1. Introduction

The production of new materials requires prior knowledge of the content of components, their mechanical characteristics, geometrical parameters of compounds and their arrangement inside a solid. Among the huge class of various materials, we focus on poroelastic materials which are widely used as acoustic absorbers and thermal insulators due to their exclusive properties [1]. In the context of design of poroelastic panels with acoustic absorption and/or transmission loss features, optimal macroscopic acoustic properties of the panel depend not only on the characteristics of bulk porous materials but also on the structural parameters such as the panel’s thickness. Moreover, the composite panel made of mixture of different poroelastic materials may have improved acoustic performances in absorption or transmission loss [2]. The goal of this work is to formulate and implement a procedure to design a poroelastic panel with a maximum acoustic absorption and optimal microstructural cell parameters found by means of the homogenisation technique.
2. Method and results

Let us consider an air-saturated poroelastic layer of constant thickness $d$ (Fig. 1). The layer has a rigid backing at one side and surrounded by the air from another. In the present study, we only consider the normal incident in the $e_3$-direction, which corresponds to the testing of materials in impedance tubes. The incident plane pressure wave, denoted by $p_i \exp(i\omega t - iq_0x_3)$, with the wavenumber $q_0 = c_0/\omega$, angular frequency $\omega$, hits the porous layer and reflects from it. We assume that the poroelastic material has periodic microstructure. Fig. 1 shows an example of a unit cell of size $L \times L \times L$ with the cylindrical pore filled with air whose the radius is $r$ and the height is $L$ along the $e_3$-direction.

![Acoustic problem for a porous layer of thickness with microstructure](image)

Figure 1: Acoustic problem for a porous layer of thickness with microstructure

We are interested in the design of a porous (homogenous or multilayer) panel with high absorption properties. We focus on the low frequency band and consider a $1/3$ octave that includes a set $S$ of 12 frequencies (in Hz): $S = \{200, 250, 315, 400, 500, 630, 800, 1000, 1250, 1600, 2000, 2500\}$. The absorption feature of the considered panel may be evaluated by a quantity called the Sound Absorption Average, which corresponds to the mean of the absorption coefficient evaluated at these frequencies: $ar{\alpha} = \frac{1}{12} \sum_{j=1}^{12} \alpha_a(\omega_j)$, where the $\omega_j$, for $j = 1, ..., 12$, are the values of the set $S$. We define the functional $J$ as the objective function and search for its maximum over the range of design parameters at microstructural scale which are taken to be different for the acoustic problem described above. For each parameter set, the effective properties are computed by using asymptotic homogenization method.

For the example presented in Fig. 1, we took the porosity $\phi$ and cell’s size $L$ as design parameters and solved optimisation problem for fixed values of $d$. Effective poroelastic properties involved in Biot’s model are not effected by the change of $L$ but remain dependent on the porosity. On the other hand, the effective permeability and compressibility can drastically vary for different $L$. The problem is stated as follows:

\[
\min_{\phi, L} -J \quad \text{such that} \quad \begin{cases} 
0.01 \leq \phi \leq 0.78, \\
10 \mu\text{m} \leq L \leq 1000 \mu\text{m}.
\end{cases}
\]  

(1)

The upper bound for $\phi$ is chosen according to the fact that there is a maximum value for the porosity $\phi = \pi/4 \approx 0.785$, when the cylindrical pore touches the walls of the solid matrix. The upper bound for $L$ is chosen in accordance with the homogenisation procedure when the separation of scales is required. The solution of the constrained optimisation problem is obtained by a direct algorithm so-called the pattern search algorithm which does not require the calculations of gradient.

In this study, on consider two kinds of matrix materials with different rigidities (epoxy and silicone rubber). The elastic properties are isotropic with Young’s modulus $E_0 = 3.5$ GPa, Poisson’s ratio $\nu_0 = 0.33$ and density $\rho^s = 1540$ kg/m$^3$. The properties of silicone rubber are $E_0 = 1.2$ MPa, $\nu_0 = 0.47$ and $\rho^s = 1300$ kg/m$^3$. Pores are assumed to be saturated with air with the following properties: $\mu = 1.71 \times 10^{-5}$ Pa·s, $\rho^f = 1.293$ kg/m$^3$, $c_0 = 331.6$ m/s, $\kappa = 2.41 \times 10^{-2}$ W/m·K, $C_p = 1006$ J/(kg·K), $\gamma = 1.403$ at $T_0 = 0^\circ$ C and $P_0 = 10^5$ Pa.
The results of optimisation procedure are presented in Fig. 2 (left). One can notice that for each value of $d$, the optimal values of $J$ are unchanged by using rigid or epoxy frame models. The rigidity of skeleton has been shown to have significantly influence on absorption coefficient of porous materials [3]. While the absorption coefficient for epoxy frames appears to be indistinguishable for the studied range of frequencies and $d = 1, 2, 5, 10$ cm (data not shown), different optimal parameters are found for the soft elastic frame made of silicone rubber. In this case, the optimal porosity does not tend to be maximum but instead takes low values whereas the cell size appears to be high. The optimal values $J_{opt}$ does not increase much but the behaviour of absorption coefficient changes quantitatively as seen in Fig. 2 (right). We observe the same peaks as in the case of epoxy frame but these are shifted to lower frequency band and occur more often. This phenomenon is caused by the resonance of the frame which becomes more pronounced for soft skeletons.

<table>
<thead>
<tr>
<th>Epoxy / Silicone rubber</th>
<th>$d$ (cm)</th>
<th>$L_{opt}$ (µm)</th>
<th>$\phi_{opt}$</th>
<th>$J_{opt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>13 / 998</td>
<td>0.78 / 0.03</td>
<td>0.03 / 0.06</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>28 / 998</td>
<td>0.78 / 0.3</td>
<td>0.06 / 0.18</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>80 / 472</td>
<td>0.78 / 0.24</td>
<td>0.14 / 0.27</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>204 / 762</td>
<td>0.78 / 0.19</td>
<td>0.3 / 0.36</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>255 / 650</td>
<td>0.78 / 0.31</td>
<td>0.42 / 0.49</td>
</tr>
</tbody>
</table>

Figure 2: (left) results of optimisation procedure for a porous layer with the microstructure shown in Fig. 1 given for the epoxy and silicone rubber frames; (right) Absorption coefficient in the studied range of frequencies and optimal design parameters with silicone rubber frame for $d = 1, 2, 5, 10, 20$ cm.

In this paper, only homogeneous porous plate with cylindrical pores has been studied for illustration and validation purposes. Further studies to consider different cell geometries and/or heterogeneous plates, e.g. plates made of multilayered, functionally-graded or periodic mixture of poroleastic materials with different contrasts [4], can be performed using the same procedure. Moreover, more enhanced optimisation algorithms could be employed to consider combination between various material’s physical properties with microstructural characteristics, e.g. higher rigidity or additional heat insulation.

REFERENCES


