Numerical and Experimental Study of a Hybrid Electro-Acoustic Nonlinear Sound Absorber in a Resonant Room

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We present a hybrid Electro-Acoustic Nonlinear Energy Sink (EA-NES) coupled to a resonant room. It is made of a baffled nonlinear membrane with its front face exposed to the noise in the room, and the rear face enclosed. The enclosure includes a feedback loop associating a microphone and a loudspeaker that control the acoustic pressure applied to the rear face of the membrane. The EA-NES action is based on the concept of Targeted Energy Transfer resulting from the nonlinear coupling between the absorber and an acoustic mode in the room (primary system). The nonlinear dynamics of the system is studied theoretically. The asymptotic study shows the possible existence of relaxation oscillations, which can be tuned by the control loop settings. The experimental study confirms theoretical predictions and shows that a hybrid EA-NES can reduce the sound level in a concrete building. Harmonic regime and Strongly Modulated Regime (SMR) are observed. The SMR responses are simulated with a good agreement.

Keywords: Nonlinear absorber, Targeted energy transfer, Periodic excitation, Acoustic resonance, Noise reduction

1. Introduction

The challenge to reduce low-frequency noise transmission through an enclosure has received much attention. Recent works focus on passive, active and hybrid active/passive devices [4]. This challenge is considered here through the relatively new concept of Targeted Energy Transfer (TET).

TET is based on a coupling between a primary system (which vibrations must be mitigated) and a nonlinear absorber. The principle is to place the coupled system on one of its nonlinear modes in order to produce quasi-reversible transfers of the vibratory energy from the primary system to the nonlinear absorber. This phenomenon is called energy pumping [1] and the nonlinear absorber is known as NES for Nonlinear Energy Sink. The basic NES generally consists of a lightmass, an essentially nonlinear spring and a viscous linear damper. In the field of structural vibration, a wide variety of NES designs has been proposed, with different types of stiffnesses (cubic, non-polynomial, non-smooth nonlinearities...). In acoustics, two types of NES have been proposed, one named acoustic NES based on an Helmholtz resonator with nonlinear behaviors [2], the other named vibroacoustic NES based on the use of a simple thin clamped structure involving geometric nonlinearity at large displacement [3]. It was demonstrated...
that a vibroacoustic NES can achieve very efficient noise reduction at low frequency. In both cases the thin clamped structure has to be part of the frontier of the closed acoustic domain, one face (named the front face) is exposed to the primary acoustic field (to be controlled) whereas the other face (the rear face) radiates outside. It results in a pressure difference applied to the membrane, which is necessary for TET. Neglecting the rear radiation impedance, the thin baffled structure is mainly coupled to the primary acoustic field.

The objective of this paper is to investigate analytically and experimentally the performance of the EA-NES considering voltage and current driving mode of the control loudspeaker in realistic conditions. Unlike previous theoretical or experimental studies, the primary system where the acoustic field is meant to be mitigated is an acoustic cavity mostly made of concrete walls.

Indeed the enclosure of the EA-NES makes the membrane interact with the primary acoustic field without the need to be placed across the outer boundary and thus solves the problem of outward radiation. A simple four DOF model is developed coupling an electro-mechanical-acoustic model for the EA-NES with a model cavity representative of its first acoustic mode. We investigate the targeted energy transfer occurring between the acoustic medium and the EA-NES during the sinusoidal forced regimes. The predictions of this model are compared with the experimental data.

The paper is organized as follows. In Section 2, we start with a short description of the system under study and the experimental setup, then we describe each element of the acoustic system, considering first each sub-structure separately, and then modeling the coupled system. In Section 2.3, we study the forced responses to harmonic excitation. The responses are estimated with the complexification averaging method, and compared with results of direct numerical integration of the equations in time domain. In Section 3, we begin with a description of the experimental set-up. Then, we check the stability analysis of the feedback loop and perform a frequency analysis under broadband excitation. In the last part we analyze the forced responses and their agreement with the model in the nonlinear regime, and we discuss the efficiency of the EA-NES.

2. System under study

The system under study is composed of an acoustic cavity that includes the EA-NES at the position $M_a$ and a source loudspeaker at the position $M_S$ as shown in Fig. 1. We are looking at the acoustic pressure at positions $M_1$, $M_2$ and $M_3$.

![Figure 1: Schematic representation of the acoustic cavity: (1) EA-NES, (2) loudspeaker source, (3) microphones.](image)
2.1 Description of the set-up

Figure 2: (a) Picture and (b) schematic representation of the hybrid electro-acoustic nonlinear absorber: (1) clamped membrane, (2) control loudspeaker, (3) control microphone, (4) amplifier with conditioning filter loop gain.

The same EA-NES as introduced in [4] is considered in this study. It is composed (see Fig. 2) of a plywood box with a circular (nonlinear) viscoelastic (latex) membrane clamped on one face. The clamped membrane with its supporting device is shown in Fig. 2(a). The device includes a sliding system used to apply a constant and permanent in-plan pre-stress to the membrane. An enclosed electrodynamic loudspeaker (BEYMA 8P300Fe/N loudspeaker, 8 Inch) named "control loudspeaker" is mounted inside the box (see Fig. 2(b)). The coupling between the membrane and the control loudspeaker is ensured acoustically by the air in a coupling box of a volume $V_e$. The volume of the rear enclosure of the control loudspeaker is $V_r$. An active controller is used to perform a pressure reduction at the rear face of the membrane using the control loudspeaker in voltage or current driving mode. The controller is an analog feedback loop that reduces the pressure measured in the enclosure $V_e$ by using a proportional gain $K$.

The EA-NES is based on the conjugate functioning of three elements: (i) the clamped membrane that interacts with the acoustic field in order to provide noise attenuation in its non-linear range; (ii) the hood by which the EA-NES can work inside a surrounding acoustic field unlike previous developed NES; (iii) the feedback loop that reduces the pressure in the hood and allows to use a small hood volume and also to tune the stiffness and damping linear behavior of the EA-NES thanks to the tuning of $K$, the loop gain.

The EA-NES is coupled to a primary system. The primary system is a rectangular parallelepiped shaped room with dimension $L_x$, $L_y$ and $L_z$ (see Fig. 1). We assume that all the walls of the room are rigid. The equation of motion as a one DOF system is obtained by performing a Rayleigh-Ritz reduction taking account of one mode on the acoustic wave equation (see for example [3]).

2.2 The non-dimensional 3-DOF system

A simplified model is developed to address the properties of the device. The model is obtained by coupling the behaviours of the membrane and the control loudspeaker.

As explained in [5], the pre-stressed membrane motion can be formulated as a one Degree Of Freedom (DOF) nonlinear oscillator. The equations of motion of the control loudspeaker follow from Newton’s second and Kirchhoff’s laws, assuming a linear behaviour. We rely on the Thiele-Small parameters of the loudspeaker.

The final non-dimensional model is obtained by grouping the equations of the EA-NES with those of the primary system. Introducing non-dimensional quantities for the acoustic pressure and the time, and
rescaling the final dimensional system with the parameter $\epsilon$ we obtain the following non-dimensional 3-DOF system

\begin{align}
\ddot{p} + \epsilon \lambda_p \dot{p} + p - \epsilon \mu_p \dot{x}_m &= -\epsilon \beta (1 + \epsilon \sigma) \sin((1 + \epsilon \sigma) t), \\
\epsilon \ddot{x}_m + \epsilon \lambda_m \dot{x}_m + \epsilon \lambda_2 m x_m^2 \dot{x}_m + \epsilon \bar{k}_{11} x_m + \epsilon \bar{k}_{12} x_{LS} + \epsilon \bar{k}_{33} x_m^3 + \epsilon \mu_m p &= 0, \\
\epsilon \gamma_{LS} \ddot{x}_{LS} + \epsilon \lambda_{LS} \dot{x}_{LS} + \epsilon \bar{k}_{21} x_m + \epsilon \bar{k}_{22} x_{LS} &= 0
\end{align}

with $p$, $x_m$ and $x_{LS}$ being respectively the pressure in the room, the membrane center displacement, and the loudspeaker membrane displacement expressed as non-dimensional quantities. All other parameters are constants except for the time $t$ in the source term. $\bar{k}_{21}$ and $\bar{k}_{22}$ depend linearly of the control loop gain $\bar{K}$, which influence is studied in this article.

2.3 Asymptotic analysis

The objective of the asymptotic analysis is to characterize analytically the forced responses of Eqs. (1) to (3) near the first resonance frequency. The asymptotic analysis is based on the complexification averaging method as discussed in detail by [1] combined with the geometric singular perturbation theory [6, 7]. The averaging method distinguishes slow and fast motion. Qualitatively, fast motion can be seen as the oscillation of the system submitted to an excitation, which amplitude is considered constant. While slow motion refers to the evolution of the amplitude of the same oscillations. The slow motion analysis permits to establish the so-called Critical Manifold (CM) of the system, in view of stability analysis. There are three variables $N_1, N_2, N_3$ associated with the three DOF of the system and their typical relationship is shown in figure 3(a) in the $(N_1, N_2)$ plane. Here, the folding points bound a dotted domain where harmonic solutions are unstable, potentially giving rise to other, more complex solutions like relaxation oscillations or chaotic motion. Some simple relaxation oscillations can be approximated by two-frequency quasiperiodic solutions, taking the form of modulated oscillations when the system is submitted to a harmonic excitation. We are interested in finding this unstable domain because it is usually where the absorber is the most efficient.

![Figure 3: (a) Typical critical manifold in $(N_2, N_1)$-plane (blue curve) with fold points (maximum-black, minimum-red) and unstable zone (dotted curve). (b & c) System with EA-NES driven by current with $\bar{K} = 7.5$ and $A_s = 0.136$ ($\beta = 0.032$). (b) Forced response (acoustic pressure) at point $M_3$: primary system without NES (black dots), Eq. (1-3) (red squares), and stable (blue diamonds) and unstable (green triangles) fixed points (c) Time responses (red curves) obtained from Eq. (1-3) in the $(N_2, N_1)$-plan for $f_S = 44.2$ Hz with the unstable fixed point (green marker).]
2.4 Parametric study of the current gain $K$

The CM are plotted Fig. 4(a) in the $(N_2, N_1)$-plane using numerical values corresponding to the experimental setup and for different values of gain $K$ for current control. The associated plot, Fig. 4(b), represents the critical excitation level $\beta_{cr}$ versus $K$ for the same conditions. For small values of $K$, the CM shows fold points whereas for $K > K_{rel} \approx 2.01$ (see Fig. 4(b)), two fold points exist (see Fig. 4(a)). Hence $K_{rel}$ defines the threshold gain from which relaxation oscillations can take place if $\beta > \beta_{cr}$ (i.e. if the excitation level is sufficient high). The critical excitation level $\beta_{cr}$ characterizes a threshold in terms of excitation level. Equivalently a threshold can be defined in terms of $N_1$-amplitude (primary system) by $\sqrt{H(N_2^1)}$ (the ordinates of the fold points, see Fig. 4(a)(black markers)). Both thresholds increase with the gain $K$.

When the control loop is set in voltage mode, the influence of $K$ is smaller and the thresholds are higher than the thresholds associated to the current control.

These thresholds give an order of magnitude for the excitation level needed in the unstable region, in view of setting the experimental conditions.

![Figure 4: System with EA-NES driven by current: (a) Critical manifold with fold points (maximum-black, minimum-red) for $K = 0$, $K_{rel}$, 4, 6, 10, 14, 18, 22 and 28 from the red curve ($K = 0$) to the blue curve ($K = 30$). (b) Critical values $\beta_{cr}$ versus $K$: the dotted red curve corresponds to the red fold points whereas the continuous black curve corresponds to the black fold points ($K_{rel} \approx 2.01$).](image-url)

Figure 4 gives a numerical example of the action of the EA-NES. We focus on EA-NES driven by current with $K = 7.5$ ($> K_{rel}$) and $\beta = 0.032$ ($> \beta_{cr}$). The acoustic pressure amplitude at point $M_3$ obtained with the adimensional model Eqs. (1) to (3) and the asymptotic analysis are compared in Fig. 4(b). Also plotted are the response of the primary system showing the efficiency of the EA-NES. It is slightly shifted to lower frequencies due to the absence of the linear part of the NES in the model of the room. The differential models were solved using Mathemtica ordinary differential equations solver NDSolve (with the choice Automatic for the option Method) with the trivial equilibrium point as initial conditions. In the area of the stable periodic solutions, the asymptotic approximation matches very well with the integrated solution. In the unstable area, the integrated solution has a low amplitude compared to the response without EA-NES. This reduction is what the EA-NES is aimed at.

When the periodic solution is unstable, the system can exhibit relaxation oscillation as observed Fig. 3(c) for $f_S = 44.2$ Hz. The time response obtained from Eq. (13) is plotted in the $(N_2, N_1)$-plane. It oscillates around the unstable range.
These observations show that the EA-NES we study should be able to reduce significantly the sound level in the room. These observations give also an indication about the source level and frequency around which the experiments should be done.

3. Experimental study

3.1 Setup and primary analysis

The experiments are conducted in a concrete parallelepipedic room, except for the ceiling which has the shape of a shortened pyramid covered by a thick wooden floor as it can be seen in Fig. 5(a). A complete description can be found in [8]. The size of the room is $L_x = 3.928$ m and $L_y = 3.05$ m with height from $L_z = 2.4$ m to $2.7$ m.

The Frequency Response Function (FRF) denoted $p(M_2)/i_s$ measured between the source loudspeaker current $i_s(t)$ and the acoustic pressure, $p(M_2, t)$, at $M_2$ with the blocked EA-NES inside the room is plotted Figure 5(b). The FRF is measured using a white noise in the frequency range $[30, 80]$ Hz as target signal $e(t)$. The first resonance frequency appears at $f \approx 43.8$ Hz corresponding to the $(1, 0, 0)$-mode and it is associated to a quality factor near to $Q_{100} \approx 133$. Note that these numerical values have been used in the previous theoretical analysis.

![Figure 5: (a) Picture of the room with the EA-NES. (b) Frequency response function $p(M_2)/i_s$ measured with the blocked EA-NES inside the room.](image)

3.2 Nonlinear analysis

We measure the response of the cavity around its $(1, 0, 0)$-mode under sinusoidal forcing defined from a target signal $e(t) = A_s \sin(2\pi f_e)$ which provides an input current signal to the source loudspeaker. Several measurements are performed, increasing the forcing amplitude $A_s$ from 0.01 to 0.25 and varying the forcing frequency for each amplitude from 42.5 Hz to 45 Hz, with a step of 0.1 Hz. The displacement at the center of the membrane of the EA-NES measured with an optical sensor Keyence LK-G152. The sampling frequency is $f_s = 8192$ Hz.

We first consider the EA-NES with current feedback control and the gain value $K = 7.5$. The acoustic pressure in the room are measured at location $M_3$ and are similar at the three microphones positions. TET
has been observed with this configuration in the numerical simulation. The experimental measurements show SMR regimes where TET is observed in good agreement with the simulations.

The influence of the parameters characterizing the active part of the EA-NES on the TET efficiency are investigated in order to validate the asymptotic analysis. The parameters are the feedback control mode of the loudspeaker (current versus voltage) and the gain $K$. The gain $K$ varies from $K = 0$ to $K = 15$. when current feedback control is considered and $K = 550$. when voltage feedback control is considered. In all cases, the stability of the feedback control system is satisfied.

In order to quantify the triggering threshold and the width of the TET, ridge lines are extracted from the acoustic pressure measured in each scan in amplitude and frequency. Each point of the ridge line is defined as the maximum of the RMS values of the acoustic pressure for the considered frequency range: a frequency scan like in Fig. 3(c) gives one point in the ridge line. We make these scans in frequency because the apparent resonance peak in the room is not well defined and varies with the source amplitude. This behavior is typical for this kind of systems.

We also present as a reference the ridge line measured at position $M_3$ with the blocked EA-NES.

According to the Fig. 5(a), the triggering threshold of the TET depends on the gain $K$ in the case of the current control as observed in the critical manifold in Fig. 4. Indeed, starting from $K = 2.5$, the triggering threshold increases from 9 Pa to 15 Pa. The same phenomenon is observed with the voltage drive but in a less pronounced way (see Fig. 5(b)). As shown in [4], the effect of the gain $K$ is to decrease the resonance frequency of the EA-NES at low level. As a consequence the needed level of excitation to synchronize the EA-NES with the $(1, 0, 0)$-mode is increasing and one can observe in the related critical manifold that the fold points also increase. One can also observe that the excitation range where occurs the limitation of the pressure depends very little on the gain $K$.

![Figure 6](image)

Figure 6: Ridge line of the RMS values of pressure measured at point $M_3$ for the system with EA-NES driven by (a) current, (b) voltage.

## 4. Conclusion

We have studied a hybrid EA-NES coupled to a resonant room. We have modeled the electromechanical dynamics of the system. The asymptotic study of the model shows the existence of a critical value above which relaxation oscillations can exist. This threshold rises along with the gain of the control loop and is affected by the kind of control loop command law (voltage or current command law).
The experimental study shows that a hybrid EA-NES can work in a concrete building. It is able to limit the sound level in the room in its working range up to 8 dB for a footprint of only 0.2% of the room volume. The different regimes observed correspond well to previous observations for NESs, including relaxation oscillations (Strongly Modulated Regime), although the system or experimental conditions in previous works were far from this study’s ones. Unlike previous acoustical studies, here the SMR responses are simulated with a good quantitative agreement, in voltage or current command law. We have also simulated and observed that the thresholds defining the working range of the NES can be tuned electrically by setting the gain $K$ of the control loop.

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References


