Vehicle pass-by-noise (PBN) testing has become of even greater importance due to stringent regulations in Europe where a reduction from 74 dB(A) to 68 dB(A) in noise levels is required by 2024. Manufacturers are in search of new techniques to predict PBN noise emission in early stages of vehicle design. Among the relevant phenomena involved in a PBN test, the ground reflection accounts for a large portion of the overall noise level. Hence, there is a need for including accurate modeling of the ground reflection in a PBN scenario. This paper proposes an analytical description of an arbitrarily moving source considering frequency dependent ground reflection. The model relies on the transient time delayed Green’s function formulation which extends the result of classical Doppler Weyl-Van der Pol equation for arbitrary source path and velocity. The image source method model the wave reflected by the ground by means of a Doppler shifting and frequency-dependent ground impedance model. The final goal is to offer an early-stage design assessment of an exterior PBN test by means of overall sound pressure levels as well as binaural sound synthesis (auralization) in view to run sound perception tests.

Keywords: sound synthesis of moving source, ground impedance

1. Introduction

Auralization combined with psychoacoustics metrics can help vehicle manufacturers and urban planners understand and quantify road noise, independently of its maximum sound pressure level [1]. In general, a single noise level metric does not provide the full picture of the annoyance and harshness of the sounds emitted by vehicles. Recent alternatives to on-track or in-room recordings rely on model-based wave propagation, which can be versatile approaches to synthesize different driving scenarios in early design stages.

Despite its simplistic definition, a monopole source is an acceptable way of representing a source in the far-field. The study of moving acoustic monopole sources has been introduced in the 1970’s [2], when special relativity was used to reduce this problem to a simpler one with non-moving source case. This was possible due the fact that the Doppler phenomenon does not occur in the Lorentz space. Such approach, however, does not easily allow an extension to arbitrary motion because of complexities in the transformation of an accelerated frame.

Developments have also been made to include the effect of time-domain locally-reacting impedance into the pressure field solution, either considering frequency-independent [3] or frequency-dependent
surface impedance \[4\]. In both cases, acoustic radiation of constant subsonic speed was considered. In parallel, developments of non-uniform paths for a monopole moving source have been made, either with reflection \[5\] or without reflection considerations \[6\].

This paper aims to extend the moving source problem by retrieving the radiated wavefield from a moving source with arbitrary velocity, while considering the frequency-dependent spherical reflection on a locally reacting ground. The exact solution of the pressure field is computationally intense to solve since the propagation time delay must be computed at every instant of the simulation. In the proposed solution, an approximated propagation time delay using a Taylor expansion is employed. The pressure field solution at the receiver can then be synthesized. In order to demonstrate its application, an example case of a pass-by-noise (PBN) scenario is considered. Current approaches for PBN synthesis involve computing the multiple source-receiver transfer functions either numerically \[7\] or by measuring it \[8\] with a microphone array in the far field.

This paper is organized in two sections. First, we begin by introducing the mathematical description of the problem. In the second section, we perform three simulation analyses, we compare the obtained iterative expression with the analytical solution for a constant velocity case. Then, we compare the proposed approach with a simple semi-static approach. Finally, examples for a PBN test are shown, one at constant speed and one accelerating.

2. Acoustic pressure solution for an arbitrary moving source

The diagram shown in Fig. 1 describes the simulation methodology using the propagation model. The source emits a time signal denoted by \( s(t) \) in stationary conditions or in the reference frame of the source. The time-domain propagation model has three main inputs: the geometry of the problem, the cinematic conditions and the road model. The moving source position is denoted \( \mathbf{r}_s(t) = (x_s(t), y_s(t), z_s(t)) \in \mathbb{R}^3 \) with \( t \in \mathbb{R} \). The evolution of the source position in time depends on the cinematic condition given by the source velocity \( \mathbf{v}(t) \) and acceleration \( \mathbf{a}(t) \). The stationary receiver position is specified by the coordinates \( \mathbf{r} = (x, y, z) \in \mathbb{R}^3 \). The ground surface is characterized by its frequency-dependent specific impedance \( Z(\omega) \). In turn, the reflected field on the ground is attenuated by a ground reflection coefficient \( Q(\omega, \theta) \), which is a function of the incidence angle \( \theta \).

![Figure 1: Schematic of the model simulating the time signal at the receiver location from a moving source.](image)

The propagation model for a moving source is obtained by the transient Green’s function which is the causal solution of the differential equation \[9\]

\[
\left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) g(\mathbf{r} | \mathbf{r}_s(t), t) = -\delta(\mathbf{r} - \mathbf{r}_s(t), t),
\]
where \( \delta(r - r_s(t), t) \) is the 4D Dirac distribution. The units for \( \delta \) and \( g \) are \( m^{-3} \cdot s^{-1} \) and \( (m \cdot s)^{-1} \) respectively. The transient Green’s function is given by

\[
g(r|s(t), t) = \frac{\delta \left( t - \frac{||r - r_s(t)||}{c} \right)}{4\pi ||r - r_s(t)||}.
\]  

(2)

The velocity potential is defined as

\[
\varphi(r, t) = \int_{\tau=0}^{\infty} g(r|s(t-\tau), \tau) s(t-\tau) d\tau,
\]

where \( s(t-\tau) \) is the source strength (with units \( m^3 \cdot s^{-1} \)) at an instant delayed by \( \tau \) seconds given by

\[
\tau(r|s(t), t) = \frac{||r - r_s(t-\tau)||}{c}.
\]

(3)

From the Dirac properties,

\[
\varphi(r, t) = \frac{s(t-\tau)}{4\pi ||r - r_s(t-\tau)||} \left[ \frac{1}{c} \mathbf{v}(t-\tau) \cdot (r - r_s(t-\tau)) \right].
\]

(4)

The pressure field derives from \( p = -\rho \partial \varphi / \partial t \) yielding

\[
p(r, t) = \frac{\rho}{4\pi} \left\{ \frac{\dot{s}(t-\tau) R(t)}{\left| R(t) - \frac{1}{c} \mathbf{v}(t-\tau) \cdot \mathbf{R}(t) \right|^2} \right.
\]

\[
+ \frac{s(t-\tau) R(t)}{\left[ R(t) - \frac{1}{c} \mathbf{v}(t-\tau) \cdot \mathbf{R}(t) \right]^3} \left( \frac{\mathbf{v}(t-\tau) \cdot \mathbf{R}(t)}{R(t)} + \frac{\mathbf{a}(t-\tau) \cdot \mathbf{R}(t)}{c} + \left( \frac{\mathbf{v}(t-\tau)^2}{c^2} \right) \right) \right\},
\]

where the shorthand notation \( R(t) = ||\mathbf{R}(t)|| \) with \( \mathbf{R}(t) = r - r_s(t-\tau) \) and the notation \( \{ \cdot \} \) for time derivative are employed. The reader is referred to Refs. [6] and [9] for the details of the derivation.

For a constant velocity, assuming the source is moving along the \( x \)-axis, the source position is given by \( r_s(t) = (x_0 + v_xt, 0, 0) \), with initial position \( x_0 \) and constant source velocity \( v_x \). Then Eq. (6) reduces to the classical Doppler Weyl-Van der Pol equation [10]

\[
p(r, t) = \frac{\rho}{4\pi} \left\{ \frac{\dot{s}(t-\tau) R(t)}{\left[ R(t)(1 - M_x \cos \vartheta) \right]^2} + \frac{s(t-\tau) R(t) v(\cos \vartheta + M_x)}{\left[ R(t)(1 - M_x \cos \vartheta) \right]^3} \right\},
\]

(7)

where \( \vartheta \) is the angle between the velocity vector \( \mathbf{v} \) and the source-receiver distance vector \( \mathbf{R} \).

For the constant velocity case, an analytical expression for the time propagation delay can be obtained

\[
\tau = \frac{M_x (x - v_xt) + \sqrt{(x - v_xt)^2 + (y^2 + z^2)(1 - M_x^2)}}{c(1 - M_x^2)}.
\]

(8)
2.1 Approximation of propagation time delays

Solving the nonlinear Eq. (4) for every time instant is impractical for real-time applications. An alternative is to find an approximate solution using an iterative scheme [11].

By first simplifying the notation $\tau (r|\mathbf{r}_s(t), t) \equiv \tau (t)$ without loss of generality, we consider the first order Taylor expansion of the propagation time delay

$$\tau (t + dt) \approx \tau (t) + dt \frac{d\tau (t)}{dt},$$

where its temporal derivative is given by

$$\dot{\tau} (t) = -\frac{\mathbf{v}(t - \tau) \cdot [\mathbf{r} - \mathbf{r}_s(t - \tau)]}{c||\mathbf{r} - \mathbf{r}_s(t - \tau)||}.$$ (10)

Simplifying once again the notation $\tau (t_i) \equiv \tau_i$, the iterative scheme becomes

$$\tau_i = \tau_{i-1} + \dot{\tau}_{i-1} dt,$$ (11)

where $dt$ is the sample time and the propagation time delay $\dot{\tau}_{i-1}$ from previous iteration is given by

$$\dot{\tau}_{i-1} = -\frac{\mathbf{v}(t_{i-1} - \tau_{i-1}) \cdot [\mathbf{r} - \mathbf{r}_s(t_{i-1} - \tau_{i-1})]}{c||\mathbf{r} - \mathbf{r}_s(t_{i-1} - \tau_{i-1})||}.$$ (12)

The initial propagation time delay $\tau_0$ is assumed to be the analytical expression obtained for constant velocity in Eq. (8).

2.2 Bounded domain solution

Equation (6) describes the pressure field of a moving source for an unbounded domain. To account for the reflection, the image source approach is applied. The total pressure field $p_{total}$ is the superposition of the directed ($p$) and the reflected ($\bar{p}$) pressure fields. The latter is attenuated by convolution with the spherical ground reflection operator $q$ so that the total pressure field becomes

$$p_{total}(\mathbf{r}, t) = p(\mathbf{r}, t) + q(t) * \bar{p}(\mathbf{r}, t),$$ (13)

where $q(t) = \mathcal{F}^{-1}\{Q(\tilde{\omega})\}$ is the inverse Fourier transform of the spherical wave reflection operator in frequency domain and $\bar{p}(\mathbf{r}, t) \equiv p([|\mathbf{r}|\mathbf{r}_s(t - \tilde{\tau})], t)$ derives from the virtual source $\mathbf{r}_s(t - \tilde{\tau})$ at each delayed instant time $\tilde{\tau}$. The image source’s propagation time delay is not the same as for the original source due the difference in distance travelled by the direct and reflected pressure wave.

The spherical wave reflection coefficient is given by [12]

$$Q(\tilde{\omega}) = R_p(\tilde{\omega}) + [1 - R_p(\tilde{\omega})] F(\bar{\varepsilon}),$$ (14)

in which $F(\bar{\varepsilon})$ is the boundary loss factor and $\bar{\varepsilon} = \varepsilon(\tilde{\omega})$ is the complex numerical distance

$$F(\varepsilon) = 1 + j \sqrt{\pi} \varepsilon \exp(-\varepsilon^2) \text{erfc}(-j\varepsilon), \quad \varepsilon^2(\tilde{\omega}) = 2 j k_0 \chi^2(\tilde{\omega}) \frac{||\mathbf{r} - \mathbf{r}_s(t - \tilde{\tau})||}{Z(\tilde{\omega})[1 - R_p(\tilde{\omega})]^2},$$ (15)

Furthermore, $\tilde{\omega}$ is the Doppler-corrected circular frequency

$$\tilde{\omega} = \omega \left[ R(t) \frac{R(t)}{|R(t) - \mathbf{M}(t - \tau) \cdot \mathbf{R}(t)|} \right]$$ (16)
and $R_p$ is the plane wave reflection coefficient [13]

$$
R_p(\tilde{\theta}; \tilde{\omega}) = \frac{Z(\tilde{\omega}) \sin \tilde{\theta} - \chi(\tilde{\omega})}{Z(\tilde{\omega}) \sin \tilde{\theta} + \chi(\tilde{\omega})}, \quad \chi(\tilde{\omega}) = \sqrt{1 - \left(\frac{k_0}{k}\right)^2 \cos^2 \tilde{\theta}}
$$

(17)

where $Z(\tilde{\omega}) = Z_{eq}/Z_0$ is the frequency-dependent impedance with $Z_{eq}$ the characteristic ground impedance and $Z_0 = \rho c$ is the air characteristic impedance derived from air density $\rho$ and speed of sound $c$ at ambient temperature.

The ground model chosen for this analysis is the Hamet model, which is a three-parameter phenomenological framework commonly used to characterize road pavements [14]. The ground surface material is represented in frequency domain by its equivalent density and bulk modulus as follows

$$
\tilde{\rho}_{eq}(\omega) = \frac{\rho K}{\phi} \left(1 - j\frac{\omega_\mu}{\omega}\right), \quad \tilde{K}_{eq}(\omega) = \frac{\rho c^2}{\phi} \frac{1}{1 - (1 - 1/\gamma)/(1 - j\omega_\theta/\omega)},
$$

(18)

with

$$
\omega_\theta = \frac{\sigma}{\rho Pr}, \quad \omega_\mu = \frac{\sigma \phi}{\rho K},
$$

(19)

where $Pr = 0.71$ is the Prandtl number for air at room temperature and $\gamma = 1.4$ is the air heat capacity ratio. The ground parameters are the porosity $\phi$, the shape factor (or structure factor) $K$ and the flow resistivity $\sigma$. One can obtain the characteristic ground impedance and ground wave number as follows

$$
Z_{eq} = \sqrt{\tilde{\rho}_{eq} \tilde{K}_{eq}}, \quad k = \omega \sqrt{\tilde{\rho}_{eq}/\tilde{K}_{eq}}.
$$

(20)

It is worth mention that the simple transformation of the reflection to time-domain is not trivial. The impedance model needs to attend three conditions to be physically admissible: it has to guarantee that for a real value excitation, the acoustic variables are real values, it has to ensure that the ground absorbs acoustic energy and it has to guarantee the causality condition. Dragna et.al [15] demonstrated the admissibility of surface impedance models of the same family as the Hamet model used in this paper.

### 3. Simulation results

In this section we simulate a point source moving according to a standard pass-by noise test [16]. The receiver is located at the fixed position $r = (x_{mid}, 7.5, 1.2)$ m, where the $x$-axis coordinate depends on the cinematic conditions of the problem. The source is moving in a straight path with constant or accelerating speed $r_s(t) = [v_xt + (1/2)a_xt^2, 0, 0, 0.5]$ m, where $a_x$ is the acceleration in the $x$-axis. The source emits a signal with harmonic frequency 200 Hz. The sample rate and sample time are 8192 Hz and $3$ s respectively.

To determine the acoustic pressure fluctuation in time we utilize the sound pressure level defined as

$$
SPL(t) = 10 \log_{10} \frac{1}{\Delta t} \int_{\Delta t} \frac{p^2(t)}{p_0^2} dt
$$

(21)

Figure[2] shows the comparison of sound pressure level between the analytical expression and the transient time delayed approach for constant velocity case. In this scenario, simulation is done without ground reflection. For low speeds, the two results are identical. As the speed increases the two solutions differs slightly, this can be due the fact the analytical solution is only valid for low speeds in order to ensure the linearity of the governing equation.
Figure 2: SPL of a constant velocity cinematic condition utilizing both the analytical and iterative solutions with x-axis constant velocity at (a) \( v_x = 14 \text{ m} \cdot \text{s}^{-1} \) and (b) \( v_x = 45 \text{ m} \cdot \text{s}^{-1} \).

Figure 3 shows the calculated pressure time signal at receiver location and its spectrogram. This figure compares a semi-stationary approach with the transient time delay proposed in this paper. In the semi-stationary case, a simple piece-wise approach was utilized. The free field Green’s function for a 3D monopole source is calculated for a number of fixed points. Then, the source is convolved in frequency domain with each point, and the resulting pressure signal in time domain is combined using an overlap-add technique. The number of fixed source points used is 55 and the parameters for the Hamet ground model were chosen arbitrarily: \( \phi = 0.15, K = 6.47, \sigma = 19407172 \text{ kPa} \cdot \text{s} \cdot \text{m}^{-2} \). On one hand, a simple overlap-add is not enough to avoid destructive interference between successive frames, as it can be observed in Fig. (3a). If more source points are used, the artifacts (noticeable in the spectrogram) can be reduced but they remain audible. Additionally, an increase in the number of points also increases the computational cost of the synthesis. On the other hand, Fig. (3b) shows a smoother time signal without noticeable interference in the time signal and artifacts in the spectrogram.

Figure 4 shows a spectrogram comparison of different cinematic scenarios, stationary source, moving source with constant velocity and accelerating. The Doppler shift can be noticed in both cases of moving source, whereas there is no shift in the motionless case. In the accelerating case, the shift is more pronounced because at the instant the source crosses the receiver, its velocity is higher than the constant velocity case. Particularly, a short increase of frequency can be noticed, which is a small Doppler shift upwards before the larger Doppler shift downwards due to the passing of the receiver. Nevertheless, the shift is audible in both cases.
4. Conclusion

The solution of the sound propagation for an arbitrary moving source considering frequency-dependent spherical reflection on locally reacting ground is presented in this paper. For the simulation results, a simple monopole source emitting a monochromatic signal was used, which could be extended to more realistic sound sources. We compared the proposed iterative expression with the analytical solution for a simpler constant velocity case, and it was shown that they differ slightly as the source speed increases. The proposed transient time delay technique was compared with a simple semi-stationary approach, and the former displays no artifacts and interference problems, which makes it suitable for auralization purposes.

Once the receiver pressure is computed, further auralization can be done such as binaural synthesis as a first step into auralizing. Additionally, an extension of the proposed technique involves representing each vehicle noise component by a number of acoustic sources by applying airborne source quantification techniques. A full vehicle represented as multiple monopole sources could be synthesized in the desired scenario with both auralization and sound pressure level as possible outputs. However, there are still challenges to be addressed such as the physical validity of the model against an experiment and the lack of other important propagation of sound mechanism such as tire-road interaction.

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References


