In order to obtain the dynamic parameters of foundation soil to exactly predict ground-bored vibrations induced by railway, a SASW field test is carried out on the basis of the theory of Rayleigh surface wave and spectrum analysis. Then the mathematical model of inversion is introduced and the inversion program is compiled on the basis of the genetic algorithm by iteratively minimizing the distance between the experimental and theoretical dispersion curves. It is found that the inversion accuracy greatly depends on the completeness of experimental dispersion curve from SASW field test. The good agreement and low inversion error prove that the dynamic parameter inversion method of foundation soil is correct and effective, which can be used to obtain the crucial soil properties before predicting the traffic-induced ground and structural vibrations.

Keywords: SASW test, dispersion curve, dynamic soil parameters inversion

1. Introduction

The transmission of waves through the soil is an essential element in the problem of railway-induced ground and structural vibrations, while wave propagation in the soil is determined by the dynamic soil properties. As a consequence, the knowledge of these dynamic soil properties plays an important role in exactly predicting ground-borne vibrations.

For the prediction of ground-borne vibrations, the soil is frequently modelled as a layered half-space, where free surface waves exist as natural modes of soil. The free surface waves result from an interaction between P-waves and S-waves due to the presence of the free surface and the interfaces between layers. Surface waves propagate in the horizontal direction and vanish with depth, and they are dispersive: their phase velocity varies with the frequency and is determined by the variation of the soil properties with depth. In this paper, the dynamic parameters inversion of foundation soil is based on the dispersion theory of Rayleigh surface wave and spectrum analysis. The strategy of inversion of soil properties consists of three steps: (1) Carry out the SASW field test to obtain the experimental dispersion curve; (2) By means of given or hypothetical soil parameters, establish the soil model and calculate the theoretical dispersive curve by some analytical methods, numerical methods, or other methods; (3) iteratively minimize the distance between the experimental and theoretical dispersion curves by some algorithm theories.
2. A SASW field test and experimental dispersion curve

The SASW field test is a non-invasive test with the assumption that the response at a sufficiently large distance from the source is dominated by dispersive surface waves travelling in a layered half-space. The fundamental principle of the SASW approach is based on the measurement of the surface wave velocity propagating through a material to estimate the corresponding shear wave velocity [1]. The working process of SASW field test is showed in Fig. 1.

A free field at the campus of Beijing Jiaotong University, China is chosen to carry out the SASW test. The layout of SASW field test is arranged as Fig. 2. The vibrations are generated at the soil’s surface using an impact hammer. The free field response is measured with accelerometers and the spacing between adjacent sensors is 1m.

For the $k$-th test case, the cross power spectral density $S_{ij}^k(\omega)$ between receivers $i$ and $j$ can be calculated as:

$$S_{ij}^k(\omega) = \frac{1}{T} \tilde{a}_i^k(\omega) \tilde{a}_j^{*k}(\omega), \quad (1)$$

where $T$ is the duration of the measurement, $\tilde{a}_i^k(\omega)$ denoted the frequency content of the acceleration at receiver $i$ for the $k$-th test case, and $\tilde{a}_j^{*k}(\omega)$ denoted the complex conjugate of $\tilde{a}_i^k(\omega)$. If $i=j$, this function is referred to as the auto power spectral density. The average cross power spectral density $S_{ij}(\omega)$ between receivers $i$ and $j$ can thus be computed as

$$S_{ij}(\omega) = \frac{1}{N} \sum_{k=1}^{N} S_{ij}^k(\omega),$$

in which $N$ is the number of test cases.
Next, the transfer function $H_y(\omega)$ and the coherence function $\Gamma_y(\omega)$ are computed respectively as:

$$H_y(\omega) = \frac{S_y(\omega)}{S_x(\omega)}.$$  \(2\)

$$\Gamma_y(\omega) = \frac{S_y(\omega)S_x^*(\omega)}{S_x(\omega)S_y^*(\omega)}.$$  \(3\)

The coherence function $\Gamma_y(\omega)$ is a measure of the experimental data quality. A unit value indicates a perfectly linear relation between the signal $i$ and $j$. A smaller coherence may indicate noise disturbing the measurements or non-linear behavior of the soil.

The receivers at 2m and 4m, 3m and 6m, 4m and 8m, 6m and 12m, 8m and 16m, 12m and 24m, 24m and 48m from the centre of the source are taken as pairs [2]. For each pair, the phase velocity of the surface wave $C_{E_R}(f)$ is estimated as:

$$C_{E_R}(f) = \frac{2\pi f \Delta x}{\Delta \varphi(f)}.$$  \(4\)

where $f$ is the engineering frequency (Hz) with $\omega = 2\pi f$; $\Delta x$ is the distance between the receivers and $\Delta \varphi(f)$ is the unfolded phase of the cross power spectral density $S_y(\omega)$. For a fixed frequency $f$, the estimation of the dispersion curve $C_{E_R}(f)$ is withheld if the criteria $\Gamma_y(\omega) \geq \Gamma_{\text{min}}$ and $\frac{\Delta x}{\lambda_{E_R}(\omega)} \leq \frac{\Delta \varphi(f)}{2\pi f}$. The first condition imposes a threshold on the coherence function to limit the influence of incoherent noise. A value $\Gamma_{\text{min}} = 0.95$ is used. The second condition ensures that the ratio of the distance $\Delta x$ and the estimated surface wavelength $\lambda_{E_R}(\omega) = \frac{2\pi C_{E_R}}{\omega}$ is within certain bounds. The lower bound $\bar{r}_{\text{min}}$ acts as a high-pass filter that limits the contribution of body waves, while the upper bound $\bar{r}_{\text{max}}$ serves as a low-pass filter to remove the high frequency components contaminated by coherent noise. Herein, $\bar{r}_{\text{min}} = 1$ and $\bar{r}_{\text{max}} = 3$ are used [3].

By means of the above analysis, the experimental dispersion curve is obtained in the frequency range between 15Hz and 100Hz, as is showed in Fig. 3 by red points. For low frequencies, the wavelength of the surface waves is large and the surface waves reach deep soil layers. These layers are generally stiff, resulting in a high phase velocity. Due to the limitation of field experiment, it is very difficult to get the dispersion curve below 10Hz. For high frequencies, the wavelength of the surface waves is smaller and the surface waves travel through shallow soil layers. These layers are generally softer, resulting in a lower phase velocity.

![Figure 3: Experimental dispersion curve.](image-url)
3. Dynamic Parameters Inversion of Foundation Soil

3.1 Theoretical dispersion curve by TLM-PML method

In the theoretical analysis of ground-born vibrations, the soil is generally modelled as a horizontally layered linear elastic halfspace. Herein, the TLM-PML method [4] is used to calculate the surface wave modes of layered halfspace and the theoretical dispersion curve of a soil with a given profile.

According to earlier rough survey [5], the soil on the SASW test site at the campus of Beijing Jiaotong University has some known parameters, listed in Table 1. The resulting theoretical dispersion curve of this site is showed in Fig.4. It can be seen that there is good agreement between the theoretical and experimental dispersion curves, which validate the effectiveness of TLM-PML method and SASW field test.

Table 1: Benchmark soil profile of free field in SASW test

<table>
<thead>
<tr>
<th>Soil layer</th>
<th>Depth /m</th>
<th>Density /(g·cm(^{-3}))</th>
<th>Elastic modulus /MPa</th>
<th>Poisson ratio</th>
<th>(C_s/(m\cdot s^{-1}))</th>
<th>(C_p/(m\cdot s^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Backfill</td>
<td>2</td>
<td>1.49</td>
<td>48</td>
<td>0.32</td>
<td>110</td>
<td>215</td>
</tr>
<tr>
<td>Miscellaneous fill</td>
<td>3</td>
<td>1.65</td>
<td>138</td>
<td>0.35</td>
<td>176</td>
<td>365</td>
</tr>
<tr>
<td>Sand</td>
<td>30</td>
<td>2.01</td>
<td>303</td>
<td>0.33</td>
<td>238</td>
<td>473</td>
</tr>
<tr>
<td>Clay</td>
<td>∞</td>
<td>2.05</td>
<td>659</td>
<td>0.29</td>
<td>353</td>
<td>648</td>
</tr>
</tbody>
</table>

![Figure 4: Experimental and theoretical dispersion curves.](image)

3.2 Dispersion curve fitness and by genetic algorithm

In fact, the soil properties as Table 1 are often undetermined in predicting traffic-induced ground vibrations. Therefore, it is necessary to get the soil parameters by inversion algorithm before exactly predicting the vibration. The inversion strategy of soil parameters is to firstly assume several crucial soil parameters, and then to calculate the theoretical dispersion curve and compare it with the experimental dispersion curve, and furthermore iteratively adjust and optimize the soil parameters in order to minimize the distance between the theoretical and the experimental dispersion curve by some algorithms.

Generally, each soil layer is characterized by a thickness \(d\), a shear wave velocity \(C_s\), a dilatational wave velocity \(C_p\), a hysteretic material damping ratio \(\xi\) (for both shear and dilatational waves), and a density \(\rho\). When the number of soil layers is great, the cost of inversion calculation for the five undetermined parameters is very high. Because the sensibility of density and the dilatational wave velocity on the dispersion curve of surface wave is much lower than that of the depth and the shear wave velocity [6], the parameters in the optimization are the layer thickness \(d\) and the shear wave velocity \(C_s\) for each layer, and the other characteristic parameters are estimated from the earlier rough survey performed at the same site.
In this paper, the other data except $C_S$ are assumed to be known and adopted from Table 1. The search range of shear wave velocity $C_S$ is 50-350 m/s. As the effect of the material damping ratio on the dispersion curve is negligible, an arbitrarily chosen value $\xi = 0.03$ is used for all layers. The inversion procedure minimizes the error between the inversion-theoretical dispersion curve $C_R^f (f)$ by inversion algorithm and the experimental dispersion curve $C_R^k (f)$. In this study, the minimization problem is solved with genetic algorithm [7], as is shown in Fig. 5. The fitness is the criteria to evaluate the minimization degree, and the fitness function depends on the object function, which can directly influence the performance of genetic algorithm. Herein, the objective function in the genetic algorithm is defined as:

$$
\text{Obj}_R(f_j) = \text{Obj}_R(f_j) + |\text{abs}(C_{R}^{k}(i, f_j)) - C_{R}^{f}(i, f_j)|.
$$

(5)

in which the range of frequency $f_j$ is 15Hz~100Hz. A logarithmic spacing scheme is used to assign a higher weight to the low frequency range. In this way, the steep variation of the experimental dispersion curve in the low frequency range is properly accounted for.

Figure 5: Inversion procedure by means of genetic algorithm.

Figures 6 gives the resulting inversion-theoretical dispersion curve (the black line). In addition, the experimental dispersion curve is also illustrated in Fig.6. By comparison, it can be found that the fitness of object function is very satisfactory for most frequencies except the low frequency range 10-20Hz and the abnormal range 55-60Hz.
The inversion results for the shear wave velocity \( C_S \) are listed in Table 2 and Fig.7. It can be seen the inversion errors for the top soil layer and the bottom layer are small, respectively 0.37% and 0.85%. However, because the experimental dispersion curve in the frequency range of 0-10Hz is missing and it may influence the inversion results of the second and the third layers, the inversion error of \( C_S \) for the second layer and the third layer is a little unsatisfactory. Therefore, the inversion accuracy greatly depends on the completeness of experimental dispersion curve.

Table 2: Comparison of actual and inversion-theoretical shear wave velocity

<table>
<thead>
<tr>
<th>Soil layer</th>
<th>Depth /m</th>
<th>Actual ( C_S ) /( \text{m/s} )</th>
<th>Inversion-theoretical ( C_S ) /( \text{m/s} )</th>
<th>Density /( \text{g/cm}^3 )</th>
<th>Inversion error /%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>110</td>
<td>110.41</td>
<td>1.49</td>
<td>0.37</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>176</td>
<td>202.49</td>
<td>1.65</td>
<td>15.05</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>238</td>
<td>257.62</td>
<td>2.01</td>
<td>8.24</td>
</tr>
<tr>
<td>4</td>
<td>∞</td>
<td>353</td>
<td>350.00</td>
<td>2.05</td>
<td>0.85</td>
</tr>
</tbody>
</table>

![Figure 7: Comparison of actual and inversion-theoretical shear wave velocity.](image)

### 4. Conclusions

In this paper, a SASW field test is carried out to obtain the experimental dispersion curve of a given site. Then the mathematical model of inversion is introduced and the inversion program is compiled on the basis of the genetic algorithm by iteratively minimizing the distance between the experimental and theoretical dispersion curves. It is found that the inversion accuracy greatly depends on the completeness of experimental dispersion curve from SASW field test. The good agreement and low inversion error prove that the dynamic parameter inversion method of foundation soil is correct and effective, which can be used to obtain the crucial soil properties before predicting the traffic-induced ground and structural vibrations.

### REFERENCES

