OPTIMISING SOIL STIFFNESS ON HIGH SPEED RAIL LINES TO PREVENT VIBRATION

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The fast movement associated with high speed trains can cause significant dynamic effects within the supporting railway track structure. The speed at which maximum dynamic response occurs is known as the 'critical velocity' and is undesirable because large rail vibrations are generated when travelling close to it. These vibrations can cause a safety concern, and also propagate to the free-field where they disturb nearby buildings. A method to minimise these vibrations is to stiffen the soil directly below the track either via soil replacement or soil improvement, however both options are expensive. Their cost can be reduced though if either the depth or stiffness magnitude of the replacement is optimised. Therefore this work develops a track-ground model using the thin-layer method, which is capable of assessing the effect of different combinations of soil improvement on track vibration levels. It is shown that if improvement is carefully designed, performance can be maximised for minimum cost. Similarly, if improvement is poorly chosen, it can result in marginal improvement, and in some cases even amplify track vibration.

Keywords: Railway vibration, soil stiffening, thin-layer element method

1. Introduction

Rail vibration is a growing problem due to increasingly stringent international standards and increased lengths of track infrastructure under construction/operation near buildings [1]. A range of numerical (e.g. 2.5D modelling [2], [3]) and experimental approaches (e.g. [4], [5]) have been proposed to predict such vibrations. The vibration spectrum from railways is comprised of low frequency ‘quasi-static’ vibration and also high frequency vibration generated due to wheel-rail irregularities ([6], [7]). This research focuses on the low frequency vibration generated when the train moves at a velocity greater than \(\approx 50\%\) of the natural wave speeds of the track-ground structure. When the train moves at 100% of this speed, it is known as the critical velocity.

In an attempt to simulate track dynamics at critical velocity, [8] used closed-form expressions to model the problem as a moving load traversing a homogenous and infinite elastic medium. The problem was then better tailored towards railway applications by [9] and [10] who used closed-form expressions for the track and Green’s functions for the soil response.
Later, [11] outlined a semi-analytical numerical model that could account for soil strata with greater complexity. It involved the track and ground being modelled using a viscoelastic beam and Green’s functions for the soil. Similarly, [12] proposed a similar-type approach, however the soil was simulated using integral transformation methods. This was also expanded upon by [13] and [14] and found to have strong agreement with field data collected at a soft soil test site in Sweden.

This work expands upon the previously discussed approaches, using an analytical model for the track and a thin-layer element method for the soil. The resulting model has fast run times and is validated using field measurements. Finally, it is used to investigate the reduction in track vibration arising from different combinations of soil remediation.

2. Numerical modelling

The model uses a semi-analytical method to compute vertical track deflections. It consists of an analytical track model, and a semi-analytical thin-layer element model for the soil. These sub-models are formulated in the frequency-wavenumber domain, and then coupled assuming a relaxed boundary condition at their interface. To do so, although the 2 models have different numbers of degrees of freedom, they are only coupled in the vertical direction. As will be shown, this approach produce accurate results, yet allows for the simulation of deep-wave propagation in an efficient manner [15].

2.1 Track model

The track is considered as a slab track and is modelled analytically, using a springs and dampers to represent railpad response. Similarly, beams are used to represent the rail and slab structures ([14], [16], [17]). The equations of motion are formulated in the wavenumber-frequency domain and shown in Equation 1.

\[
\begin{bmatrix}
    E_I r k_x^4 + k_p^* - \omega^2 m_r \\
    -k_p^* \\
    E_I s k_x^4 + k_p^* - \omega^2 m_s + k_{eq} \\
\end{bmatrix}
\begin{bmatrix}
    \ddot{u}_r(k_x, \omega) \\
    \ddot{u}_s(k_x, \omega) \\
\end{bmatrix}
= \begin{bmatrix}
    \ddot{p}(k_x, \omega) \\
    0 \\
\end{bmatrix}
\] (1)

\[\text{Figure 1: Slab track model layout}\]
2.2 Soil model

Soil response is computed using the thin-layer element method. To do so it is discretized in the vertical direction using infinitely long horizontal elements thus avoiding the need for lateral absorbing boundaries [18]. Each element consists of 3 nodes as shown in Figure 2, which ensures that the stresses and strains are accurately captured within the soil domain. For cases where an embankment is modelled [19], it is included as an additional soil layer. Like the track, the equations of motion are formulated in the frequency-wavenumber domain as shown in Equation 2, where $K$ and $M$ are the global stiffness and mass matrices respectively, $U$ is displacement, $\omega$ is frequency and $P$ is the load.

$$([K] - \omega^2[M])U = P$$  \hspace{1cm} (2)

2.3 Equivalent soil stiffness

The track and soil are coupled using a complex equivalent stiffness ($k_{eq}$) defined in the wavenumber-frequency domain, as shown in Equation 3. This uses the principals of the equilibrium of loads and the compatibility of displacements along the track-ground interface [20]. To implement this, first the TLM model (Equation 2) is computed for a soil on which there is a moving load over a width equal to the track. This computes the soil Green’s function, which is then used in Equation 3 to compute the equivalent stiffness. Finally, this is then inserted into Equation 1 to calculate the track displacements.

This coupling is important when train speed is high because high levels of wave propagation occur across the track-ground interface. Therefore accurately simulating the soil wave propagation is vital to achieving accurate track displacements.

$$k_{eq}(k_x, \omega) = \frac{2\pi}{\int_{-\infty}^{+\infty} \tilde{u}_{zz}^C(k_x, k_y, z = 0, \omega)C_{tg} dk_y}$$  \hspace{1cm} (3)
3. Validation

To validate the model, data was collected on a live railway line in Carregado, Portugal [21]. A series of geophysical tests were performed to characterise the soil and receptance tests were performed to characterise the track. Then, during train passage, a laser based system was used to record track displacements of the rail.

The comparison between the TLM model results and the field data are shown in Figure 3 and Figure 4. Figure 3 shows the time history response of the rail, while Figure 4 compares the corresponding frequency spectrum. From Figure 3 it is clear that the numerical model quite accurately predicts the track behaviour – the shape and magnitude of response is reproduced well. Similarly, in terms of frequency content (Figure 4), the key dominant frequencies are consistently predicted in terms of both frequency and magnitude.

![Figure 3: Validation time history](image1.png)

![Figure 4: Validation of frequency spectrum](image2.png)
4. Analysis

To analyse the effect of soil stiffness improvement on railway track displacements, a soft soil test case is considered. It is infinitely deep with the properties shown in Table 1, which results in a critical velocity of 90m/s. The train speed is 300km/h (91.7m/s) meaning it is very close to the critical velocity and therefore soil stiffness remediation is needed. The track properties are shown in Table 2.

15 different improvement options are considered: five different depths spanning equally between 1-5m depth, and for each, three different stiffness magnitudes are considered: 150, 200, 250MPa.

<table>
<thead>
<tr>
<th>Layer thickness (m)</th>
<th>Young’s modulus (MPa)</th>
<th>Poisson’s ratio</th>
<th>Density (kg/m³)</th>
<th>Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil</td>
<td>∞</td>
<td>45MPa</td>
<td>0.35</td>
<td>1800</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.03</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Railpad</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_p$ (N/m)</td>
<td>$5.5 \times 10^8$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_p$ (Ns/m)</td>
<td>$2.5 \times 10^5$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Soil properties

Table 2: Track properties

<table>
<thead>
<tr>
<th>Slab track</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$EI_r$ (Nm²)</td>
<td>$1.26 \times 10^7$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_r$ (kg/m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h_{slab}$ (m)</td>
<td>0.35</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{slab}$ (MPa)</td>
<td>$3 \times 10^4$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2b$ (m)</td>
<td>2.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$ (kg/m³)</td>
<td>2500</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5 shows the effect of the 15 changes on maximum rail displacement. The original case where no remediation is performed, results in 1.66mm displacement, and is denoted by the dashed black line. It is seen that adding additional improvement at large depth below the ground has less of an effect than at the surface. For example, adding 1m of improved soil to the non-remediated case results in a greater percentage improvement compared to adding 1m of improved soil at a depth of 3m. This is because the surface wave energy imparted by the track into the ground is primarily located near the earth’s surface.

When considering the effect of stiffness magnitude, stiffer remediation results in lower track deflections. This is as expected, however it is also seen that the percentage in improvement increases with depth. Interestingly though, when only 1m of soil improvement is used, it has the undesired effect of increasing track displacements, rather than reducing them. This is true when the stiffness magnitude is 150 or 200MPa, however if the improvement depth is increased to 2m, this changes and track displacements are again reduced.

This local increase can be explained by analysing the relationship between train speed and rail deflections, (i.e. the dynamic amplification factor) for the 1m improvement case as shown in Figure 6. It is seen that the 330km/h train speed is slightly past the critical velocity when no improvement is performed. However, after improving the top 1m of soil, the critical velocity shifts to an elevated speed which is actually closer the critical velocity compared to the ‘no remediation’ case. Then, because greater dynamic amplification occurs at approximately the critical speed, the track displacements then also increase.
5. Conclusions

Ground-borne vibration from railways is a growing problem, particularly with the rapid global deployment of high speed rail infrastructure. When trains travel at greater than approximately 50% of the natural wave speed of the track-ground structure, dynamic amplification is magnified. This can propagate to large distances from the track and effect nearby structures. To reduce this vibration, soil improvement below the track can be undertaken. To investigate this, this paper first outlines a numerical model to compute dynamic amplification in the presence of high dynamic effects. To do so the track is modelled analytically and the soil is modelled using the thin-layer method. The model is validated using field data collected on a railway line in Portugal, and shows strong agreement with the collected data.

The model is then used to investigate the effect of 15 soil improvement options for a soft soil site. Five different improvement depths and three different improvement stiffness’ are considered. It is shown that soil improvement must be undertaken with care and that a poorly designed solution may result in further displacement amplification rather than reduction.
6. Acknowledgements

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