COMPONENTS OF VARIANCE IN REVERBERATION TIME MEASUREMENT – PART 1: FIELD TESTING ROOMS OF LIGHTWEIGHT CONSTRUCTION

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This paper uses analysis of variance (ANOVA) and a specific design of experiment (DOE) and construction choice to isolate the component of variance associated with the part (or room) being measured along with individual contributions of measurement uncertainty from the measurement system and the person making the measurement. It demonstrates how the gauge repeatability and reproducibility (GRR) technique can be used to identify the variability in reverberation time (RT) measurement over the frequency range 100-3150Hz as an individual part of the process associated with field sound insulation testing in the UK. The experiment uses the measurement of reverberation time associated with the ‘interrupted source method’ from EN ISO 354: 2003

Keywords: Reverberation Time, ANOVA, Repeatability, Reproducibility

1. Introduction

The design of any scientific experiment must not only document and include details of the design of the experiment and the measurement procedure but must also attempt to attach a measurement error to the empirical results. Indeed some emphasise that an experiment is not complete until an analysis of the final result has been conducted [1]. This is good practice as it allows the informed reader to understand, at a basic level, the likely variability in the measurement process and appreciate the precision which can be attached to the experimental procedure.

This paper looks at the uncertainty associated with the field measurement of reverberation time and focuses on this as a component of the pre-completion sound insulation testing requirements of the Building Regulations for England and Wales. Field tests are the ubiquitous method of demonstrating compliance with the sound insulation performance standards and the definitive method of demonstrating conformity with the minimum sound insulation values should compliance be contested. Annex B of Approved Document E [2] Part of the field test procedure requires the measurement of the reverberation time in the room in order to correct for the rooms’ effect on the sound insulation performance of the surface being measured. The sound insulation value calculated in each 1/3rd octave band is then ‘standardized’ \( (D_{nT}) \) to a 0.5 second reverberation time using the methods as detailed in the international standards [3, 4] for determining the airborne and impact performance of the surface.

This correction for reverberation time effect is detailed in equation (1):

\[
D_{nT} = D + 10 \log \frac{T}{T_0} dB_i
\]  

Where:
D is the level difference,
T is the reverberation time measured in the receiving room;
T₀ is the reference reverberation time; for dwellings, T₀ = 0.5 s.

In this paper we are interested in the measurement uncertainty in measuring ‘T’.
It is worthwhile noting the following from the standards: ‘NOTE 1 The standardizing of the level
difference to a reverberation time of 0.5 s takes into account that in dwellings with furniture the reverberation
time has been found to be reasonably independent of the volume and of frequency and to be
approximately equal to 0.5 s. With this standardizing, DnT is dependent on the direction of the sound
transmission if the two rooms have different volumes.’. See Burgess & Utley et al and Diaz et al [5, 6].

This paper analyses the data from a special design of experiment (DoE) called a Gauge Repeatability
and Reproducibility (GRR) experiment that was carried out on a residential block of flats of light-
weight timber construction where measurement of reverberation times were made in 6 unfurnished
bedrooms. In this GRR the room is the part being measured and its size and shape was ‘blocked’ in this
instance by choosing rooms of identical dimensions to determine the rooms’ effect on reverberation
time from apparently identical parts.

The measurement uncertainty is split into components of variance to identify the uncertainty contribution
of the instrumentation, test engineer (operator) and the part (or in this case receiver room) being
measured using advanced analysis of variance (ANOVA) which is described in more detail below.

## 2. Site survey – Room and construction details.

The airborne sound insulation tests were carried out on a residential scheme which was nearing
completion. The floor elements were between matched room pairs and five individual operators were
used, each with their own test systems. An aim of this approach was to minimise systematic deviations
in the measurement and thus the variability in measurement test data. Each “test system” was employed
to test six notionally identical room pairs, or ‘parts’, to determine the performance of the separating
floor. In all, the six room pairs were tested by each tester (reproducibility) on three separate occasions
(repeatability) over the space of three days. This resulted in eighteen test results for each system.

The measurement procedure was designed to represent the noise.co.uk Ltd UKAS Accredited meth-
ods statement using a static sampling approach during a normal operational day. The test method for
measurement of reverberation time used the ‘interrupted source’ method from the standard [7]. Each
test operator complied with the following procedure: position the Loudspeaker in the receive room tak-
ing care to ensure that the speaker is more than 0.5m from any of the room boundaries. Ensure that the
microphone positions used to measure the reverberation time are set up to be greater than 2m from the
source, 1.5m from each other and 1m from any room boundary where practicable. Take a Reverbera-
tion Time (RT) measurement. Repeat the RT measurement step for a further minimum 5 runs, making
6 readings in total. Measurements should be taken using at least one speaker position, with two read-
ings at three selected microphone positions.

The tests were carried out in a small bedroom that measured 5.035m long x 2.975m wide x 2.4m
high with an approximate room volume of 27m³. Six identical room pairs were selected across the de-
development. See Fig 1 below.

The rooms were of bare wood floor, plasterboard wall and ceilings and were unfurnished. The floor
element being measured on the test was a timber construction which from the top down consisted of:
18mm flooring boards; plasterboard layer; resilient batten min depth 70mm; 25 mm mineral wool be-
tween battens; 18mm wood based board; 253mm metal web joists; 100mm mineral wool between
joists; 2 x 10mm plasterboard, coupled with 25mm metal resilient bars; see Fig 2. The internal and
separating walls were of lightweight plasterboard construction.
Figure 1: Matched room pairs, room shape & volume and location on site plan.

Figure 2 Light weight floor section whose sound insulation was being measured during the test.

Drawing on earlier research on identifying the components of variance in the field measurement of sound insulation by Whitfield and Gibbs [8-10] the experimental approach uses (ANOVA) and a (GRR) test method. The usefulness of these methods is mentioned by Mandel [11] and Tsai [12] and the previous use of ANOVA in acoustic research is not without precedent, see Taibo and Glasserman de Dayan [13] and it has also been used for measurement of reverberation time, all be it in the context of measuring the absorption of an element in a laboratory environment see Davern and Dubout P [14, 15].

The main advantages of ANOVA are listed by Deldossi and Zappa [16] and include the ability to determine the contribution of the operator and part and operator by part interaction. A key contribution to the development of GRR was written by Montgomery and Runger [17, 18] and culminated in a monograph on the subject, including its special applications by Burdick et al [19]. in which the ANOVA design, for the purpose of this research, is described as a Balanced Two Factor Crossed random model with interaction. It informs this research on achieving an accurate and reliable estimate of the variability in the measurement process due to the part, the operator and the instrument. It is this model and additional information provided by Montgomery [17, 18, 20] and Burdick et al [19] which forms the analytical framework, to separate out and quantify the components of variance in reverberation time measurement.

3. GRR

The GRR has a particular DoE which relies on a number of gauge “operators” to measure a number of test specimens (parts) a repeated number of times. In this DoE due to the onerous test procedure required to capture one result (test) 5 UKAS accredited sound insulation test operators were used, each
with their own test kit and tasked at measuring 6 floor specimens (parts) 3 times each. The model is
detailed in equation (2):

\[ Y_{ijk} = \mu + O_i + P_j + (OP)_{ij} + E_{k(ij)} \]  \hspace{1cm} (2)

Where \( i = 1, 2, \ldots, p \) : \( j = 1, 2, \ldots, o \) : \( k = 1, 2, \ldots, r \) and;
- \( p \) = number of parts,
- \( o \) = number of operators and;
- \( r \) = number of replications and;
- \( O_i, P_j, [(OP)_{ij}], \) and \( R(k(ij)) \) are random variables representing the effects of the operator, parts,
operator by part interaction and the replications on the measurement and \( \mu \) is an overall mean. Clearly, in
the experiment described here \( p = 6, o = 5 \) and \( r = 3 \)

The definition of reproducibility in the GRR is covered in Burdick et al [19] and incorporates the inter-
action term and is shown in equation (3): The combined Gauge variance components are shown in
equation (3) and the total variance shown in equation (4) which describes the total measurement un-
certainty associated with the field testing of this particular part.

\[ \sigma^2_{\text{reproducibility}} = \sigma^2_{O} + \sigma^2_{PO} \]  \hspace{1cm} (3)
\[ \sigma^2_{\text{gauge}} = \sigma^2_{\text{repeatability}} + \sigma^2_{\text{reproducibility}} \]  \hspace{1cm} (4)
\[ \sigma^2_{\text{total}} = \sigma^2_{\text{gauge}} + \sigma^2_{\text{part}} \]  \hspace{1cm} (5)

It should be noted that in a GRR the reproducibility term does not contain the repeatability term by
definition. This is different to the method of assessment in BS5725 [21] where repeatability is embe-
ded in the reproducibility term resulting in reproducibility always being greater than repeatability. In
GRR. The reproducibility can be separated out into two components of variance, defined as the oper-
ator variance (\( \sigma^2_{O} \)) and the operator by part interaction (\( \sigma^2_{PO} \)). This is an important feature of ANOVA
because it detects any interaction the operator has with the part being measured. In some cases the in-
teraction term can be significant, and dominant as demonstrated by Whitfield and Gibbs [22] and it
would remain hidden if using the BS5725 method of calculating repeatability ‘r’ and reproducibility
‘R’. Typical RT results for a single operator Figure 3 and across operators Figure 4 are shown below:
It is possible to now use the repeated measured values with the reproduced values of other operators to reduce the total uncertainty of measurement into its component parts. The results of the ANOVA analysis are graphically illustrated below in Figure 5 for total variance and in Figure 6 for individual components.

**Figure 5:** Total variance in the measured results

**Figure 6:** Total GRR with (r) (instrument), (R) (Operator) and part to part (Room) variances.
In Figure 5 the total variance of the RT measurement process ranges between 0.012 and 0.036 seconds and shows no real trend across the frequency range. These variances are broken down in Figure 6 to show the components associated with the instrument (repeatability ‘r’) the operator (Reproducibility ‘R’) and part to part (Room). It is noted that the ANOVA of the data determined there was no significant indication of interaction between the operator and the part being measured so the variance associated by ‘reproducibility’ is equal to the variance of the operator alone with the interaction term reduced to zero. The ability to detect interaction is an important feature of ANOVA because it detects any interaction the operator has with the part being measured. In some cases the interaction term can be significant in a measurement routine, and at certain frequencies also be dominant as demonstrated by Whitfield and Gibbs [17] and it would remain hidden if using the BS5725 method of calculating repeatability ‘r’ and reproducibility ‘R’ or assuming an approach using GUM [23].

What’s notable from GRR data shown in Figure 7 is that the variance associated with the instrument (r) is generally higher at the lower frequencies falling after peaking at 0.024 seconds @ 200Hz to 0.006 seconds @ 2500 & 3150Hz. It is likely partly due to the limitations of the sound level meter and RT response at lower frequencies and the less diffuse sound field in the relatively small rooms.

This is in contrast to the variance attributable to the part being measured which shows the variance levels are lower than the variance associated with the instrument between 100-500Hz and gradually trend upwards with frequency after 200Hz (0.001 sec) to 0.012 sec @630Hz where it crosses and dominates over the variance associated with the instrument.

The variance associated with the operators (R) shown in Figure 6 fluctuates with frequency but is below 0.011 sec across the frequency range 100-3150Hz. The dominant contributor to the total uncertainty associated with the measurement of reverberation time in this experiment is therefore dependent on frequency and is due to either the instrument being used or the part being measured.

4. Conclusions

The calculation of measurement uncertainty in the measurement of reverberation time in the field testing of sound insulation can be carried out using a specialist DoE based on ANOVA. This allows the
components of variance to be broken down into the uncertainty associated with the instrument (r), that associated with the operator (R), and that associated by the part being measured.

In order to identify how the part, in this case the room in which the reverberation time is measured, contributes to the overall uncertainty of measurement this factor can be ‘blocked’ by choosing identical rooms of the same shape size and finish varying only by the workmanship used to create them. It is noted that the GRR type experiment uses the same experimental effort as that required by traditional means of identifying ‘r’ and ‘R’ e.g. BS5725, but allows the researcher to splice out a significantly more detail from the same data about the contributors to the overall uncertainty including any interaction between the operators making the measurement and the part being measured. The blocking of the room effect allows further information to be obtained from the GRR experiment in that the dominant contribution to the total variance appears to be frequency dependent with the variance associated with the instrument component dominating at lower frequencies 100 – 500Hz, and the part to part variance being more influential at higher frequency. This is summarised in Table 1 & 2.

### Table 1: Components of variance table for reverberation time measurement across the frequency range 100-3150Hz

<table>
<thead>
<tr>
<th>Frequency</th>
<th>( \sigma^2 )</th>
<th>( \sigma_r^2 )</th>
<th>( \sigma_R^2 )</th>
<th>( \sigma_p^2 )</th>
<th>( \sigma_p^2 )</th>
<th>( \sigma_{Total}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100Hz</td>
<td>0.003</td>
<td>0.002</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.014</td>
</tr>
<tr>
<td>125Hz</td>
<td>0.006</td>
<td>0.005</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.012</td>
</tr>
<tr>
<td>250Hz</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.006</td>
</tr>
<tr>
<td>500Hz</td>
<td>0.002</td>
<td>0.002</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.004</td>
</tr>
<tr>
<td>1000Hz</td>
<td>0.003</td>
<td>0.003</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.007</td>
</tr>
<tr>
<td>1250Hz</td>
<td>0.004</td>
<td>0.004</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.009</td>
</tr>
<tr>
<td>2500Hz</td>
<td>0.005</td>
<td>0.005</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.011</td>
</tr>
<tr>
<td>5000Hz</td>
<td>0.006</td>
<td>0.006</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.014</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Frequency</th>
<th>( \sigma^2 )</th>
<th>( \sigma_r^2 )</th>
<th>( \sigma_R^2 )</th>
<th>( \sigma_p^2 )</th>
<th>( \sigma_p^2 )</th>
<th>( \sigma_{Total}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100Hz</td>
<td>0.011</td>
<td>0.011</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.014</td>
</tr>
<tr>
<td>125Hz</td>
<td>0.018</td>
<td>0.018</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.022</td>
</tr>
<tr>
<td>250Hz</td>
<td>0.024</td>
<td>0.024</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.033</td>
</tr>
<tr>
<td>500Hz</td>
<td>0.023</td>
<td>0.023</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.032</td>
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<tr>
<td>1000Hz</td>
<td>0.008</td>
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<td>0.001</td>
<td>0.000</td>
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<td>0.007</td>
</tr>
<tr>
<td>1250Hz</td>
<td>0.005</td>
<td>0.005</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.005</td>
</tr>
<tr>
<td>2500Hz</td>
<td>0.003</td>
<td>0.003</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.003</td>
</tr>
<tr>
<td>5000Hz</td>
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<td>0.002</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.002</td>
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</tbody>
</table>

### Table 2: Standard deviation table for reverberation time measurement across the frequency range 100-3150Hz

<table>
<thead>
<tr>
<th>s-d Secs</th>
<th>( \sigma^2 )</th>
<th>( \sigma_r^2 )</th>
<th>( \sigma_R^2 )</th>
<th>( \sigma_p^2 )</th>
<th>( \sigma_p^2 )</th>
<th>( \sigma_{Total}^2 )</th>
</tr>
</thead>
<tbody>
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<td>0.010</td>
<td>0.010</td>
<td>0.005</td>
<td>0.000</td>
<td>0.000</td>
<td>0.015</td>
</tr>
<tr>
<td>125Hz</td>
<td>0.013</td>
<td>0.013</td>
<td>0.006</td>
<td>0.000</td>
<td>0.000</td>
<td>0.016</td>
</tr>
<tr>
<td>250Hz</td>
<td>0.015</td>
<td>0.015</td>
<td>0.006</td>
<td>0.000</td>
<td>0.000</td>
<td>0.016</td>
</tr>
<tr>
<td>500Hz</td>
<td>0.017</td>
<td>0.017</td>
<td>0.006</td>
<td>0.000</td>
<td>0.000</td>
<td>0.017</td>
</tr>
<tr>
<td>1000Hz</td>
<td>0.010</td>
<td>0.010</td>
<td>0.005</td>
<td>0.000</td>
<td>0.000</td>
<td>0.013</td>
</tr>
<tr>
<td>1250Hz</td>
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<td>0.013</td>
<td>0.006</td>
<td>0.000</td>
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<td>0.014</td>
</tr>
<tr>
<td>2500Hz</td>
<td>0.015</td>
<td>0.015</td>
<td>0.006</td>
<td>0.000</td>
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<td>0.015</td>
</tr>
<tr>
<td>5000Hz</td>
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<td>0.017</td>
<td>0.006</td>
<td>0.000</td>
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<td>0.018</td>
</tr>
</tbody>
</table>

More data from GRR experiments is desirable to investigate further the uncertainties associated with the measurement of RT in field test situations and include other room types, constructions and sizes. There is also the possibility to extend the frequency range of interest to include both higher and lower frequency bands e.g. 50 Hz – 5KHz.
REFERENCES

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