Pumping systems are used for transmission of fluids in various technological processes. Quite frequently the pumping unit in the pumping process is of a displacement type. Using the same type of pump in different systems results in pressure pulsations, which become the cause of increased vibroacoustic loading and failure of a unit components. Therefore, it is important to know the pressure pulsations in the fluid power system at the design stage. We have applied the theory of electrodynamic analogy and the equations of an acoustic quadrupole to propose a method for calculating pressure pulsations in fluid power systems based on models of dynamic primitive geometries such as “capacitance”, “pipe” and “resistance”. The proposed method allows to estimate the dynamic characteristics of the fluid power system at the design stage, and also allows to select the geometric parameters of the system ensuring minimal pressure pulsations.

Keywords: hydraulic system, impedance method, pressure oscillations reduction

1. Introduction

Pumps are usually the flow rate source in fluid power systems. In most cases, these are the pumps that cause increased dynamic loads of the hydraulic system.

There are static and dynamic characteristics of pumps. Static characteristics, in contrast to dynamic ones, do not take into account the non-stationarity of the pump working flow. The most effective method for determining the dynamic characteristics of the pumps is the impedance method which makes it possible to obtain the necessary dependencies, taking into account the characteristics of the system connected to the pump.

Pulsations of the flow, being a result of the pump flow rate formation process, tend to reduce the efficiency and service life of both the pump and the fluid power system [1,2,3]. Flow rate pulsations are directly related to pressure pulsations. Reducing pressure pulsations in complex pipelines is of particular importance in high-pressure fluid power systems [4]. Determining the level of pulsations in the fluid power system at its design and development stages allows to take measures to eliminate them.

The issue of determining the characteristics of a pulsating medium flow in a fluid power system is reduced to determining lumped parameters of inclusions (impedances of inclusions) - boundary conditions at the beginning and at the end of a straight pipe section [5]. The essence of the impedance method is that the boundary conditions are combined into one condition with the help of impedance inhomogeneity - a linear combination of pressure \( p \) and flow \( q \). Thus, the impedance of inclusion is understood as the total impedance equal to the ratio of the dynamic pressure to the dynamic velocity in the section where the inclusion is established.
Mathematical model of a quadrupole developed in electrical engineering [6] was first used to calculate high-amplitude pulsations of the medium in a split-path pipeline systems by Grizodub O.N. [7] and was later developed in the research of Berdnikov V.V., Glikman B.F., Shorin V.P. [8,9,10] et al. Solving problems by using this model allows not only to simplify computational operations and apply a number of general relations valid for a large group of linear circuits, but also to use the values of the parameters of the simplest elements of the system calculated or experimentally determined in advance.

2. Impedance method. Hydraulic system decomposition

The most common oscillatory system is an electrical circuit. Expertise in the field of electrical circuits, with the help of analogies, is known to be applied for solving problems in the field of fluid power, where the hydraulic oscillatory system is replaced by a similar electrical system. This task is simplified and reduced to designing an electrical circuit. An electrical circuit consists of interconnected simpler circuits, called branches or circuitry. The branches consist of elements (resistance, inductance and capacitance). In electrical circuits, the variables are one-dimensional and change over time. In fluid power systems that have small dimensions compared with the wavelength, vibrations can also be considered one-dimensional.

The analogue of electric current i, voltage u and resistance r in fluid power is the variable component of the flow rate g, pressure p and impedance Z, respectively.

Fluid power systems consist, as a rule, of the source of pulsations, pipelines and inclusions. The dynamic characteristics of the source of vibration can be determined by the method outlined in [10]. The dynamic characteristics of pipelines depending on their length, are considered either in lumped parameters or in distributed ones [9,10,11]. Dynamic characteristics of inclusions were developed in [10,12,13]. We will consider the dynamic characteristics of the pump, pipeline and main inclusions of the fluid power system.

Pulsations in the pressure of the working fluid are known to be the result of the interaction of the pump with the attached system [10,14,15,16] in accordance with the diagrams in Figure 1.

![Diagram for calculating the parameters of the pulsating flow of the fluid at the outlet of the pump](image)

**Figure 1:** Diagram for calculating the parameters of the pulsating flow of the fluid at the outlet of the pump

The diagram presents:

\[ Z_s = |Z_s|e^{j\varphi_s} \] - internal impedance of the pulsation source;
$Z_l = |Z_l|e^{j\phi_l}$ - internal impedance of the test-bench system;

$p(q)$ – amplitude of pressure pulsations (flow) of the fluid.

The amplitude of pressure pulsations at the outlet of the pump is:
- for equivalent pressure source:

$$p_s = p\frac{Z_l}{Z_{s+l}Z_i}$$  \hspace{1cm} (1)

- for equivalent source of the flow:

$$p_s = q\frac{Z_{s+l}Z_i}{Z_{s+l}Z_i}$$  \hspace{1cm} (2)

In formulas (1) and (2) we consider:

$p = |p|e^{j\omega t}$ – complex amplitude of pulsations generated by an ideal source of pressure pulsations;

$q = |q|e^{j\omega t}$ – complex amplitude of pulsations generated by an ideal source of flow;

The dynamic characteristics of the pump are determined, as a rule, experimentally [10, 15, 17, 18, 19, 20], and less frequently - theoretically [21]. The dynamic characteristics of the pump include:

- internal resistance of the pulsation source $Z_s$ and amplitude of pressure pulsations $p_s$.

The frequency characteristics of the elements can be obtained by writing the equations of the relationship between the parameters at the input (notation 1) and output (notation 2) in the form of a quadrupole equation:

$$\begin{bmatrix} p_1 \\ g_1 \end{bmatrix} = A \begin{bmatrix} p_2 \\ g_2 \end{bmatrix},$$  \hspace{1cm} (3)

where $p_1$ and $p_2$ are pulsation pressure components at the inlet and outlet,

$g_1$ and $g_2$ are pulsation pressure components at the inlet and outlet.

Parts with lumped parameters include those which characteristic size $l$ is 6...10 smaller than the wavelength of the pulsations $\lambda \left( \frac{\lambda}{l} \ll 6 \ldots 10 \right)$. These are, as a rule, elements of hydraulic equipment, tanks, dampers. For units with distributed parameters one can attribute a pipeline.

Most parts of the hydraulic system are interconnected sections of the pipeline. Matrix $T$ of pipeline length $l$ has the form:

$$T = \begin{bmatrix} ch\gamma & Z_{w}sh\gamma l \\ \frac{1}{Z_{w}}sh\gamma l & ch\gamma \end{bmatrix},$$  \hspace{1cm} (4)

where $Z_w$ – pipeline wave resistance,

$\gamma$ - wave propagation constant,

$l$ - pipeline length.

Capacitive parts of the pipeline systems (filters, extended short sections of the pipeline, etc.) are considered as reactive impedances in most computational models and their dynamic properties are determined by impedance $Z_n$. The matrix $E$ of capacity $V_C$ has the form:

$$E = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_c} & 1 \end{bmatrix},$$  \hspace{1cm} (5)

where $Z_c$ is impedance capacity.

Dynamic properties of narrow pipes and channels (resistances) are determined by impedance $Z_{ch}$. Matrix $K$ of a channel with a length $l$ and a diameter $d$ has the form:

$$K = \begin{bmatrix} 1 & Z_{ch} \\ 0 & 1 \end{bmatrix},$$  \hspace{1cm} (6)
where $Z_{ch}$ - channel impedance.

Most parts of the fluid power system can be represented through the above parts or their combinations.

The impedance calculation method is suitable for determining steady state pulsations in a fluid power system. According to [22, 23] the amplitude-frequency characteristics of pneumatic pipelines calculated by the impedance method, differ from the experimental characteristics in the region of resonant frequency range by no more than 2 dB, and the calculated and experimental resonant frequencies - by no more than 10%.

3. Impedance method. Modelling and numerical examinations of the hydraulic system

As an example of the application of the impedance method, we consider a simplified diagram of a hydraulic system with a power source (two-pole), a filter and a load (two-pole) and a tank (two-pole), as well as an electrodynamic analogue of the diagram (Figure 2).

![Circuit diagram of the hydraulic system](image)

Figure 2: Circuit diagram of the hydraulic system for calculating the parameters of the oscillatory flow of fluid at the outlet of the pump with an equivalent source of pressure pulsations

In Figure 2, the pump, filter and throttle are represented by the model in concentrated parameters, and two pipeline sections - as a model with distributed parameters.

Taking into account the accepted assumptions, the equations connecting the complex amplitudes of the pulsations $p$ and volumetric flow rate $q$ of the working medium in the characteristic sections of the information chain have the form [24]:

$$ Z_{ch} $
\[
\begin{align*}
\begin{bmatrix}
    p_0 \\
    g_0
\end{bmatrix} &= \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix}
    p_1 \\
    g_1
\end{bmatrix} \\
\begin{bmatrix}
    p_1 \\
    g_1
\end{bmatrix} &= \frac{1}{j\frac{v_c}{\rho c^2}} \begin{bmatrix}
    1 \\
    1 + j\frac{v_c}{\rho c^2} R_f
\end{bmatrix} \begin{bmatrix}
    p_2 \\
    g_2
\end{bmatrix} \\
\begin{bmatrix}
    p_2 \\
    g_2
\end{bmatrix} &= \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix}
    p_3 \\
    g_3
\end{bmatrix} \\
\begin{bmatrix}
    p_3 \\
    g_3
\end{bmatrix} &= \begin{bmatrix} 1 & R_{th} \\ 0 & 1 \end{bmatrix} \begin{bmatrix}
    p_4 \\
    g_4
\end{bmatrix} \\
p_4 &= 0,
\end{align*}
\]

where \( q_i, p_i \) – complex amplitude of the volume flow and pressure pulsations in the \( i \) sections of the fluid power circuit; \( A_{1,2} = ch\gamma l_{1,2}, B_{1,2} = Z_{0}sh\gamma l_{1,2}, C_{1,2} = \frac{1}{Z_{0}}sh\gamma l_{1,2}, D_{1,2} = ch\gamma l_{1,2} \) – pipeline transfer ratios; \( j = \sqrt{-1}; V_C \) – reduced cavity volume; \( \omega \) – angular velocity; \( \rho \) – fluid density, \( c \)– sound speed; \( R_f \)– active resistance (impedance) of the filter, \( R_{th} \)– adjustable throttle resistance (impedance).

5. Summary and conclusions

In this article we have described the impedance method for determining the dynamic characteristics of the fluid power system. The method allows to visualise the system in diagrams, to consider the parts of the fluid power system in lumped and distributed parameters. The impedance method allows to select the geometric parameters of the system (at the design stage) providing minimal pressure pulsations.

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REFERENCES

2. Ovsyannikov, B.V., Borovskij, B.I., Teoriya i raschyot agregatov pitaniya zhidkostnyh raketyh dvigatelej, Mashinostroenie, Moscow (1971).
10. Shorin, V.P., Ustranenie kolebanij v aviacionnyh truboprovodah, Mashinostroenie, Moscow (1980).
12 Vladislavlev, A.P., Messerman A.S. Elektricheskoj modelirovanie dinamiki sistem s raspredelennymi parametrami, Energiya, Moscow (1978).
24 Rzhevkin, S.N., Kurs lekcij po teorii zvuka, MGU, Moscow (1960).