Optimal vibration isolation semi-active control algorithm based on full state feedback

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Damping plays an important role in vibration control. To find the best control strategy for vibration isolation system using magneto rheological (Mr) damper, the existing relative speed damping control strategy and acceleration damper control strategy are analyzed. On this basis, the relative speed-acceleration joint damping control strategy and the optimal vibration isolation semi-active control algorithm based on full state feedback are designed. The simulation results show that semi-active control algorithm based on full state feedback has the best vibration isolation effect among four situations.

Keywords: Semi-active full state feedback

1. Introduction

In the past two decades, with the help of computer, signal processing, sensors, actuators and other technologies, the active vibration control technology has developed rapidly. The most basic and common feedback control strategies include displacement-speed combined feedback based on state feedback, and speed feedback control based on output state feedback. In some mobile controlled objects, such as vibration reduction, civil engineering, robot control and other related vibration measurement systems, the most commonly used sensor is acceleration sensor. This is because the acceleration sensor has many advantages such as high measurement accuracy, frequency bandwidth, light weight, and small volume, which makes the signal tested in such a system an acceleration signal. Due to the widely use of acceleration sensors in vibration active control, the accumulated errors of numerical integration of acceleration signals to obtain velocity or displacement and the time delay of filters will become very large, so the study of the relative speed-acceleration joint damping feedback control becomes important in semi-active vibration control system recently.

In this section, the motion state of floating raft vibration isolation system is analyzed, and the appropriate switch control strategy based on magnetorheological (Mr) damper is selected to control it. Advantages and disadvantages of relative speed damping control strategy are analyzed firstly. Then strong points and shortcoming of acceleration damper control strategy are analyzed. On this basis, semi-active switching control strategy of relative speed-acceleration joint damping is put forward.
In order to give full play to the continuously adjustable damping force of Mr damper and obtain the best vibration isolation effect, the full-state feedback optimal controller is introduced into semi-active control. The output current is continuously adjusted to give full play to the damping adjustable advantages of Mr damper. The simulation results proved that optimal semi-active vibration isolation control based on full-state feedback is the best algorithm for vibration isolation system using Mr damper.

2. Model of vibration isolation system

Figure 1 is a schematic diagram of the floating raft vibration isolation system of a ship. The floating raft system is a double-deck vibration isolation system. It is consisted of equipment, upper isolator, intermediate raft body, lower layer isolator and foundation. The floating raft system is mounted on hull base. That is, hull base is the foundation of floating raft system. The flexibility of hull base is considered in order to design accurately.

Where \( m_1 \) represents the mass of the equipment, \( k_1 \) and \( c_1 \) denote the stiffness and damping of the upper isolator, \( m_2 \) represents the mass of the intermediate valve body, \( k_2 \) and \( c_2 \) denote the stiffness and damping of the lower layer isolator, \( m_3 \), \( k_3 \) and \( c_3 \) represent the equivalent mass, equivalent stiffness and equivalent damping of the flexible foundation respectively.

3. Semi-active switch control

Switching control is the simplest in semi-active control, and its essence is equivalent to feedback bimodal regulation based on some state signal. At present, the control effect of switch control has been achieved in the aspects of automobile suspension, seismic resistance of buildings, bridge base and cable damping, and so on. Among them, ceiling damping control (Sky-Hook, SH), floor shed damping control (Groud-Hook, GH), acceleration damping control (Acceleration Drive Damping, ADD) and relative velocity damping control (Rakheja-Sankar, RS) are common.
3.1 Semi-active switching control based on relative velocity

Relative velocity damping control (Rakheja-Sankar, RS) was proposed by Rakheja [1] in 1985 and is mainly used in semi-active control of vehicle shock absorbers. The vehicle vibration absorber is similar to the floating raft isolation system, so the floating raft switch control strategy can be designed based on the idea of relative damping control. The essence of relative damping control is to weaken the transfer of vibration energy by adjusting the elastic force conversion ability of the upper spring.

In the floating raft vibration isolation system, the intermediate valve body is taken as the research object. If the outside world does work on the spring \((x_i - x_j)(\dot{x}_i - \dot{x}_j) > 0\) and the spring absorbs energy from the outside, the damper input current should be adjusted to zero ensuring that as much energy as possible is absorbed into elastic potential energy by the spring. And when the spring works on the outside \((x_i - x_j)(\dot{x}_i - \dot{x}_j) \leq 0\), the spring releases the energy to the outside world. At this time, the input current of the damper should be adjusted to the maximum to ensure that the damper absorbs as much elastic potential energy as possible from the spring. Because the actual damping force of Mr damper is affected by many factors, such as current, external velocity and displacement, current is chosen as semi-active control variable. Therefore, the relative speed semi-active switch control strategy is as follows:

\[
I_{\text{semi}} = \begin{cases} 
0 & (x_i - x_j)(\dot{x}_i - \dot{x}_j) > 0 \\
I_{\max} & (x_i - x_j)(\dot{x}_i - \dot{x}_j) \leq 0 
\end{cases}
\]

(1)

Where \(I_{\text{semi}}\) represents semi-active control output current and \(I_{\max}\) represents maximum operating current of Mr damper.

The floating raft system is simulated with 0-30Hz sweep frequency, and the sinusoidal signal is used, the amplitude is 500N. The simulation result is shown in Figure 2.

![Figure 2: Comparison of acceleration response of base](image)

Figure 2 shows that the relative velocity damping switch control only reduces the resonance peak of the first order resonance frequency. Before and after the first order resonance frequency, the relative velocity damping switch control actually weakens the vibration isolation effect of floating raft.

In the vicinity of the second order resonance frequency, the relative velocity damping switch control shows a better vibration isolation control effect. In the higher frequency range, the relative velocity damping switch control has a lower vibration isolation effect than the non-controlled floating raft control, but the decrease is not significant.

3.2 Semi-active switch Control based on acceleration

Acceleration damping control (Acceleration Drive Damping, ADD) is a semi-active control strategy proposed by Savaresi [2] in 2005. It has been proved to be a simple and reliable optimal control strategy.

In the floating raft vibration isolation system, if the damping force direction is opposite to the equipment acceleration direction \(a_i(\dot{x}_i - \dot{x}_j) > 0\), the input current should be increased as much as possible to restrain the equipment acceleration. If the direction of damping force is the same as the direction of
equipment acceleration \((a_1(\dot{x}_1 - \dot{x}_2) \leq 0)\), the input current should be minimized to prevent the increase of equipment acceleration. At the same time, the current is chosen as the semi-active control variable, and the acceleration semi-active switch control strategy is as follows:

\[
I_{\text{semi}} = \begin{cases} 
I_{\text{max}} & \quad a_1(\dot{x}_1 - \dot{x}_2) > 0 \\
0 & \quad a_1(\dot{x}_1 - \dot{x}_2) \leq 0
\end{cases}
\]  

(2)

0-30Hz sweep frequency is also selected for simulation, and the force sinusoidal signal is used to excite the signal, the amplitude is 500N. The simulation results are shown in figure 3.

![Figure 3: Comparison of acceleration response of base](image)

Figure 3 shows that the acceleration damping switch control enhances the isolation effect before the first order resonance frequency, but decreases the isolation effect of the first order resonance frequency after the first order resonance frequency.

In the vicinity of the second-order resonance frequency, the acceleration damping switch control shows a better vibration isolation control effect. In the higher frequency range, the vibration isolation effect of the acceleration-damping switch is lower than that of the non-controlled floating raft, and the decrease is larger than that of the relative velocity damping control.

3.3 Relative velocity combined acceleration semi-active switch control

Analyzing the two semi-active control strategies, it is found that the relative velocity semi-active switch control reduces the peak value of the base response from the first order resonance frequency, but increases the base response before and after the first order resonance frequency. Although the vibration suppression effect of acceleration semi-active switch control is obvious before the second order resonance frequency, but after the second order resonance frequency, the vibration isolation effect is obviously reduced.

Therefore, the two control strategies are considered to be combined to control. When the output current determined by the two control strategies is the same, the common decision value is chosen as the output. If the two decisions are inconsistent, the maximum current value is taken as the output from the point of view of as much energy consumption as possible. Therefore, the relative velocity-acceleration combined semi-active switch control strategy is as follows:

\[
I_{\text{semi}} = \begin{cases} 
I_{\text{max}} & \quad a_1(\dot{x}_1 - \dot{x}_2) > 0 \\
0 & \quad a_1(\dot{x}_1 - \dot{x}_2) \leq 0 \\
& \quad \|\dot{\mathbf{x}}_1 - \dot{\mathbf{x}}_2\| < 0 \\
& \quad \|\mathbf{x}_1 - \mathbf{x}_2\| \geq 0
\end{cases}
\]  

(3)

The simulation results of three switching algorithms for 0-30Hz frequency sweep show that the force sinusoidal signal is excited. The amplitude is 500N. The result is shown in Figure 4.
It can be seen that the joint damping control reduces the vibration isolation effect before the first order resonance frequency and near the second order resonance frequency simultaneously. The vibration isolation effect of the higher frequency range is lower than that of the non-controlled floating raft, but the decreasing amplitude is smaller than the acceleration damping. Therefore, the semi-active algorithm of joint damping control has the advantages of both relative velocity damping control and acceleration damping control.

4. Optimal semi-active vibration isolation control based on full-state feedback

The joint damping control does not feedback all the state signals to adjust the damping force, which makes the damping force which is most suitable for the state of the system cannot be given in some frequency bands. At the same time, the switch control can only adjust the current of 0 and the maximum of the current, and does not give full play to the advantages of continuously adjustable damping force of Mr dampers. In order to obtain the best vibration isolation effect, the state feedback optimal controller is introduced to adjust the output current in semi-active control, so as to give full play to the advantage of damper damping adjustable.

4.1 Basic theory of full state feedback optimal control

If the state space equation of the system is

\[ X = AX + BU \]
\[ Y = CX + DU \]  

The quadratic expression of the performance index of the system is

\[ J = \frac{1}{2} \int_0^T (X'QX + U'R^TPU)dt \]

Where Q and R are called the weight matrixes of the optimal algorithm. They are positive semidefinite matrixes. U is the output optimal control force, and the linear relation between U and X is satisfied.

\[ U = -KX \]

Where K represents the state feedback gain matrix, and

\[ K = R^+B^TP \]

Where P is a positive definite symmetric matrix, which can be obtained by solving the Riccati equation. The Riccati equation is as follows:

\[ PA + A^TP - PBR^+B^TP + Q = 0 \]
The output $U$ with the minimum value can be obtained by substituting the $P$ value of the solution with the substitution of Equation 7 and 6.

### 4.2 Full state feedback optimal semi-active control algorithm for vibration isolation

As shown in Figure 1, the vibration kinematics equation is established as follows:

$$
\begin{align*}
  m_1 \ddot{x}_1 + k_1 (x_1 - x_2) + c_1 (\dot{x}_1 - \dot{x}_2) &= F - F_u \\
  m_2 \ddot{x}_2 + k_2 (x_2 - x_1) + c_2 (\dot{x}_2 - \dot{x}_1) &= F_u \\
  m_3 \ddot{\chi}_3 &= 0
\end{align*}
$$

The dynamic equation is changed into the state space equation without considering the external disturbance. If the state variable $\mathbf{x} = [x_1 \ x_2 \ \dot{x}_1 \ \dot{x}_2 \ \ddot{x}_1 \ \ddot{x}_2]^T$ and the acceleration of the base $\mathbf{y} = [\ddot{x}_3]^T$ as the output of the system, the state space equation 10 is shown as follows:

$$
\begin{align*}
  \dot{\mathbf{x}} &= A\mathbf{x} + B\mathbf{v} + B_2\mathbf{w} \\
  \mathbf{y} &= C\mathbf{x}
\end{align*}
$$

Order $\mathbf{u} = \mathbf{v} + B^*\mathbf{b}_w$, form 10 is transformed into a standard form of state space:

$$
\begin{align*}
  \dot{\mathbf{x}} &= A\mathbf{x} + B\mathbf{u} \\
  \mathbf{y} &= C\mathbf{x}
\end{align*}
$$

The quadratic form of the performance index of vibration isolation system is established, that is, there are

$$
J = \frac{1}{2} \int (X^TQX + U^TRU) dt
$$

By observing the performance index, it can be found that the quadratic form of the system takes into account both the system response and the control energy, that is, the fast response of the system requires a large input of energy. There is a contradiction between the two, but the performance of Mr damper ensures its ability of fast response and low energy dissipation. Therefore, the optimal output force can be obtained by taking a smaller value of $R$ and a relatively large value of $Q$, which can satisfy the need of optimal control of the system.

It is important to note that the feedback control force based on the full state feedback optimal control method is provided by an unrestricted main actuator. As Mr damper is a semi-active component, the direction of output controllable damping force is determined by the external state and is not controllable. Therefore, the magneto-rheological damper cannot completely realize the feedback control force obtained by the full state feedback optimal control method.

Only when the feedback control force obtained by the full state feedback optimal control method is opposite to the velocity direction of the magneto-rheological damper, that is, when the feedback control force is the same as the magneto-rheological damper damping force, the magneto-rheological damper can be equivalent to the main actuator. At this point, the full state feedback optimal control method is effective.

If the optimal control force obtained by the full state feedback optimal control method is $f_c$, the equivalent range of Mr damper is given by combining with acceleration damping switch control.

$$
I_{semi} = \begin{cases} 
  a_i(\dot{x}_1 - \dot{x}_2) & 0 < I_{semi} < \infty \\
  a_i(\dot{x}_1 - \dot{x}_2) > 0 & f_c(v_1 - v_2) < 0 \\
  a_i(\dot{x}_1 - \dot{x}_2) < 0 & f_c(v_1 - v_2) \geq 0
\end{cases}
$$

Where $I(f_c)$ represents the current obtained by the feedback compensation inverse model when the output damping force is $f_c$. 

\[a\]
4.3 Simulation and analysis

The full state feedback optimal vibration isolation algorithm is compared with the relative velocity-acceleration semi-active switching algorithm and the uncontrolled floating raft. An uncontrolled raft with a current of 0 can be considered to have added a fixed damping. The excitation uses a sinusoidal force signal with an amplitude of 500N, and the result is shown in Figure 5.

![Figure 5: Comparison of semi-active isolation control algorithm](image)

From Figure 5, it can be seen that the vibration isolation performance of floating raft isolation system using the full state feedback optimal vibration isolation algorithm is obviously improved.

At the first order resonance frequency, the optimal vibration isolation effect is basically the same as that of the relative velocity-acceleration combined semi-active switching algorithm, and the vibration near the resonant frequency of the floating raft is well suppressed.

In the frequency band above the second order resonance frequency, the difference between the optimal vibration isolation effect and the uncontrolled floating raft isolation effect is very small. The resonance effect is consistent with the resonance effect of I=0A, because the magnetorheological damper itself contains a certain damping, even if the current is 0, it is equivalent to adding part of the damping in the original system.

When we choose the MRD, we should select the MRD with the minimum initial damping and the maximum adjustable coefficient. So that even if the power is cut off, the damping of magnetorheological damper can also exert the passive control effect to suppress the resonance frequency band without worsening the vibration isolation effect in the high frequency band.

It can be seen from the above comparison that the floating raft vibration isolation system using the full state feedback optimal vibration isolation algorithm makes full use of the adjustable capability of the magnetorheological damper and realizes the optimization of the raft vibration isolation capability in the full frequency range.

5. Conclusion

The simulation results show that semi-active control algorithm based on full state feedback has the best vibration isolation effect among four situations.

In the later period, there are two directions worth studying. The first is to control the time delay well and the second is the decoupling control problem of the high-dimensional time delay system.

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