PASSIVE CONTROL SOUND TRANSMISSION THROUGH DOUBLE-PLATE STRUCTURES BY USING MASS-SPRING-DAMPER SYSTEM

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It is well-known that the acoustic performance of double-plate structures deteriorates rapidly around the mass-air-mass resonance frequency. In this paper, a mass-spring-damper (MSD) system connected between incident and radiating plate is used to control the sound transmission in low frequency ranges. A full structural-acoustic coupling modal model is developed to analyze the vibroacoustic behaviour of the double-plate structures with MSD system. And the numerical calculation results are presented. The calculated results show that, tuning the natural frequency of the MSD system exact to the mass-air-mass resonance frequency cannot guarantee the maximum improvement on transmission loss. Optimal natural frequency depends on the mass of the MSD system. The calculation results also show that the sound transmission loss of a double-plate structure can be improved significantly by using optimally tuned MSD system. In presented results indicate that an overall improvement 12 dB below 1000 Hz can be achieved when the mass of the MSD system equals to 10% weight of the double-plate structure.

Keywords: Double-plate structure, sound transmission loss, mass-spring-damper (MSD) system

1. Introduction

Double-plate structures are widely used in the aerospace and building industries when good sound insulation characteristics have to be achieved. However, the main drawback of such structures is a decrease in low-frequency transmission loss around the mass-air-mass resonance [1 – 3]. It means that today’s sound isolation performance of the double-plate structures still requires significant improvement especially in regard to low frequencies, which dominate traffic and aircraft noise. In an effort to improve the transmission loss through the double-plate structures, many techniques have been developed, which can be broadly classified into active control and passive control methods.

One possible solution is by means of active control methods [4 – 6]. However, there are some limitations seriously compromising the active control system for practice applications, such as requirement of additional power support for control energy, complex controller design, robustness problem, etc.

Another possible solution is to use the passive control method. For examples, it is possible to apply an arrangement of optimally tuned Helmholtz resonators (HRs) to increase the acoustical damping
level inside the cavity between the double-plates. A theoretically model and experimental verification for passively control finite double-plate structures using HRs was introduced in our previous papers [7, 8]. The numerical and experimental results show that the optimally tuned HRs can indeed be used to effectively control the sound transmission through a double-plate structure. Idrisi et al.[9] used heterogeneous(HG) blankets, which consist of poroelastic material with small embedded masses, to control sound transmission through the double-plate structures.

In this study, a MSD system is imposed to improve the sound transmission loss around mass-air-mass resonance frequency. First, the governing equations of a fully coupling structural-acoustic-acoustic system by using modal coupling method are established; then, the effects of the MSD system parameters on the sound transmission are discussed, and the optimally tuned natural frequency and mass of the MSD system are discussed in detail; finally, some useful conclusions are drawn and illustrated by numerical results.

2. System modelling

A model is demonstrated in Fig. 1 to describe the mechanical behaviour of a double-plate structure with a MSD system. Two parallel plates with length $L_x$ and width $L_y$, denoted by incident plate and radiating plate, are located in an infinite rigid baffle. A MSD system is installed between two plates at $(x_0, y_0)$.

$$\nabla^2 p - \frac{1}{c_o^2} \frac{\partial^2 p}{\partial t^2} = 0 \quad \text{with} \quad \frac{\partial p}{\partial n} = \begin{cases} \rho_o \frac{\partial^2 w_i}{\partial t^2} & \text{on incident plate } (z = 0) \\ - \rho_o \frac{\partial^2 w_r}{\partial t^2} & \text{on radiating plate } (z = L_z) \\ 0 & \text{otherwise} \end{cases}$$

Figure 1: (a) Double-plate structure installed between incident and radiating plate; (b) Free body diagram of the MSD system

Because there is no excitation source in cavity, the acoustical field of the cavity can be described in homogeneous wave equation by wave equation

where $\rho_o$ and $c_o$ are the density and sound speed of the air, respectively. $p$ is the sound pressure in cavity. $w_i$ and $w_r$ are displacements of incident plate and radiating plate, respectively.

Due to the reaction force of the spring-damper-mass system, the vibration of the incident and radiating plate are governed by the following equations

$$D_i \nabla^4 w_i + m_i \frac{\partial^2 w_i}{\partial t^2} = p^{in} - p(z = 0) - F_i(x_0, y_0).$$

(2)
\[ D \nabla^4 w_r + m_r \frac{\partial^2 w_r}{\partial t^2} = p(z = L_z) + F_r(x_0, y_0). \]  

(3)

where \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \), \( D_k = \frac{h_k^i E_k}{12(1-\nu_k^2)} \), \( m_k = \rho_k h_k \) (\( k = i, r \)). subscript \( i \) and \( r \) denote the incident plate and radiating plate, respectively. \( D, m, h, E, \nu \) and \( \rho \) are the bending stiffness, the mass per unit area, the thickness, the Young’s modulus, the Poisson’s ratio, and the density of the plate, respectively. \( F \) is the reaction force due to the MSD system. \( p^i_m \) is the excited sound pressure on the incident plate. In this study, \( p^i_m \) is assumed as the random incident wave field.

The velocity distribution of the plates and the cavity pressure can be represented by a series of expansions

\[ v_i(x, y) = \sum_m \Phi_{k,m}(x, y)\eta_{k,m} = \Phi_i^T \eta_k \quad (k = i, r). \]  

(4)

\[ p(x, y, z) = \sum_n \Omega_n(x, y, z)P_n = \Omega^T P. \]  

(5)

where \( \eta_{k,m} \) and \( P_n \) are the \( m \)th structural modal coordinates and the \( n \)th sound pressure modal coordinates, respectively. \( \Phi_{k,m}(x, y) \) and \( \Omega_n(x, y, z) \) are the \( m \)th uncoupled structural mode shape and the \( n \)th acoustic mode shapes, respectively.

Substituting Eqs. (4) and (5) into Eqs. (1 – 3), and using the orthogonal properties of the mode shape functions. And taking the damping ratio of plates and fluid in cavity into account, the complete set of equations for the double-plate system can be expressed as matrices form

\[ P = Y_p (L\eta_i - L\eta_r). \]  

(6)

\[ \eta_i = Y_i (P^{ext} - L_i^T P - F_i). \]  

(7)

\[ \eta_r = Y_r (L_r^T P + F_r). \]  

(8)

where \( P^{ext} \) is the generalised structural modal vector due to the exciting source (random incident wave in this case). \( Y_p, Y_i, Y_r \) are diagonal matrix, their diagonal elements are

\[ Y_p(n,n) = \frac{2\omega^2}{V} \frac{j\omega}{(\omega^2)^2 - \omega^2 - 2j\zeta^p \omega_n^p}. \]  

(9)

\[ Y_k(m,m) = \frac{1}{M_m^k} \frac{j\omega}{(\omega_m^k)^2 - \omega^2 + 2j\zeta_m^k \omega_m^k}, \quad (k = i, r). \]  

(20)

\[ L_k(n,m) = \int_S \Omega_k(x, y, z)\Phi_{k,n}(x, y)ds \quad (k = i, r). \]  

(31)

where \( \omega_m^k \) (\( k = i, r \)) and \( \omega_n^p \) are the uncoupled plate and cavity natural frequency, respectively. \( \zeta_m^k \) (\( k = i, r \)) and \( \zeta_n^p \) are the modal damping ratios of the plates and cavity, respectively.

\( F_i \) and \( F_r \) are the structural modal vector due to the MSD system. From Fig. 1, it is easy to obtain \( F_i \) and \( F_r \), such as,

\[ F_i = \Phi_i(x_0, y_0) \left[ \left( \frac{k_m}{j\omega} + c_m \right)[y_m - v_i(x_0, y_0)] \right]. \]  

(42)
where \( v_m \) is the velocity of the mass.

Notice that the vibration of the MSD system can be expressed as a single degree of freedom system, that is,

\[
j \omega m_m v_m + 2 c_m v_m + \frac{k_m}{j \omega} v_m = \left( \frac{k_m}{j \omega} + c_m \right) v_r(x_0, y_0) + v_r(x_0, y_0).
\]

where \( m_m, c_m \) and \( k_m \) are the mass, damping and stiffness of the MSD system.

From Eqs.(12 – 14), \( F_i \) and \( F_r \) can be obtained.

\[
F_i = \Phi_i(x_0, y_0) \left( \frac{k_m}{j \omega} + c_m \right) v_r(x_0, y_0) + v_r(x_0, y_0).
\]

Recall Eqs.(6 – 8), we get

\[
P = P_{\text{without}} + K_i F_i + K_r F_r = P_{\text{without}} + A_i v_i(x_0, y_0) + A_r v_r(x_0, y_0).
\]

where \( P_{\text{without}} = \left[ I + Y_p L_i Y_r^T + Y_p L_i L_i^T \right] F_{\text{ext}} \), and \( P_{\text{without}} \) is the acoustic modal pressure in cavity without MSD system. \( K_i = \left[ I + Y_p L_i Y_r^T + Y_p L_i L_i^T \right] Y_i L_i Y_i \), \( K_r = \left[ I + Y_p L_i L_i^T + Y_p L_i L_i^T \right] Y_i L_i Y_i \).

By using Eqs.(4, 5, 17), the modal velocity with MSD system can be expressed as

\[
\eta_i = \left[ I - G_i \Phi_i^T(x_0, y_0) \right] \eta_{\text{without}} + G_i \Phi_i^T(x_0, y_0) \eta_i.
\]

\[
\eta_r = \left[ I - U_r \Phi_r^T(x_0, y_0) \right] \eta_{\text{without}} + U_r \Phi_r^T(x_0, y_0) \eta_r.
\]

\[
P = P_{\text{without}} + A_i \Phi_i^T(x_0, y_0) \eta_i + A_r \Phi_r^T(x_0, y_0) \eta_r.
\]

where \( G_i = -Y_i L_i A_i + Y_i \Phi_i(x_0, y_0) \left( \frac{k}{j \omega} + c \right) (M - 1) \), \( G_r = -Y_i L_i A_r + Y_i \Phi_i(x_0, y_0) \left( \frac{k}{j \omega} + c \right) M \), \( U_i = -Y_i L_i A_i + Y_i \Phi_i(x_0, y_0) \left( \frac{k}{j \omega} + c \right) M \), \( U_r = -Y_i L_i A_r + Y_i \Phi_i(x_0, y_0) \left( \frac{k}{j \omega} + c \right) (M - 1) \).

And \( \eta_{i, \text{without}} \) and \( \eta_{r, \text{without}} \) are the modal velocities without MSD system for incident and radiating plate, respectively. And

\[
\eta_{i, \text{without}} = Y^T_i \left( P_{\text{ext}} - L_i^T P_{\text{without}} \right), \quad \eta_{r, \text{without}} = Y_i L_i P_{\text{without}}.
\]

Combining Eqs.(18 – 20), we can get the fully coupled structural-acoustic responses for the double-plate structure with MSD system.

In the classical acoustics, the performance of the structure insulation is usually expressed in terms of its sound transmission loss \( TL \), which is defined as the sound power \( W_i \) incident on the incident plate divided by the sound power \( W_r \) radiated by the radiating plate.
where $W_i$ and is the sound power incident on the incident plate. $W_r$ is the sound power radiated by the radiating plate.

Some of the results quoted in the following sections will be expressed in terms of the frequency averaged transmission loss, defined as

$$TL_{avg} = 10 \log_{10} \left( \frac{W_{inc,avg}}{W_{rad,avg}} \right)$$

with $W_{avg} = \frac{1}{N} \sum_{n=1}^{N} W_{an}$. \hspace{1cm} (23)

where $[\omega_1, \omega_3]$ is the frequency range of interest.

3. **Numerical calculations**

A double-plate structure with the mass-air-mass resonance frequency at 155Hz studied by Ref. [12] with the finite element model is restudied. The boundary conditions for both plates are simply-supported. The physical parameters of the structure are as follows: $L_x \times L_y \times h = 350\text{mm} \times 220\text{mm} \times 1\text{mm}$; the depth of cavities $L_z = 76.2\text{mm}$. The density, Young’s modulus and Poisson ratio of each plate are 2814 kg/m$^3$, 71 $\times$ 10$^3$ N/m$^2$ and 0.33, respectively. The structural and acoustical damping ratios are assumed as 1%. The incident plate is excited by a random incident acoustic wave of 1Pa amplitude. The MSD system is assumed located at the center of each plate.

3.1 **Spring stiffness effect on sound transmission loss**

Before analyzing the control performances produced by the MSD system, it is interesting to consider the effects produced by the spring of the MSD system.

Assume that the values of the mass and damper of the MSD system are zero. At this time, only a spring is connected between incident and radiating plate. Figure 2 shows the effect of the values of the spring stiffness on sound transmission loss.

![Image](image.png)

**Figure 2:** The sound transmission loss under various values of the spring stiffness.

In Fig. 2, the vibration distributions at 69Hz (the first natural frequency of uncoupled structural mode) and 155Hz (the mass-air-mass resonance frequency without the MSD system) are also presented.
From Fig. 2, it is found that the sound transmission loss is independence to the spring values around 69 Hz. This seems surprising at first glance but the cause is due to the fact that the plates move in phase. It indicates that the distance between two plate around 69Hz remains the same and the spring force is zero around 69Hz. Furthermore, it can be found that introducing a spring between plates cannot improve the sound transmission loss. This conclusion agrees well to Ref.[2] and is quite understandable because the spring can be seen as a mechanical link in this case.

3.2 Mass of the MSD system effect on sound transmission loss

According to classical tuned-vibration absorber theory, if the mass is given, the optimal natural frequency and damper values for the MSD system are

\[
\omega_{m}^{opt} = \frac{\omega_{t}}{1 + \mu}, \quad \zeta_{m}^{opt} = 2m_{0} \omega_{t} \sqrt{\frac{3\mu}{8(1 + \mu)^{3}}}
\]

(24)

where \(\omega_{t}\) is the target frequency, \(\mu = m_{s}/m_{r}\), and \(m_{r}\) is the mass of the base structure.

Notice that the sound transmission loss reduces quickly around the first natural frequency (69Hz) of the uncoupled structural mode and the mass-air-mass resonance frequency (155Hz), as shown in Fig. 2. So the MSD system should be targeted to control these two frequencies. The numerical results are presented in Fig. 3 with the optimal parameters of the MSD system given by Eq.(24).

Figure 3: The sound transmission loss \(TL\) when the MSD system is targeted to control (a) the first uncoupled structural mode; (b) the mass-air-mass resonance mode

From Fig. 3, it can be found that the MSD system tuning to the first uncoupled structural mode can obtain the better control performance. Because in this case, not only the target first uncoupled structural mode can be controlled, the sound transmission around mass-air-mass resonance is also damped. However, when the MSD system is targeted to control mass-air-mass resonance, the sound transmission loss around the first uncoupled natural frequency cannot be improved. From Fig. 3, it can be found that the MSD system connected between incident and radiating plate can affect not only the targeted mode, but also the other modes, due to the in phase and out-of-phase vibration between two plates (as shown in Fig. 2). It means that the optimal natural frequency obtained from Eq.(24) might not
be the best solution for double-plate structures. We should search a new solution to optimize the natural frequency of the MSD system. It will be discussed in next section.

3.3 The optimal tuned natural frequency and mass of the MSD system

In traditional MSD system applications, the MSD systems are mainly used for vibration control. So a large mass of the MSD system is always favourable for vibration control case. However, in this study, the main aim is to improve the sound transmission loss at low frequencies, it means that the sound power of the radiating plate should be controlled. To obtain the optimal natural frequency and mass of the MSD system, we use the frequency averaged sound transmission loss $TL_{avg}$ as the global cost function, i.e.: 

$$\max f(m_m, \omega_m) = TL_{avg} \text{ subject to } m_m \leq 0.3m_T \text{ and } 10Hz \leq \omega_m \leq 200Hz.$$  \hspace{1cm} (25)

First of all, assume that the natural frequency of the MSD system is given. Figure 4 shows the $TL_{avg}$ improvement as a function of the mass of the MSD system. It is clear that the $TL_{avg}$ improvement does not linear increasing with the mass. There is an optimal mass for each given natural frequency. For further increasing the mass, the $TL_{avg}$ improvement will be reduced. It means that adding mass of the MSD system does not necessary increase the sound transmission loss. From Fig. 4, it can also be found that the natural frequency of the MSD system targeted to the mass-air-mass resonance mode cannot obtain the best $TL_{avg}$ improvement. For example, when the natural frequency of the MSD system is tuning to 69Hz or 90Hz, two clear maximum of 12dB and 11.7dB $TL_{avg}$ improvement are observed. However, when the natural frequency of the MSD system is tuned to mass-air-mass resonance frequency (155Hz), the best $TL_{avg}$ improvement is only 8dB.

![Figure 4: The improvement of $TL_{avg}$ as a function of both mass and tuned natural frequencies of the MSD system (Circle marks: the optimal mass for each case).](image)

To further investigate the control performance of the MSD system, the improvement of $TL_{avg}$ as a function of both mass and tuned natural frequencies of the MSD system is presented in Fig. 5. Unlike the traditional tuned-vibration absorber theory, it can be found that the optimal natural frequency depends on the mass of the MSD system. It means that the optimal natural frequencies can be far removed from the mass-air-mass resonance frequency. As mass increases, the optimal natural frequency changes from 100Hz to 20Hz. Furthermore, it can be found that an $TL_{avg}$ improvement 12 dB can be achieved when the natural frequency of the SMD system is 85Hz and $m_m/m_T=10\%$. 

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4. Conclusions

In this study, the passive control of sound transmission through a double-plate structure using a MSD system connected between incident and radiating plate is presented. The numerical results show that the sound transmission loss can be improved significantly if the mass and natural frequency of the SMD system is optimal tuned. Furthermore, it is found that the optimal natural frequency is a function of the mass of the MSD system. It indicates that the mass and natural frequency of the SMD system should be optimized simultaneously to maximize the improvement of the transmission loss obtained the best the improvement of the sound transmission loss over a wide frequency bandwidth.

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