SIDE EFFECTS OF USING COIL SPRINGS AS AN APPROXIMATION OF FREE BOUNDARY CONDITIONS IN MODAL ANALYSIS

Miroslav Janota
Czech Technical University in Prague, Faculty of Mechanical Engineering, Prague, Czech Republic
email: m.janota@rcmt.cvut.cz, orcid id: 0000-0002-6513-200X.

In the modal tests used to verify the finite element model, free boundary conditions are usually simulated. It is generally known that any kind of boundary conditions affect the measured modal parameters. The modal damping rather than the modal frequency is affected more significantly. Due to this, the use of coil springs for fixturing the measured parts seems to be optimal in terms of damping. Coil springs themselves have very little damping. However, during a series of experiments, coil springs have been shown to negatively affect the vibration of the test piece and hence its modal parameters. The content of this paper is a description of this phenomenon and how to avoid it.

Keywords: Modal Analysis, Boundary Conditions, Coil Springs

1. Introduction

Experimental modal analysis is a measurement method that serves to experimentally identify modal parameters, which are damped natural frequencies, damping ratios and mode shapes. For detailed description see [1], [2] or [3].

Experimental modal analysis is done especially for the following reasons: 1) to identify the dynamic properties of the machine or component being measured, in relation to its behaviour during operation, 2) to compute different variants of machine parts or the effect of using different materials, and 3) to verify structural calculations. Particularly in the second and third cases, boundary conditions have a significant effect on the measurement results. In general, we can divide the boundary conditions as follows:

- free boundary conditions: these conditions are impossible to achieve but they are wanted because they are easy to simulate
- real support conditions: measured structure is somehow affected by its supports
- free boundary conditions approximation:
  - laying or suspension of measured structure on flexible elements
  - support measured structure in places nodal points of chosen mode

Influence of boundary conditions on modal parameters was discussed by Avitabile in [12], Brillhart in [10] and Carne in [8] and [9]. While Avitabile and Brillhart describe their long-term experience, Carne deduced that the most affected parameter is damping ratio. In the worst case the difference between real and measured value was over 1000 percent! In order to minimize effect of support condition...
he recommends that the first flexible mode frequency be at least ten times the highest rigid body mode frequency.

From this perspective, the use of steel coil springs seems to be an optimal solution. The springs, as steel parts, have a small damping and there is no problem to reach the recommended frequency ratio between rigid and flexible body mode. In our work a number of experiments were carried out in which coil springs were used for supporting of tested coupons. After the evaluation of the measurement results, it was clear that the use of coil springs was not a good choice. The purpose of this paper is to describe the problem that has arisen during testing. This resulted in the devaluation of measured data. This paper is divided as follows. In chapter 2 there is given a brief description of the used apparatus and measurement methods. In chapter 3 there are described case studies where the problem has arisen. In chapter 4 the cause of the problem is described and finally a summary is made in chapter 5.

2. Description of used apparatus and measurement technique

In all experiments, a PULSE or NI-USB 9234 analyser, a modal hammer and one or three-axis accelerometers were used. The weight of the modal hammer and accelerometers was selected according to the size and weight of the measured coupon. With the increasing mass of coupon, the more massive the hammer was needed, and vice versa. The lighter part, the lighter the accelerometer was needed to avoid the so-called "mass loading" effect. The apparatus was calibrated using a calibration vibrator and a calibration mass.

During the measurement, time signals from the hammer (force) and accelerometers (acceleration) were recorded. The frequency response function was calculated from the ratio of the cross-spectrum of force and acceleration and the force auto-spectrum, for detailed information see [2].

<table>
<thead>
<tr>
<th>Sensor description</th>
<th>Manufacturer</th>
<th>Type</th>
<th>S/N</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modal hammer</td>
<td>Brüel&amp;Kjaer</td>
<td>8206-003</td>
<td>54988</td>
<td>1.069 mV/N</td>
</tr>
<tr>
<td>Modal hammer</td>
<td>Endevco</td>
<td>2303</td>
<td>1075</td>
<td>0.225 mV/N</td>
</tr>
<tr>
<td>Tri-axial accelerometer</td>
<td>Brüel&amp;Kjaer</td>
<td>4524B</td>
<td>34298</td>
<td>x: 97.34 mV/g</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>y: 95.71 mV/g</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>z: 96.27 mV/g</td>
</tr>
<tr>
<td>Uniaxial accelerometer</td>
<td>PCB</td>
<td>352A21</td>
<td>36757</td>
<td>9.730 mV/g</td>
</tr>
</tbody>
</table>

3. Case studies

Over the past years, a number of tests have been carried out to evaluate the effect of particle component composites on improving the dynamic properties of machine tool rams. A detailed description of the experiments was made by Vrba in [4]. Since structural calculations were always carried out before experiments, free conditions were required. Because of the variability of the measured parts, coil springs with different dimensions were chosen. Their parameters are described in Table 2.

<table>
<thead>
<tr>
<th>name of the spring</th>
<th>d [mm]</th>
<th>D [mm]</th>
<th>no [-]</th>
<th>lo [mm]</th>
<th>k [kN/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>small coil spring</td>
<td>2.0</td>
<td>17.2</td>
<td>7.5</td>
<td>38.8</td>
<td>3.8</td>
</tr>
<tr>
<td>middle coil spring</td>
<td>3.8</td>
<td>28.0</td>
<td>5.3</td>
<td>44.0</td>
<td>17.9</td>
</tr>
<tr>
<td>large coil spring</td>
<td>9.2</td>
<td>50.0</td>
<td>7.8</td>
<td>138.5</td>
<td>72.8</td>
</tr>
<tr>
<td>G = 79 269 MPa</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho = 7900 \text{ kg.m}^{-3} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \nu = 0.3 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where: d wire diameter \( \text{D/R} \) mean coil diameter/radius
3.1 Testing of coupons made from particle component composite

As a first step, coupons in the form of long beams made from particle component composites were tested. The samples were deposited on the small coil springs described in Table 2 and on the rubber mats which were placed in the nodes of the first bending mode. The goal was to minimize the effect of supports on this mode. The dynamic compliance measured on the beam for both kind of support is shown in Figure 1. It can be seen from this figure that when measuring with springs, there was a certain kind of dispersion of one mode. If the measurements were made only with springs, it wouldn’t be possible to say that there was any kind of problem or inaccuracy of results.

3.2 Testing of steel profiles filled with particle component composite

Subsequently, steel welders in the shape of the "O" profile were measured. They were filled with different variances of particle composites. At the beginning, profiles were laid on a four middle coil springs located in the corners of the "O" profiles. The assumption was that the springs would not affect modal parameters. At the end, it was necessary to change spring position and move them to the nodes of the first bending mode. After that, there was an improvement, which can be seen in the Figure 2. Without changing spring position it would be difficult to say whether this is a reality or a false result. Other possible error indicator is MAC (Modal Assurance Criterion), which is shown in the Figure 3. Mode shapes of dissipated frequencies have the same MAC value, which is always indication that something has gone wrong. MAC is a number that is used to compare two shapes of vibration. MAC ranges from 0 to 1. If the MAC is 1, the compared shapes are identical and vice versa.

3.3 Testing of machine tool rams filled with particle component composite

Finally, a real machine tool part was measured. It was a thin-walled steel welder of ram. These rams are much stiffer than the "O" profiles, so they were expected to be more resistant against "coil spring effect". Rams were laid on four large coil springs. Measured dynamic compliances are shown in the Figure 4. In order to improve measured results, it was necessary to hang rams on a long rope.

3.4 Evaluation of experiments

The result of the use of coil springs as supports during modal tests was distortion or dissipation of particular modes. This phenomenon does not depend on coil spring dimension. It is hardly detectable if measured is not repeated under different support conditions. In the next chapter a possible explanation is provided.
Figure 1: Dynamic compliance of particle component composite coupon

Figure 2: Dynamic compliance of steel profile filled with particle component composite

Figure 3: MAC of mode shapes of steel profiles filled with particle component composites
4. **Explanation of the effects of coil springs on the resulting dynamic performance**

The basic mistake was to consider the real coil spring as its ideal replacement. This means to neglect its mass and assume only stiffness. The real coil spring has both stiffness and weight. We can neglect damping. This means that coil springs have resonances. This problematics is described by a number of publications. Here we can mention Becker [11] and Yildirim [5]. At the beginning of their publications, both refer to the basic formula for calculating the coil spring natural frequency. This formula is written in Eq. 1. At the same time, they add that this is an inaccurate calculation. However, if a quick calculation is required with a minimum of input parameters, the formula can also be used. In order to calculate frequencies depending on spring compression, it is also necessary to use Eq. 2 to Eq. 5. Input parameters are: the dimensions of the spring in the unloaded state, its material constants, and the deformation of the spring in the loaded state. The result is the natural frequency of compressed spring. A comparison of the calculated and measured values for the small and medium springs is given in Table 3. Coil springs were tested so that they were clamped into the hand vise. They were excited by a fingertip. Vibration response was measured by uniaxial accelerometer clamped on the jaw. Measured spectral densities of coil spring vibration are shown in the Figure 6. They are superimposed on frequency response functions from Figure 1, Figure 2 and Figure 4. One can clearly see that coil springs affect measured frequency response functions.

The results Table 3 show that the coil spring natural frequency depends on its deformation. The real (=measured) change is larger than the calculated one. This results in important knowledge that the behaviour of the springs during the tests is nonlinear. The springs are being visibly deformed during modal tests. This could be an explanation of the "distortion" of some modes. Frequency of distorted mode is the same as the natural frequency of used coil spring. Distortion is given by frequency variation caused by spring deformation. Verification of the linearity was carried out when measuring a steel beam with dimensions of 520 x 63 x 20 mm. Its dynamic compliance was measured at different levels of hammer hit. The steel beam was used to eliminate the non-linearity of the particle component composite coupon. The resulting dynamic compliance is shown in the Figure 5. It can be seen that the behaviour of the spring varies depending on the size of the input force.
\[ f_{co} = \frac{1}{8 \pi n_o c R}, \text{where } c = \frac{R}{r} \]  
\[ \tan(\alpha_o) = \frac{l_0}{2 R n_o \pi}; \]  
\[ \sin(\alpha) = \left(1 - \frac{l}{l_0} \right) \sin(\alpha_o) \]  
\[ n_0 = \frac{(1 + \nu) \cos^2 \alpha + \sin^2 \alpha}{(1 + \nu) \cos \alpha \cos \alpha_o + \sin \alpha \sin \alpha_o} \]  
\[ f_c = \frac{1}{8 \pi n_c R} \]  

The drawing above was taken from [11].

Table 3: Comparison of measured and calculated coil spring natural frequencies

<table>
<thead>
<tr>
<th></th>
<th>small coil spring</th>
<th>middle coil spring</th>
<th></th>
<th>large coil spring</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>l [mm]</td>
<td>fm [Hz]</td>
<td>fc [Hz]</td>
<td>error</td>
<td>l [mm]</td>
<td>fm [Hz]</td>
</tr>
<tr>
<td>35</td>
<td>329.5</td>
<td>313.4</td>
<td>-5.0 %</td>
<td>40</td>
<td>387.7</td>
</tr>
<tr>
<td>30</td>
<td>329.8</td>
<td>313.5</td>
<td>-5.0 %</td>
<td>35</td>
<td>392.5</td>
</tr>
<tr>
<td>25</td>
<td>343.3</td>
<td>313.6</td>
<td>-9.0 %</td>
<td>30</td>
<td>409.6</td>
</tr>
<tr>
<td>l: length of compressed spring</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fm: measured natural frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fc: calculated natural frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>error: error of calculation, see (6)</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

\[ error = \frac{(f_c - f_m)}{f_m} \]  

Figure 5: Verification of coil spring linearity
small coil spring behaviour during testing of coupons made from particle component composite

middle coil spring behaviour during testing of steel profiles filled with particle component composite

large coil spring behaviour during testing of machine tool rams filled with particle component composite

Figure 6: Effect of coil springs on measured FRFs
5. Conclusion

The main recommendation resulting from the experiments described above is not to use steel coil springs to support measured parts during modal analysis. This is due to their resonances and nonlinear behaviour. If necessary, it is possible to estimate the natural frequency of used coil spring using equations Eq. 1 to Eq. 5 and be sure that the upper limit of frequency range of interest lies below this frequency.

6. Acknowledgements

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