The objective of active vibration control of plain bearings is to extend their operating speed range. The control is carried out by a proportional controller that controls the position of the bearing bushing according to the deviation of the bearing journal position. The piezo actuators are used for bearing bushing motion. Increasing the signal-to-noise ratio of the journal position measurement is achieved with the use of the bandpass filter of the second order in the feedback loop which is tuned to the instability frequency of the whirl type. The center frequency of the bandpass filter is tuned to the rotor rotational speed. At the end of the paper, the results of experiments showing the extension of the bearing rotational speed range without rotor position instability are presented.

Keywords: journal bearing, active vibration control, piezo actuator, signal-to-noise ratio (SNR), bandpass filter

1. Introduction

Journal hydrodynamic bearings (alternatively called sleeve bearings or plain bearings for radial load) are a standard solution to support rotors. Their advantage is a possibility to carry the high radial load and to operate at high rotational speeds. The disadvantage of the journal bearings is the excitation of unwanted rotor vibrations by whirling of the journal in the bearing bushing. The bearing journal be-
comes unstable as the journal axis begins to perform a circular motion that is bounded only by the walls of the bearing bushing. When the speed threshold is exceeded, the axis of the bearing journal starts to circulate, causing the rotor to vibrate. These vibrations are called whirl. A passive way of how to suppress vibrations consists in adjusting the shape of the bearing bushing, such as lemon or elliptical bore of the bushing, or use of tilting pads. Even though there are several solutions based on mentioned passive improvements this article deals with the use of the active vibration control (AVC) with piezo-actuators as a measure to prevent instability. The principle of an actively controlled bearing and the correspondent technical drawing is illustrated in Fig. 1. Research on actively controlled bearings using piezo actuators began about ten years ago. The main publications are articles and papers\textsuperscript{1,2,3}. Except to the VSB Technical University of Ostrava, where a functional prototype of this system was put into operation, it was nowhere else build a similar system. This paper informs about further improvements to this system.

The objective of the entire research is to suppress the shaft vibrations and increases boundary of instability to a higher shaft rotation speed by using a suitable control algorithm and piezoelectric actuators. The entire control algorithm being programmed in the dSPACE control system as a real-time system makes it necessary to resolve several other sub-tasks. One of these tasks is to apply an automatically tuneable bandpass second-order filter of the IIR type which should improve the efficiency of the active vibration control system. The proportional feedback controller provides attenuation of the disturbances over a wide frequency range while the bandpass filter in parallel with the proportional feedback controller allows a selective increase of the proportional gain of the controller in the narrow frequency range where the bearing journal tends to vibrate.

![Figure 1: Actively controlled journal bearing.](image)

2. The frequency of external disturbance that causes instability

Rotor instability of the whirl type means a phenomenon in which the journal axis circulates within the bearing bushing at a reduced speed relative to the rotor rotational speed of $f_{\text{rotor}}$. The frequency of the mentioned circulation is named $f_{\text{whirl}}$. This frequency depends on the journal's frequency according to this approximate formula

$$f_{\text{whirl}} = (0.42 \text{ to } 0.48) \times f_{\text{rotor}}. \tag{1}$$

The full (two-sided) cascade spectrum of the journal axis motion for a run-up from 0 to 12000 rpm is shown in Fig. 2. The spectra demonstrate the presence of a frequency of whirling in the measured signals during increasing rpm after crossing the threshold of instability. The full spectrum is calculated using the FFT of the coordinates of the journal axis in a complex plane that is perpendicular to the journal axis. For example, the FFT of the time signal decomposes the signal to harmonic components; the full spectrum represents the decomposition of the journal motion inside the bearing bushing bore on elementary orbits with different rotational frequencies of the phasor pair, see a book\textsuperscript{4}. 

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\textsuperscript{1} K. Mayer, J. S. Kavanagh, and N. J. O'Keeffe, "Active vibration control of a journal bearing using piezo-actuators," in Proc. 13th Int. Conf. on the Mathematics of Finite Elements and Applications (MAFELAP), 2011.


The positive frequencies correspond to the phasor rotation in the positive direction, and the negative frequencies correspond to the phasor rotating in the opposite direction. An example of the full spectrum of the journal motion without any control demonstrates the elementary orbits of the journal axis which rotates at +/- 0.45X fraction of the rotor rotational speed is shown in Fig. 2. The onset of the instability of the whirl type is excited by an external disturbance at that frequency. It is a reason for closing the feedback only for the narrow frequency band around the $f_{\text{whirl}}$ frequency, which also increases the signal-to-noise ratio for the feedback signal.

### 3. Bandpass filter

The bandpass filter is designed as a digital filter. For the control of active vibration damping, a second-order band-pass filter will be used to avoid the unnecessarily long phase delays in the feedback loop. The second-order bandpass filter has the transfer function

$$G(s) = \frac{Y(s)}{X(s)} = K \frac{T_0 s}{T_0^2 s^2 + 2\xi T_0 s + 1},$$

where the cut-off frequency $\omega = 1/T_0$ is selected so that the filter allows to pass the component which excites the instability. The Bode plot of the second-order bandpass filter is shown in Fig. 3.

![Bode plot of the bandpass filters frequency function $\xi = 0.05$ (left column) and $\xi = 0.5$ (right column).](image)

The frequency function of the bandpass filter to gain $K = 1$ and time constant $T_0 = 1$ and two different damping ratios $\xi$ are plotted in Fig. 3. The gain at the centre frequency of the bandpass filter, at the angular frequency $\omega = 1/T_0$ is as follows

$$G(j\frac{1}{T_0}) = \frac{jK}{-1 + j2\xi + 1} = \frac{K}{2\xi}.$$
For angular frequency $\omega = 0$ and $\omega \rightarrow \infty$, the gain is close to zero. The width of the permeable frequency band is dependent on the damping ratio $\xi$. It is not desirable for the robustness of the filter bandwidth to have a small damping ratio but to be sufficiently flat. Frequency change does not cause a large change in filter gain. From the two examples of the damping ratio in Fig. 3, $\xi = 0.5$ is recommended.

4. Signal-to-noise ratio

The signal-to-noise ratio $S/N_{dB}$ (abbreviation SNR) depends on the number of bits of the A / D converter and the converter error, which is most often the rounding error or the actual lower bit number than declared. This error is assumed to be a white noise type. The frequency spectrum of the conversion error is shown in Fig. 4. Spectrum "area" is equal to a variance of $\sigma_x^2$. Because the converter is part of the measurement chain, the signal-to-noise ratio determines the measurement accuracy over the entire frequency range from zero to Nyquist frequency ($f_s/2$) which is called the baseband frequency range $BB$. Signal power ($PWR$) will be used to define $S/N_{dB}$

$$S/N_{dB} = 10 \log \left( \frac{\text{Signal PWR}}{\text{Noise PWR}} \right) = 10 \log \left( \frac{\text{Signal PWR}}{2\pi f_s/\text{BW} / f_s} \right) = 10 \log \left( \frac{\text{Signal PWR}}{\sigma_x^2} \right) + 10 \log \left( \frac{f_s}{2 \times \text{BW}} \right). \quad (4)$$

If the frequency range is reduced to $BW$ bandwidth with the use of the ideal filter, then the signal-to-noise ratio also increases according to the following formula

$$S/N_{dB} = \left[ S/N_{dB} \right]_{BB} + 10 \log \left( f_s / (2 \times \text{BW}) \right). \quad (5)$$

![Figure 4: Signal-to-noise ratio $S/N_{dB}$](image)

The bandpass filter is not ideal as expected above. An estimate of this ratio for the second order filter will now be presented. The transfer function in the $s$-plane needs to be converted to the $z$-plane by the bilinear transformation given by $s = 2/T_s (1 - z^{-1})/(1 + z^{-1})$

$$G(z) = \frac{Y(z)}{X(z)} = K \frac{(1-z^{-2})}{(2(R+\xi)+1/2R) + (1/R-4R)z^{-1} + (2(R-\xi)+1/2R)z^{-2}} = K \frac{b_0 + b_2 z^{-2}}{a_0 + a_1 z^{-1} + a_2 z^{-2}} \quad (6)$$

$$a_0 = 2(R + \xi) + 1 / (2R) \quad b_0 = 1$$
$$a_1 = 1/R - 4R \quad b_1 = 0$$
$$a_2 = 2(R - \xi) + 1 / (2R) \quad b_2 = -1 \quad (7)$$

where the $R = T_0/T_S$ ratio determines the sampling rate relationship to the center frequency of the bandpass filter. It is assumed that the sampling frequency $f_s = 1/T_s$ is much larger than twice the center frequency of the bandpass filter $f_c = 1/2\pi T_0$. In this case, you do not need to enter a pre-warping frequency correction.

Suppose the measured signal at the controller input is affected by additive white noise with the variance of $\sigma_x^2$. The power spectral density in the frequency interval from zero to the Nyquist frequency is in the $z$-plane $S_{xx}(\omega) = \sigma_y^2$. The variance at the output $\sigma_y^2$ of the linear time-invariant system of the
transfer function $G(z)$ is calculated with the use of the integral over any circle centred at zero in the complex plane

$$
\sigma_y^2 = \frac{1}{2\pi j} \int \hat{G}(z)G(z^{-1})S_{xx}(z) \frac{dz}{z},
$$

which is based on the literature. The ratio of the output variance $\sigma_y^2$ to the input variance $\sigma_x^2$ is given by the formula

$$
\frac{\sigma_y^2}{\sigma_x^2} = \frac{(a_2 + a_0(a_2y_0 - a_0y_2) + a_1y_1(a_0 - a_2))}{(a_2 - a_0(a_2 + a_1 + a_0)(a_2 + a_1 + a_0))},
$$

where

$$
\begin{align*}
y_0 &= b_0b_2/a_0, \\
y_1 &= (b_0b_1 + b_1b_2)/a_0 - a_1b_0b_2/a_0^2, \\
y_2 &= (b_0^2 + b_1^2 + b_2^2)/a_0 - (a_2b_0b_2 + a_1(b_0b_1 + b_1b_2))/a_0^2 + a_2^2b_0b_2/a_0^3
\end{align*}
$$

The $\sigma_y^2/\sigma_x^2$ ratio corresponds to the $2 \times BW/f_S$ ratio. Thank the bandpass filter, the signal-to-noise ratio for the journal position measurement is increased by the difference $\Delta SNR = 10 \log(\sigma_y^2/\sigma_x^2)$. Examples of SNRs for $T_0/T_S = 5, 10$ and $20$ ratios are given in Table 1. As discussed below, the controller contains two parallel blocks, one of which is of proportional type and the other is a bandpass filter. The effect of the bandpass filter block on the controller output is set by the $K$ gain. Changes in the SNR value apply only to the bandpass filter. By comparison, one bit of the A/D converter increases SNR by 6 dB.

<table>
<thead>
<tr>
<th>Damping ratio [-]</th>
<th>$T_0/T_S = 5$</th>
<th>$T_0/T_S = 10$</th>
<th>$T_0/T_S = 20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta SNR$ in dB</td>
<td>-3.92</td>
<td>0.09</td>
<td>3.14</td>
</tr>
<tr>
<td></td>
<td>6.23</td>
<td>9.13</td>
<td>12.09</td>
</tr>
<tr>
<td></td>
<td>10.45</td>
<td>13.23</td>
<td>16.13</td>
</tr>
<tr>
<td></td>
<td>13.84</td>
<td>16.44</td>
<td>19.24</td>
</tr>
</tbody>
</table>

5. **Active vibration control**

The $f_{whirl}$ frequency is a centre frequency to which the bandpass filter is tuned. As we assume the rotor is not operated at a constant rotational speed, so automatic adapting the centre frequency of the bandpass filter is required. The bandpass filter of the active vibration control system is created in the time domain with the use of Simulink which is implemented in the dSPACE signal processor. The Laplace transfer function, given by Eq. (2), has to be also converted to the second-order differential equation with coefficients which magnitude depends on the centre frequency. After conversion, the mathematical model of the filter is given by the following differential equation:

$$
\frac{d^2y}{dt^2} = (0.45 \times 2\pi f_{rotor})^2 \left[ \frac{\kappa}{0.45 \times 2\pi f_{rotor} \frac{dx}{dt} - \frac{2\xi}{0.45 \times 2\pi f_{rotor} \frac{dy}{dt}}} \right].
$$

As can be seen in Fig. 5, the bandpass filter is directly programmed in the Simulink environment. The input for the filter reset is omitted to make block diagram simpler. The gain $K$ and damping ratio $\xi$ (ksi) are considered as constants. The expression of $0.45 \times 2\pi f_{rotor}$ is replaced by the identical $\omega_{whirl}$ angular frequency. Decreasing the value of the damping ratio $\xi$ the width of the bandpass of the filter also decreases. The implementation of the bandpass filter in the control algorithm is depicted in Fig. 5.

For the correct function of the retune filter, it is necessary to measure the rotational speed $\omega_{rotor}$ without error signal. The Kalman filter smoothes the instantaneous rotational speed signal that inputs to the tunable bandpass filter because the measurement of rpm is lightly corrupted by a random error.
rotational speed is evaluated from the tacho-signal which is a string of pulses. The linear interpolation of the intersection of the trigger level with the rising edge of the pulses enhances the interpolation accuracy of the calculation of the length of the time interval between the pulses of the tacho-signal. The remaining errors smooth the Kalman filter.

Figure 5: Simulink model of the tunable bandpass filter.

The block diagram of active vibration control is shown in Fig. 6. Parallel connection of the bandpass filter and proportional controller allow experimentation with the weighting factors of these feedbacks. The connection of this control circuit is patented.

Figure 6: The system of the active vibration control system with a bandpass filter.

6. Experiments

The instability of the rotor motion results from the property of the oil film. The operating range of the journal bearing speed can be extended by the use of active vibration control, i.e., by introducing feedback from the journal position to the controller that determines the bushing position. There are three possible combinations of the arrangement of the controller feedback. The experiment enables determining the type of feedback that maximizes the operating speed range. As a set point of the control loop, the time ramp at a constant rate of the rpm increase was selected. To compare the effectiveness of active vibration control with the operation and without this system, it is necessary to know the maximum possible rotor speed without vibration of the whirl type. Almost every time, the instability of the journal motion occurs before reaching a speed of 3000 rpm. During unstable movement, the bearing journal moves along a practically circular path with a diameter that is given by clearance in the bearing. Many tests have confirmed that the movement of the bearings in the bushing is
unstable when the rotational speed exceeds the limit of 2500 rpm. Radial clearance and oil temperature are the same.

As mentioned above, different combinations of the connections of the bandpass filter and proportional controller were tested. Table 2 lists the results of the experiments. Data in the table come from a doctoral thesis\(^9\). First, feedback was tested only with a bandpass filter without the parallel proportional controller (the gain of the proportional controller is set to zero). Instability of the journal motion occurs when 6553 rpm is exceeded. In the proportional control, the instability occurred after crossing approximately 7407 rpm.

<table>
<thead>
<tr>
<th>Feedback</th>
<th>Out of control</th>
<th>Bandpass filter</th>
<th>Proportional control</th>
<th>Proportional control and bandpass filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stability margin</td>
<td>less than 3000 rpm</td>
<td>6500 rpm</td>
<td>7400 rpm</td>
<td>9300 rpm</td>
</tr>
</tbody>
</table>

In Fig. 7, the control with the combination of the proportional controller and the bandpass filter is presented. The cursor indicates the onset of instability. Variable X of the cursor data is the time in seconds and Y is the value in rpm for diagrams showing the stability margin. The scale for the vertical position of the bearing journal is in reverse order to show the visible upward stroke of the rotor axis after the run-up. By combining the proportional controller and the bandpass filter, the best results are achieved. It is clear that active vibration control is adapted to improve the functional properties of bearings by applying electronics to replace passive measures against whirl vibration that cannot be quickly and inexpensively manufactured in comparison to the cylindrical bushing. The prototype of an actively controlled bearing is the first fully functional device in the world\(^{10}\).

![Figure 7: Active vibration control with the combination of the bandpass filter and the proportional controller\(^9\).](image)

### 7. Conclusions

This paper describes the implementation of the proportional controller and the second-order bandpass filter to compensate disturbances in a limited frequency band. The bandpass filter increases the feedback gain in the narrow frequency band and in this way increases the threshold rotation speed at which the ‘whirl’ type instability occurs. The experiments proved that the filter has a positive effect on active vibration control. The highest rotational speed at stable operation (approximately 9300 rpm) was obtained by combining the proportional controller and the bandpass filter. The stable operational range
is three or four times greater than for the sliding bearing without any active vibration control. With the use of the proportional feedback and without the filter, we could achieve a speed of approximately 7400 rpm.

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