THREE DIMENSIONAL ACOUSTIC CARPET CLOAK BASED ON AN ULTRATHIN METASURFACE

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The manipulation of acoustic wave has great significance in engineering related to acoustic device design. Recent study reveals that acoustic metasurface can easily change the wave front by a periodic phase array composed of microunits, which provide us a branch approach in physical application. Here, we proposed a design of a 3D ultrathin acoustic carpet cloak based on acoustic metasurface in deep sub-wavelength range. The microunit of the metasurface is constructed by two helmholtz resonators nested together and we can change the reflection phase by adjusting the relative angle of the two helmholtz resonators. The carpet can completely conceal the object because the designed metasurface can compensate the phase distortion caused by the object. The operating frequency is about 1200Hz, numerical simulations show the carpet functions well in normal and small-angle oblique incidences. Moreover, the thickness of the metasurface is only approximately λ/19, so we can design the carpet cloak in expected shape with almost no increase in target size.

Keywords: metasurface, ultrathin, three-dimensional, carpet cloak

1. Introduction

Cloak refers to a device which can hide the object from an incident wave. This concept was originally realized by covering the object with the absorbing materials in nature on the surface of the object, dramatically reducing the scattering waves. But it does not work well in low frequency with a limited size particularly in acoustic field. In recent years, with the rapid development of metamaterial, carpet cloak was proposed based on coordinate transformation theory by using metamaterial [1-3]. This kind of carpet cloak allows the acoustic waves smoothly bypass the hidden object and keeps the wave front unchanged after passing through the object by covering it with metamaterials. A huge amount of research works has been done and validated its excellent performance by experiments [4-6]. However, the cloak based on coordinate transformation theory has a bulky size compared to the object to be hidden and hard to avoid some singular parameters [1-8]. To address those problems, metasurface-based carpet cloak, which can minic the reflected behavior like the plane ground, is proposed and gets heated attention [8-11].

Metasurface, an artificial interface capable of modulating wave front in homogeneous medium by a layer of units with subwavelength size, was first predicted by Federico Capasso in 2011 using generalized Snell’s law and later verified by Yu via the V-shape antenna structure in electromagnetic realm [12,13]. Inspired by the pioneer works in electromagnetic field, the metasurface was introduced to acoustic field soon afterward due to the similarity between electromagnetic wave and acoustic wave. Thanks to its...
preeminent manipulation in wave front, a lot of exotic devices such as meta-lens, one-way tunnel and carpet cloaks have been proposed, which opens up new possibilities for acoustic applications [14-22].

In this paper, we derived the phase compensation of the object of arbitrary shapes in different position from the generalized Snell’s law and confirmed its validity via a 3D conical carpet cloak. This 3D conical carpet cloak comprises of an array of two helmholtz resonators with the feature of annular split cavities nested together, which can change the reflected phase by tuning the cavity inside mechanically and can be easily fabricated by 3D printing technology. In addition, the thickness of metasurface we designed is only about $\lambda/19$.

![Figure 1: the schematic of the metasurface and carpet cloak](image)

2. Theory

The propagation of wave on the interface between different medias follows the Snell’s law in classic acoustics, which informs us that the wave will reflected at the same angle as its incident angle. But introducing an abrupt local phase shift along the interface can control the direction of the reflected wave at the subwavelength scale has proved by the generalized Snell’s law, which breaks the traditional manner to predict the wave profile in two natural medias and provides new applications in acoustics, particular in ultrathin carpet cloaks [13]. Here we will derivate the phase compensation from the generalized Snell’s law of reflection which can be expressed as:

$$k_0(\sin \theta_r - \sin \theta_i) = \frac{d\phi}{dx}$$

where $\theta_r$ is the reflected angle, $\theta_i$ is the incident angle, $k_0 = 2\pi/\lambda$ is wave number in free space, $\lambda$ is the wavelength and $d\phi/dx$ is the local phase gradient, respectively. All these has displayed in Fig. 1(a). This formula indicates that we can manipulate the reflected angle by introducing an abrupt local shift.

From the equation above, we can analyze a simplified model of a conical carpet cloak shown in Fig. 1(b). Assuming that a plane acoustic wave impinging on the cloak of the angle $\theta$ tilted to the ground ($x-o-z$ plane), the wave will scatter around because of the inclined plane by a tilted angle $\alpha$ if there no metasurface. Yet, we expect that the acoustic wave can be reflected with the angle $\theta$ after impinging on the inclined plane to conceal the object, which means that the incident angle of the wave relative to the carpet cloak $\beta_i = \theta - \alpha$ and the reflected angle $\beta_r = \theta + \alpha$, corresponding to $\theta_i$ and $\theta_r$ in Eq. (1), respectively. So, we can conclude that:

$$k_0(\sin \beta_r - \sin \beta_i) = \frac{d\phi}{dh(x)} \sin \alpha$$

where $dx = dh/\sin \alpha$ and simplification of Eq. (4) yields:

$$\phi = -2k_0h(x)\cos \theta$$
So we can choose artificial structures of different reflected phase response according to the height of the conceal object to compensate the phase difference of the bump, thus recovering the reflected waves as if they are reflected from the plane ground.

Figure 2: (a) the cross-section of the designed carpet cloak, (b) the physical appearance of the conical carpet cloak, (c) the dimensions of the microunit and the phase response in different parameters

3. Carpet cloaking with metasurface

3.1 Design method

The three-dimensional conical carpet cloak has shown in Fig. 2(b), which is comprised of a series of microunits with different parameters of nested resonators to compensate the phase in different height calculated by Eq. (3) and we showed its cross-section in Fig. 1(a). The microunits consist of two cavities nested together with two columns and the overall size is \( w \times h \), corresponding to the width and height of the microunit, respectively. The internal shape of the cavity outside is circular and the radius of it is \( R = 6.5 \) mm, the cavity inside has the radius \( r = 4.5 \) mm and the gap between them is \( d = 1 \) mm, which is the propagating channel of the acoustic wave and each cavity has a groove with a width of \( w_0 = 1 \) mm. In addition, we can change the length of the channel by adjusting the relative angle of the two slits to realize a reflected phase shift approximately covering \( 2\pi \) range.

In order to retrieve the phase information of the microunits with different parameters, a full wave simulation was performed to research the reflection behaviour of the designed nest structure by sweeping the relative angle \( \phi \) at 1200 Hz. Figure. 2(c) displays the result that the reflected phase gradually changing with \( \phi \). The background medium in the simulation is air with the speed and density \( c = 343 \) m/s, \( \rho = 1.29 \) kg/m\(^3\) and the material of the microunits is PLA which is hard enough to be seen as acoustic hard boundary and easy to be fabricated by 3D printing technology. The periodic boundary conditions are employed in the direction perpendicular to the wave path to reduce the boundary effects and only the lossless case is considered for the time being.

Moreover, to satisfy the phase compensation in different positions of the carpet cloak, we should choose different \( \phi \) of the microunits. Because the unit has a subwavelength size, the reflected wave excited by it can be seen as a point source. Therefore, the height of the center of the microunits can be used to be the basis to select parameter \( \phi \) corresponding to the required phase shift. Here, we designed two kinds of units with \( w = 15 \) mm and 30 mm, respectively and the same \( h = 15 \) mm to improve the design accuracy by widening phase coverage. The bottom radius of the conical carpet cloak is about 0.78 m and the height of it is about 0.46 m. The thickness of the carpet cloak is same as that of the microunits, about
19 times smaller than the wavelength at the design frequency 1.2 kHz, so it has enough space to conceal the object. We designed the carpet cloak for a particular case that the incident angle is zero.

### 3.2 Performance of the carpet cloak

To evaluate the performance of our designed ultrathin metasurface carpet cloak, we have conducted a set of simulations using COMSOL Multiphysics 5.4 for the case when a plane wave impinges normally on an empty ground, the uncloaked object placed on the ground and the object with metasurface placed on the ground. All results were calculated by the 2D axisymmetric model to reduce the memory requirements and we displayed the plots in $x$-$o$-$z$ plane. The reflected pressure and phase field of three cases are presented in Fig. (3) at the central operating frequency 1.2 kHz. As expected, the sound field is distorted when the acoustic wave is reflected from the object with no metasurface, which will be a significant difference from the surroundings and can be easily detected. In comparison, the object covered with our designed metasurface can completely reconstruct the scattered sound field, showing the ideal reflected wave field compared with the case that impinges on the ground.

**Figure 3:** (a) the pressure field on a normalised wave and (b) the phase field on a normalised field of three cases

Despite its superior performance at the designed frequency, we found that the carpet cloak was also acceptable near the designed frequency in the course of our research. To further provide a method to evaluate the capacity of the carpet cloak restoring the wave front at different frequency and expressed in an intuitive way. Here, we proposed a reference coefficient $\gamma(f)$ to character the phases deviation of the reconstructed field and defined as:

$$\gamma(f) = \frac{1}{n} \sum_{i=1}^{n} \frac{\text{Phase}_{\text{max},i} - \text{Phase}_{\text{min},i}}{2\pi}$$

where $\text{Phase}_{\text{max},i}$ and $\text{Phase}_{\text{min},i}$ refer to the maximum and minimum peak value along a given wave front numbered by $i$, which is perpendicular to the designed wave direction and $n$ is the total number of wave fronts we explored. We calculated the frequency from 1 kHz to 1.4 kHz with the step of 10 Hz and displayed the results in Fig. 4. The value $\gamma(f) \approx 0.08$ at the designed frequency is very close to the value of the case that the wave incident on the ground, which means the carpet cloak completely conceals the object. In addition, the carpet cloak has a bandwidth of 100 Hz in the case of $\gamma(f) \leq 0.2$. 

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Figure 4: different reference coefficient $\gamma(f)$ in the case of ground, no cloak and with cloak as a function of frequency

Previous research verified the proposed carpet cloak has a good performance for a normal incident wave at the designed frequency. Is it possible to play the job at an oblique incidence? By observing Eq. (3), we can find that the $\phi$ is almost invariable when the incident wave has a small-angle change. Therefore, we discussed the applicable angle range of the carpet cloak. Figure 5 depicts the sound field with different incident angle of three cases. For the $15^\circ$ incident angle, the scatter field project nearly the same reflected direction as the sound wave reflected from the ground and when the incident angle is $25^\circ$, there exist a little reflected wave out of the right path in the left edge and distinct wave front discontinuity appeared at the top.

Figure 5: the pressure field on oblique incidence wave at the angle $5^\circ$, $15^\circ$, $25^\circ$ at the cases of ground, no cloak and with cloak corresponding to (a), (b), (c), respectively

As shown in Fig. 6, We further presented the radiation pattern of our design of the ground, the uncloaked object placed on the ground and the object with metasurface placed on the ground at different
incident angle in a polar coordinate. Such radiation patterns are calculated in far field 16 m, which is obtained by the formula: \( D = 2 \frac{d^2}{\lambda} \) where \( d \) is the diameter of the carpet cloak and \( \lambda \) is the wavelength to prove the angular dependence of the reflected waves in sound pressure level. Obviously, the radiation pattern of the cloak provides a similar pattern compared with the case of ground (blue line) when the angle of incidence is 5° and 15°. But the sound pressure level increases significantly with cloak (red line) and is close to pattern of no cloak (dark line), which means that the carpet cloak can not conceal the object when the incident angle is large than 25°.

\[ \text{Figure 6: the radiation pattern on oblique incidence wave at the angle 5°, 15°, 25° at the cases of ground, no cloak and with cloak corresponding to (a), (b), (c), respectively} \]

4. Conclusion

In conclusion, an ultrathin metasurface carpet cloak was proposed with the microunits of only \( \lambda/19 \) in thickness at the working frequency 1.2 kHz. We numerically verified the validity of the carpet cloak. The designed device could successfully conceal the object of arbitrary shapes under a three-dimensional conical cloak by providing a precise local phase modulation to restore the disordered reflection wave front. The device can not only work in the designed frequency, but also has a good performance in the nearby frequencies and can tolerate oblique incident wave within about 25°.

Moreover, the current design is just a verification scheme, aiming to open up a new way for acoustic cloak based on metasurface. In fact, we can choose the microunits to compensate the phases of any shapes, not only the cone. What’s more, this kind of design can be easily adapted to the underwater applications.

REFERENCES


