Modelling of piezoelectric energy harvesting systems from vibration point of view is of interest of many researchers and plays an important role in the estimation of output energy for these systems. There are a vast number of studies focused on vibrational modelling of piezoelectric bimorph (or unimorph) harvesters. All of these studies mentioned the effect of damping on the power output of the harvester. The less mechanical damping leads to the higher power output [1]. In the spite of great impact of damping on power output, as far as the authors are aware of, there is no deep study about different damping mechanisms in piezoelectric energy harvesters.

Keywords: Damping Mechanisms, Piezoelectric, Energy Harvesting, Structural Damping, Viscous Damping.

1. Introduction

Modelling of piezoelectric energy harvesting systems from vibration point of view is of interest of many researchers and plays an important role in the estimation of output energy for these systems. There are a vast number of studies focused on vibrational modelling of piezoelectric bimorph (or unimorph) harvesters. All of these studies mentioned the effect of damping on the power output of the harvester. The less mechanical damping leads to the higher power output [1]. In the spite of great impact of damping on power output, as far as the authors are aware of, there is no deep study about different damping mechanisms in piezoelectric energy harvesters.

For energy harvesting applications, mostly researchers used viscous damping model in their approach for single-degree-of-freedom models, and for multi-degree-of-freedom modelling, they extended the concept of viscous damping and used Rayleigh proportional damping. Based on the literature [1]–[3], damping modelling techniques used for energy harvesting applications are viscous damping, which is proportional to the velocity. DuToit et al. [3] used a formulation for damping ratios from a previous study by Hosaka et al. [4], which internal friction, air-damping, squeeze force, and support loss had been reported as energy dissipation mechanisms of a cantilever beam. In DuToit formulation just the viscous damping is modelled, which is proportional to the velocity of vibrating mass. In the Finite Element formulation presented by Junior et al. [2], proportional Rayleigh damping matrix was considered.
Except one study that recently compared the structural and viscous damping [5], there is no research that taken the structural damping into account as a separate term in the vibrational equations of motion for the harvester. Just in the recent short paper by Cooley et al. [5], the role of structural damping in the modelling of piezoelectric energy harvesting has been mentioned. In the experimental work by Sodano et al. [6], they investigated the power output from PZT and MFC materials bonded to an aluminium shim with double sided tape. They mentioned the role of tape damping into decreasing power output, but no correction factor or modelling has been presented to consider damping. Crandall [7] mentioned the acoustic radiation and internal damping mechanisms as for damping in vibration of a beam. Crandall [7] stated that internal damping is a frequency-dependent factor and is a function of the material, so it is not correct to consider a general constant value for friction damping, as considered by Hosaka model [4]. Also, Crandall [7] mentioned the dependency of air damping to frequency.

In this research, the role of damping mechanisms in output energy from piezoelectric energy harvesting are studied as not previous studies addressed this issue. Afterwards, a model for damping in piezoelectric harvesters will be presented that considers viscous and structural damping mechanisms. Moreover, a more comprehensive discussion about different mechanisms in structural damping is presented as in the previous studies this has not been studied. Section 2 gives a current 1-D modelling approach for piezoelectric harvesters. In section 3, the drawbacks of this methods are reviewed and a new model considering different damping mechanisms are presented. Section 4 presents a numerical error to emphasis on the effect of damping mechanisms for output power from the harvester.

2. Current 1-D Modelling

Error! Reference source not found. presents the schematic of 1-D model [3], which comprises of a piezoelectric mass with internal resistance $R_p$ with a proof mass simply connected to a load resistor $R_l$. In this model, the entire structure is electromechanically coupled, unlike unimorph beams in which a part of the harvester is metal substrate.

![1-D electromechanical model](image)

The coupled equations for the system shown in Fig. 1 can be derived from equations in [3] as

$$\ddot{x}(t) + 2\xi_n\omega_n\dot{x}(t) + \omega_n^2x(t) - d_{31}V_p(t) = -\ddot{x}_b(t),$$  \hspace{1cm} (1)

$$R_{eq}\ddot{V}_p(t) + V_p(t) + m_{eff}R_{eq}d_{31}\dot{x}(t) = 0,$$  \hspace{1cm} (2)

where $\ddot{x}_b$ is the base excitation acceleration, $x$ is the relative displacement of harvester tip respect to the base, $V_p$ is the output voltage, $\omega_n$ is the undamped natural frequency of harvester, $\xi_n$ is the mechanical damping ratio, $d_{31}$ is the piezoelectric coupling coefficient in 3-1 mode, $R_{eq}$ is the equivalent electric resistance, $C_p$ is the capacitance of the piezoelectric, and $m_{eff}$ is the effective mass of piezoelectric layer. The overhead dot indicates the time derivative. The capacitance is defined in terms of dielectric constant $K$, the permittivity of free space ($\varepsilon_0 = 8.85 \text{nF/m}$), piezoelectric area $A_p$ and thickness $t_p$ with $C_p = K \varepsilon_0 A_p / t_p$.

In general, if only viscous damping is considered for energy dissipation from the system, energy dissipation can be modelled as
where $\{\dot{x}\}$ is the velocity vector.

In the concept of finite element, proportional damping matrix is a linear combination of mass and stiffness matrices with constant coefficients. If $[M]$ and $[K]$ are global mass and stiffness matrices of a multi-degree-of-freedom vibrating system, then proportional damping matrix considered for modelling in cantilever piezoelectric energy harvesting system will be presented as

$$[C] = \alpha [M] + \beta [K],$$

where $\alpha$ and $\beta$ are Rayleigh damping coefficients [2]. This damping matrix will act as a viscous damper as it is proportional to velocity. Rayleigh damping coefficients should be determined with experimental vibration tests. In the study by De Marqui Junior et al. [2], no discussion about these coefficients has been presented, just these figures presented as the known variables.

### 3. Proposed model

In the study by DuToit et al. [3], the piezoelectric layer has just been considered, without any substrate layer or contact layer, so the Hosaka damping model may be applicable for their case study. However, as in most piezoelectric energy harvesting, a piezoelectric layer is attached at a substrate surface to form a unimorph or similarly bimorph. In these configurations, damping model presented in Eq. (3) will not be useful anymore, as the viscoelastic damping from the contact layer should be considered. In addition, in the Eq. (1), the vibrating system is only the piezoelectric material. However, in most piezoelectric harvesters, the piezoelectric layer is attached onto a substrate surface. Thus, the model by Eq. (1) and Eq. (2) cannot be used for these cases as there is a proportion of the device which does not contribute to the power generation. So, a model is needed to distinguish the portion of mass, which contributes to vibrating motion, and the portion for power generation.

By adding the effect of substrate using a mass coefficient and a general form of damping, the equations of motion for a piezoelectric harvester with substrate and piezoelectric layer can be shown as

$$m_p\ddot{x}(t) + f_d + kx - \alpha_{mass}kd_3V_p(t) = -f_b(t)$$

$$R_{eq}C_p\dot{V}_p(t) + V_p(t) + m_{eff}R_{eq}d_{31}\omega_0\ddot{x}(t) = 0$$

where $\alpha_{mass} = m_p/mb$ is the compensation factor, $m_b$ is the device mass, $m_p$ is the piezoelectric element mass, $f_d$ is the energy dissipation, and $f_b$ is the base excitation force. The energy dissipation can be related to air resistance against beam vibration, $f_{d-air}$, and internal structural damping, $f_{d-str}$. Air-damping force can be estimated based on viscous damping model, as it has been done by [7] and [4]. However, the internal damping is much more complex for piezoelectric harvesters, as the beam is a composite beam with materials exhibiting elastic and viscoelastic behaviours at the same time. In the following, the tools for appropriate energy dissipation for a piezoelectric harvester will be presented.

When a cantilever oscillator vibrates in the air, there is an air-resistance force, which cause to dissipate energy from the dynamic system. This energy dissipation is proportional to velocity and hence, as it can be modelled as a dashpot, it is emerged as a viscous damping term. Hosaka et al. [4] investigated the energy dissipation of a macro oscillator in the air with the assumption that beam length is much larger than the other dimensions and also width is much larger than thickness. By these assumptions, $f_{d-air}$ can be expressed as

$$f_{d-air} = \frac{3\pi\rho_0 w^2 + \frac{3}{4}\pi w^2 (2\rho_0 \mu\omega)^{1/2}}{\rho_0 w^2 h}m_b \xi = 2\zeta_\omega \omega m_{s} \dot{x},$$
where $\mu$ is the air dynamic viscosity, $w$ is the beam width, $h$ is the overall beam thickness, $\zeta_m$ is the viscous damping coefficient, $\rho_b$ and $\rho_a$ are the density for beam and air, respectively.

For internal energy dissipation, the energy dissipation is often introduced as the complex term of stiffness and is defined as [8]

$$f_{d \rightarrow ir} = jk \eta_m s x,$$

where $\eta$ is the structural damping coefficient. Here, the aim is to extract an expression for $\eta$.

Overall composite materials represent a higher energy dissipation due to the viscoelasticity of the polymeric matrix [9]. Chandra et al. [10] mentioned four main factors for energy dissipation in composites, which are viscoelasticity, interphase, viscoplastic, and thermoelastic. Viscoelastic nature of matrix or fiber creates viscoelastic damping. The region adjacent to the fibres along the length will create interphase damping. In the case of applying high vibration or stress a degree of non-linear damping due to the present of high stress is evident, which is called viscoplastic damping. Thermoelastic damping is due to the heat flow from compression stress zone to the tensile stress zone. Thermoelastic damping depends to the amplitude and frequency of applied load, sample thickness and number of cycles and is more important for metal composites [10]. A piezoelectric energy harvester consisted of one or two orthotropic piezoelectric layers, an isotropic substrate layer, and an adhesive viscoelastic layer. Due to the nature of the harvester, viscoelastic and interphase damping mechanisms are evident in the model. In addition, due to cyclic loading and the presence of metal in the harvester, thermoelastic damping should be taken into the account. So, the internal structural damping coefficient can be shown as

$$\eta = \eta_{\text{visc}} + \eta_{\text{phas}} + \eta_{\text{thermo}},$$

where $\eta_{\text{visc}}$, $\eta_{\text{phas}}$ and $\eta_{\text{thermo}}$ are difficult coefficients, which it is not easy to evaluate an analytical expression for them. So, it is recommended to simulate $\eta$ as

$$\eta = \bar{\eta} + \hat{\eta},$$

where $\bar{\eta}$ is the structural coefficient count as energy dissipation for the material and $\hat{\eta}$ is the damping due to the interphase and thermoeelastic mechanisms. Here, in this modelling method, $\bar{\eta}$ is estimated for the materials and then $\hat{\eta}$ is tuned in such a way that the experimental data match the output from the analytical method. It is worth mentioning that tuning $\hat{\eta}$ should be regarded as an iteration process.

Internal energy dissipation for homogenous metal materials was proved that does not depend to the stress level but to the frequency, although some small dependency observed for glass/epoxy composites [11]. If the stress-dependency of structural damping is ignored, it can be shown that internal energy dissipation is identical to the theoretical loss factor due to transverse heat flow [7]. Hence, the internal structural coefficient can be estimated as [7]

$$\bar{\eta} = \frac{\alpha^2 ET}{c_v} \frac{\omega / \omega_b}{1 + (\omega / \omega_b)^2},$$

where $\alpha$ is thermal expansion coefficient, $T$ is temperature, $E$ is modulus of material, $c_v$ is specific heat, and $\omega$ is vibrating frequency, and $\omega_b$ is the material relaxation frequency, which is given as

$$\omega_b = \frac{\pi}{2} \frac{k}{c_v h^2},$$

where $k$ is material conductivity.

To sum up, the equations of motion shown by equations (5) and (6) can now be given by

$$\ddot{x}(t) + 2\zeta_m \omega_n \dot{x}(t) + \omega_n^2 (1 + j \eta) x - \alpha_{\text{max}} \omega_n^2 d x \dot{V}_p(t) = -\ddot{x}_a(t),$$

where $\zeta_m$ is the viscous damping coefficient.
where \( \omega_h \) is defined as \( \sqrt{k/m_p} \).

If a harmonic excitation is assumed, \( \ddot{x}_p(t) = \ddot{X}_p e^{j\omega t} \), then the displacement and voltage will be a harmonic function with the same frequency but with complex magnitude, e.g. \( V_p(t) = V_p e^{j\omega t} \) and \( x(t) = X e^{j\omega t} \). By substituting these expression into equations (13) and (14),

\[
\left[ \left( \frac{\omega_n^2 - \omega^2}{\omega_n^2} \right) + j \left( 2\xi_n \omega_n \omega + \eta \omega_n^2 \right) \right] \ddot{X} - \alpha_{m,a} \omega_n \ddot{x}_d Y_p e^{j\omega t} = -X e^{j\omega t} \\
\left[ (1 + jR_{eq} C_p \omega) \ddot{V}_p + jm_{eq} \alpha_{m,a} \omega_n \omega \ddot{X} \right] e^{j\omega t} = 0
\]

(15)

Then, by defining the dimensionless frequency and load with \( \Omega = \omega / \omega_n \) and \( r = R_{eq} C_p \omega_n \), the steady state solutions for output voltage as a function of input frequency can expressed as:

\[
\left| \frac{V_p}{X} \right|^2 = \frac{m_{eq} \alpha_{m,a} \omega_n \Omega}{\sqrt{\left[ \left( \frac{1 - \Omega^2}{\Omega^2} \right) - r \Omega (2\xi_n \Omega + \eta) \right]^2 + \left( (2\xi_n \Omega + \eta) + r \Omega (1 - \Omega^2) \right)^2}}
\]

(17)

Using the calculated voltage from Eq.(17), the magnitude of output power from piezoelectric harvester can be calculated with \( P_p = \left| \frac{V_p}{X} \right|^2 / R_i \). By defining a new electromechanical coupling coefficient \( k_r^2 \) [3], power can be estimated from Eq. (18).

\[
\left| \frac{P_p}{\left( \frac{X}{\alpha_n} \right)} \right|^2 = \frac{m_{eq} r k_r^2 \Omega^2}{\omega_n \left[ \left( \frac{1 - \Omega^2}{\Omega^2} \right) - r \Omega (2\xi_n \Omega + \eta) \right]^2 + \left( (2\xi_n \Omega + \eta) + r \Omega (1 - \Omega^2) \right)^2}
\]

(18)

As it can be seen from Eq.(18), the energy dissipation term is \( 2\xi_m \Omega + \eta \), which has two parts, one is constant at all frequencies and the other changes with \( \Omega \).

### 4. Numerical example

Now, to clear the role of damping mechanisms, a numerical example is presented. This numerical example is the analysis of a bimorph piezoelectric energy harvester with dimension 120×30 mm. Piezoelectric layer is a PZT-5A Piezoceramic with elastic modulus of 66.0 GPa, Poisson’s ratio of 0.33 and density of 7.75 g/cm³ with the thickness of 0.27 mm. The electromechanical coupling properties of the piezoelectric layer are as follows: \( d_{33} = 374 \) pC/N, \( d_{31} = -171 \) pC/N, and \( C_{p} = 177.07 \) nF/m. Centre shim is made of brass with thickness of 0.14 mm, elastic modulus of 105 GPa, Poisson’s ratio 0.3 and density of 9.0 g/cm³. In order to estimate the natural frequency for this harvester, COMSOL Multiphysics software under license number 12073023 was used. As it was shown in Fig. 2, the piezoelectric beam is meshed with Tetrahedral elements and the natural frequency of the harvester was obtained \( \omega_h = 23.1 \) Hz.

---

**Figure 2: Finite Element analysis of bimorph for natural frequency**
Now based on the presented method in Eq. (18), output power from piezoelectric harvester is compared for different cases. Three different systems in terms of energy dissipation were considered which are denoted with lightly, medium and highly damped system according to their damping coefficient. In lightly, medium and highly damped system $\zeta_m + \eta$ are 0.02, 0.04 and 0.1, respectively. Moreover, load resistances were considered to be $1e3$ or $1e6 \, \Omega$ for all these three cases. At each case, three combination of damping mechanisms were considered, namely structural damping only, viscous damping only or half combination of both damping mechanisms. Power ratio for lightly, medium and highly damped systems are shown in Fig. 3, Fig. 4 and Fig. 5, respectively. It is worth mentioning that power ratio is the power normalized with the case of viscous damping only. As it can be seen from Fig. 3, the output power in the case of structural damping only is 4 times and in the case of combined damping mechanisms are 1.8 times higher than the viscous only case, independent of the load resistance. These figures are approximately the same for medium and highly damped systems in Fig. 4 and Fig. 5. There is only a slight shift in the resonant frequency due to increasing damping, as can be seen by comparing Fig. 3 and Fig. 5.

Figure 3: Effect of different damping mechanisms on power for lightly damped system

Figure 4: Effect of different damping mechanisms on power for medium damped system
The variation of power respect to damping mechanisms emphasis that it is important to select the correct form of damping mechanisms unless the viscous only damping model will underestimate output energy by piezoelectric energy harvester. DoToit et al. [12], presented a model for predicting output power from piezoelectric only by considering viscous damping and their experimental verification showed that output power in the resonant was underestimated with this model. Fig. 6 shows the output power from their piezoelectric harvester using analytical model and experimental data. It is obvious that their model could not predict output power close to resonance. This can be due to inappropriate consideration of only viscous damping mechanisms.

5. Conclusion

This paper dealt with the problem of proper damping modelling in piezoelectric energy harvesters. After reviewing current popular damping modelling approach, viscous only damping mechanism, a damping model was presented that considers both viscous and structural damping mechanisms. Viscous damping was related to the energy dissipation from the system by air resistance while structural damping comprises of energy dissipation through elastic waves inside material, interphase and thermoplastic mechanisms. An analytical estimation for structural damping also was presented. A numerical example was presented that showed the role of different damping mechanisms in output power from piezoelectric harvester. It was shown that if only viscous damping is taken into the account, then output power at resonance will be underestimated considerably.
REFERENCES


