\frac{h}{2}}^{\frac{h}{2}} [\mathbf{Z}_e]^t [\mathbf{Q}] [\mathbf{Z}_e] \, dz \]  

(35)

\[ [\mathbf{Z}_u] = \omega^2 \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho [\mathbf{Z}_u]^t [\mathbf{Z}_u] \, dz \]  

(36)

Equations (33) and (34) are written in the following form.

\[ \hat{E}_{u,T} = \iint [\mathbf{s}]^t [\mathbf{Z}_{e,u}] [\mathbf{s}] \, dA \]  

(37)

Equation (37) has been written in combined form such that depending on the calculation performed for derivatives of strain or kinetic energies (\( \hat{U} \) or \( \hat{T} \)), matrices \([\mathbf{Z}_e]\) or \([\mathbf{Z}_u]\) is considered. After computation of matrices \([\mathbf{Z}_e]\) and \([\mathbf{Z}_u]\), derivatives of kinetic and strain energies with respect to parameters \( A_{mn} \) are obtained using Eq. (37). Any nonzero elements of \([\mathbf{Z}_e]\) and \([\mathbf{Z}_u]\), are denoted by \( R_{ij}^{(n)} \) and \( R_{ij}^{(n)} \), respectively. The matrix \([\mathbf{Z}_e]\) contain the elements of the extensional stiffness matrix \( R_{ij}^{(1)} \), bending-extension coupling matrix \( R_{ij}^{(2)} \) and bending stiffness matrix \( R_{ij}^{(3)} \).

The final results for system coefficients are computed by replacing Eq. (18) in Eq. (37) and are presented here in the open form. Since the configurations of tapered laminates are made of (0/90) ply layup, elements \( Q_{16} \) and \( Q_{26} \) from reduced stiffness matrix are zero meaning that in-plane normal and shear are decoupled. The expressions for stiffness and mass coefficients are given within the brackets in the following two expressions for strain and kinetic energies.

\[
\begin{align*}
\hat{U} = \sum_{i=1}^{l} \sum_{j=1}^{j} \left( \int_{0}^{L} R_{12}^{(3)} X_m \ddot{X}_i \, dx \int_{0}^{L} \dddot{Y}_n \, dy + \int_{0}^{L} R_{11}^{(3)} \dddot{X}_i \, dx \int_{0}^{L} \ddot{Y}_n \, dy + \int_{0}^{L} R_{12}^{(3)} \dddot{X}_m \, dx \int_{0}^{L} \ddot{Y}_n \, dy ight) A_{ij} \\
+ 4 \int_{0}^{L} R_{22}^{(3)} \dddot{X}_m \, dx \int_{0}^{L} \ddot{Y}_n \, dy 
\end{align*}
\]

(38)

\[
\begin{align*}
\hat{T} = \omega^2 \sum_{i=1}^{l} \sum_{j=1}^{j} \left( \int_{x=0}^{x=L} R_{12}^{(3)} \dddot{X}_m \, dx \int_{y=0}^{y=L} \ddot{Y}_n \, dy + \int_{x=0}^{x=L} R_{11}^{(3)} X_m \ddot{X}_i \, dx \int_{y=0}^{y=L} \dddot{Y}_n \, dy + \int_{x=0}^{x=L} R_{12}^{(3)} X_m \dddot{X}_i \, dx \int_{y=0}^{y=L} \ddot{Y}_n \, dy ight) A_{ij} \\
+ \int_{x=0}^{x=L} R_{11}^{(1)} X_m \dddot{X}_i \, dx \int_{y=0}^{y=L} \dddot{Y}_n \, dy 
\end{align*}
\]

(39)
4. FREE VIBRATION ANALYSIS

Considering \( \bar{\Omega} \), in Eq. (38), for the fixed values of \( m \) and \( n \), the indices \( i \) and \( j \) are counted up to the upper bound of the summations \( I \) and \( J \). Therefore, there are \( I \times J \) number of terms that are written in the form of a row matrix multiplied by a column matrix \([A]\) containing \( I \times J \) number of parameters, \( A_{ij} \). By repeating the operation for all possible values for \( m \) and \( n \), there are produced \( I \times J \) number of row matrices written one beneath the next one forming a matrix with the size of \( I \times J \) by \( I \times J \).

This \( I \times J \) by \( I \times J \) matrix produced from \( \bar{\Omega} \) is called stiffness matrix and denoted by \([K]\). In a similar manner for \( \hat{\Omega} \), a matrix with the same size is formed and called mass matrix denoted by \([M]\). Considering \( \bar{\Omega} = \hat{\Omega} \):

\[
[K]_{IJ\times J}[A]_{IJ\times J} = \omega^2[M]_{IJ\times J}[A]_{IJ\times J} \tag{40}
\]

In order to obtain a non-trivial solution from eigenvalue problem given by Eq. (40):

\[
det([K] - \omega^2[M]) = 0 \tag{41}
\]

By solving the eigenvalue problem in which \( \omega^2 = \omega^2 \) and the column matrix \([A]\) are eigenvalues and eigenvectors, respectively, natural frequencies and mode shapes are determined. The square root of the smallest eigenvalue is the fundamental frequency.

The fundamental frequency is obtained for the described taper laminate (Fig. 1) for different boundary conditions. Considering Eq. (18), admissible functions \( X_i(x) \) and \( Y_j(y) \) corresponding to the boundary conditions SSSS, CCCC and CCFF are in the Table 1. The clamped edges for the CCFF boundary condition correspond to \( x = 0 \) and \( y = 0 \) lines and free edges to \( x = L \) and \( y = L \) lines.

| Table 1: Admissible functions corresponding to the boundary conditions [6, 7] |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| SSSS                           | CCCC                           | CCFF                           |
| \( X_i = \sin\left(i \pi \frac{x}{L}\right) \) | \( X_m = \cos\left(\lambda_i \frac{x}{L}\right) - \cosh\left(\lambda_i \frac{x}{L}\right) - \gamma_i \left[\sin\left(\lambda_i \frac{x}{L}\right) - \sinh\left(\lambda_i \frac{x}{L}\right)\right] \) | \( X_m = \cos\left(\lambda_i \frac{x}{L}\right) - \cosh\left(\lambda_i \frac{x}{L}\right) - \gamma_i \left[\sin\left(\lambda_i \frac{x}{L}\right) - \sinh\left(\lambda_i \frac{x}{L}\right)\right] \) |
| \( Y_j = \sin\left(j \pi \frac{y}{L}\right) \) | \( Y_n = \cos\left(\lambda_j \frac{y}{L}\right) - \cosh\left(\lambda_j \frac{y}{L}\right) - \gamma_j \left[\sin\left(\lambda_j \frac{y}{L}\right) - \sinh\left(\lambda_j \frac{y}{L}\right)\right] \) | \( Y_n = \cos\left(\lambda_j \frac{y}{L}\right) - \cosh\left(\lambda_j \frac{y}{L}\right) - \gamma_j \left[\sin\left(\lambda_j \frac{y}{L}\right) - \sinh\left(\lambda_j \frac{y}{L}\right)\right] \) |
| \( i, j = 1 \)                  | \( i, j = 2 \)                  | \( i, j = 3 \)                  | \( i, j = 4 \)                  |
| \( \lambda_i \)                | \( \lambda_j \)                | \( \lambda_j \)                | \( \lambda_j \)                |
| \( 4.730,040,08 \)              | \( 7.853,204,6 \)               | \( 10.995,607,8 \)              | \( 10.995,540,7 \)              |
| \( \gamma_i \)                 | \( \gamma_j \)                 | \( \gamma_j \)                 | \( \gamma_j \)                 |
| \( 0.982,502,2 \)               | \( 1.000,777,3 \)               | \( 0.999,966,4 \)               | \( 1.000,033,553,2 \)           |

5. NUMERICAL RESULTS

Two tapered laminated square plates with configuration of \( (0/90)_{9S} \) and \( (0/90)_{3S} \) at the left and right ends, respectively, made of resin and unidirectional plies of NCT-301 Graphite-Epoxy material
with ply thickness of $125 \times 10^{-6}$ m are considered (Fig. 1). The lengths of the laminates are dependent on the taper angle and are 85.944 cm and 17.188 cm for 0.1° and 0.5° taper angles, respectively.

The composite layer is considered as a transversely-isotropic material and the corresponding stiffness matrix is denoted by $[C'']$. The mechanical properties of NCT-301 Graphite-Epoxy material are given in Table 2.

Table 2: Mechanical Properties of NCT-301 Graphite-Epoxy [5] Ply and epoxy resin

<table>
<thead>
<tr>
<th>Mechanical Properties of Unidirectional NCT-301 Graphite – Epoxy</th>
<th>Resin</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1 = 113.9 \text{ GPa}$</td>
<td>$E = 3.93 \text{ GPa}$</td>
</tr>
<tr>
<td>$G_{12} = 3.137 \text{ GPa}$</td>
<td>$G_{23} = 2.852 \text{ GPa}$</td>
</tr>
<tr>
<td>$v_{12} = 0.288$</td>
<td>$v_{21} = 0.018$</td>
</tr>
<tr>
<td>$\rho_{\text{ply}} = 1480 \text{ kg/m}^3$</td>
<td>$\rho_{\text{resin}} = 1000 \text{ kg/m}^3$</td>
</tr>
</tbody>
</table>

Solving the eigenvalue problem given by Eq. (41), the first natural frequencies obtained for the square plates with the staircase taper configuration with different boundary conditions are given in Table 3. Numerical solutions using the finite element software ANSYS® were also obtained and compared with the present Ritz solutions. The finite element solution has been obtained using the four-node element SHELL 181 in ANSYS® and converged meshes of 2808 and 195 elements have been obtained for the plates of side length 85.944 cm and 17.188 cm, respectively. The finite element model developed in ANSYS® is shown in Fig. 2. First mode shape of the plate of side length 85.944 cm is shown in Fig. 2(b) for different boundary conditions.

Figure 2: Finite element model of the tapered laminate: (a) Converged mesh and (b) First mode shapes

Table 3: Fundamental frequencies of plates (rad/s) for different boundary conditions

<table>
<thead>
<tr>
<th>BC</th>
<th>Length/Mean Thickness = 286.5</th>
<th>Length/Mean Thickness = 57.3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Length (cm)/Angle (deg) = 85.944/0.1°</td>
<td>Length (cm)/Angle (deg) = 17.188/0.5°</td>
</tr>
<tr>
<td>Ritz Method</td>
<td>Finite Element Method</td>
<td>Ritz Method</td>
</tr>
<tr>
<td>SSSS</td>
<td>108.0</td>
<td>108.5</td>
</tr>
<tr>
<td>CCC</td>
<td>229.2</td>
<td>230.0</td>
</tr>
<tr>
<td>CCFF</td>
<td>52.2</td>
<td>52.3</td>
</tr>
</tbody>
</table>
6. CONCLUSION

Tapered composite plate with staircase taper configuration has been considered. The displacements, strains, and stresses based on CLPT are expressed, and then, the mass and stiffness matrices were developed based on the Ritz method for the tapered plates with different boundary conditions. From the analysis of the results, it is concluded that:

➢ The fundamental vibration frequency for the CCCC plate is the highest and that of the CCFF plate is the lowest, having a ratio of 4.39. The frequencies of CCCC and SSSS plates have a ratio of 2.12.
➢ The plies close to the midplane do not significantly contribute to increasing the fundamental frequency even though their inertial (mass) contribution is the same as that of other plies.
➢ The resin which is the weaker material is used in the regions close to the midplane, and the composite plies are used in farther layers. Therefore, composite plies contribute with higher capacity in increasing the stiffness of the plate and the resin does not take part notably in tailoring the stiffness yet used instead of plies to avoid any significant increase in the plate’s weight. Hence, in order to maximize the fundamental frequency, the materials with higher stiffness property should be used for the external layers of the plate, and weaker materials should be used for inner layers close to the midplane.

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REFERENCES