IN-SITU CHARACTERIZATION OF A VIBRO-ACOUSTIC SUB-DOMAIN WITH THE PTF METHOD

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For vibro-acoustic systems, such complex as a vehicle, modelling and understanding of acoustic transmission path are not straightforward tasks. Substructuring approaches can be very helpful for NVH engineers in different ways. The most interesting capabilities of substructuring approaches are (i) solving smaller sub-problems and (ii) mixing different modelling “s solvers” (closed form solutions, numerical simulations or experiments). Therefore, using substructuring approach allows reaching higher frequencies or solving bigger systems and allows combining numerical and experimental descriptions of sub-problems. The latter point is particularly interesting when dealing with subdomains that remain difficult to model with numerical tools (assembly, trim, sandwich panels, porous materials, etc.). Impedance or mobility are at the basis of several substructuring methods like the Patch Transfer Functions (PTF) method. For this kind of methods, the (acoustic) impedance on the coupling surface characterizes the subdomain. This impedance is intrinsic of the subdomain and does not depend on the coupling to any other subdomain. The impedances are computed on “uncoupled” subdomains. This becomes problematic when one wants to obtain these impedances from measurements (a vehicle cabin cannot be uncoupled from the structure of the car for instance).

The present works deals with a method developed to characterize the acoustic impedance of a passive subdomain directly using responses of the whole system (with coupled subdomains) and using an inverse approach. The theoretical background is presented and an original way to face ill-posedness of the problem is proposed. Finally, the method is validated on an experimental test case.

Keywords: substructuring method, acoustic impedance, inverse method, regularization

1. Introduction

To solve and compute the response of a complex system, substructuring approaches are often used. Some of them introduce the concept of mobility or impedance to couple subdomains [1, 2]. It allows splitting the whole system into smaller coupled subdomains that leads to two main advantages:
- Splitting the system into smaller subdomains often makes the solving easier. It is the case for example for finite element methods. This allows speeding up computations or increasing the frequency band.
- As the subdomain are characterized independently, any suitable method (finite elements, measurements, analytical solutions, etc.) can be used to characterize one of them. As an example, a
subdomain characterized by finite elements might be coupled to a subdomain characterized by measurements. This versatile property of substructuring approaches is one of their main advantages.

In vibro-acoustics, the so-called Patch Transfer Function (PTF) method has been proposed [3, 4] to couple subdomains through their surface acoustic impedance. This method has been used for example to find efficient positioning of absorbing materials in a cavity [5].

However, for some reasons like the complexity of the subdomain (subdomains which include trim for example), the subdomain can sometimes not be modelled by numerical tools. In that case, the only way to introduce the subdomain in the computation of the response of the whole system is to characterize it experimentally. This is not a simple task as the subdomain has to be physically isolated from other subdomains of the whole system [6-8]. This can become impractical for system such complex as a car.

In the present article, a method is proposed to characterize a passive subdomain (without any acoustic source inside) by means of measurements performed on the whole coupled system. This allows to characterize the subdomain without any particular setup. However, as an inverse method, the approach is ill-posed. Rather than using classical Tikhonov’s regularization, we propose an alternative regularizing technique based on the use of median metric. Finally, the proposed method is validated experimentally.

2. Substructuring approach using Patch Transfer Function method

The Patch Transfer Function method is well adapted when dealing with coupled acoustic subdomains. As presented in [9], no constraining coupling assumption (like weak coupling) is needed to couple subdomains. Let’s consider for example the simple case of a rectangular box $C$ of dimensions $L_x \times L_y \times L_z$ as shown in Figure 1(a). One can split this cavity into two smaller ones, cavities $A$ and $B$. Let’s consider that a point source is acting in cavity $A$ and that one wants to compute the frequency response of the whole system (cavity $C$) at position $L$. The two subdomains are coupled through a coupling surface divided into rectangular patches of surfaces $\Gamma_i$.

![Figure 1: Sketch of a cavity of dimensions $L_x \times L_y \times L_z$, filled with air. An acoustic point source is located at point $S$ and a microphone is placed at point $L$. The frequency response at point $L$ can be computed (a) on the whole cavity or (b) by using a substructuring method like the PTF approach, dividing cavity $C$ in smaller subdomains $A$ and $B$.](image)

To solve this problem, one can first express the pressure $p_i^A$ applied on patch $i$, viewed from uncoupled subdomain $A$ as

$$p_i^A = \tilde{p}_i^A + \sum_{j=1}^{N} Z_{ij}^A v_j^A. \quad (1)$$

where $\tilde{p}_i^A$ is the direct pressure field due to the point source on the coupling surface considered as rigid, $v_j^A$ is the potential velocities of patches $j$ ($j = 1 \ldots N$) and $Z_{ij}^A$ is the acoustic impedance between patch $i$ and patch $j$. Considering all the patches of the coupling surface, one can write Eq. (1) in a matrix form as

$$p^A = \tilde{p}^A + Z^A v^A. \quad (2)$$
Similarly, for subdomain $B$, the pressure on patch $i$, viewed from uncoupled subdomain $B$ writes

$$p^B_i = \sum_{j=1}^{N} Z^B_{ij} v^B_j,$$

as no source is acting in subdomain $B$. In a matrix form, the system of equations writes

$$\mathbf{p}^B = Z^B \mathbf{v}^B.$$

One has to pay attention to the fact that matrix $Z^B$ is intrinsic to the subdomain and is the same whatever the subsystem this subdomain might be coupled to.

To couple the two subdomains, one expresses the following coupling conditions

$$\mathbf{p}^A = \mathbf{p}^B = \mathbf{p}^{\text{coup}}$$

and

$$-\mathbf{v}^A = \mathbf{v}^B = \mathbf{v}^{\text{coup}}.$$

where $\mathbf{p}^{\text{coup}}$ and $\mathbf{v}^{\text{coup}}$ are respectively the pressures and the velocities that would be measured directly on coupled system $C$ if were possible. Considering that these pressures and velocities can’t be measured (or simulated) directly on the coupled system (it’s the aim of substructuring methods), one has to deduce them from Eqs. (2) and (4). The primary unknown is the coupling velocity $\mathbf{v}^{\text{coup}}$ that can be found solving the following system of equations

$$\mathbf{v}^{\text{coup}} = (Z^A + Z^B)^{-1}\tilde{\mathbf{p}}^A.$$

Then, the pressure anywhere in the subdomains can be computed using equations similar to Eqs. (3) and (5). Thus, using this substructuring approach, one can compute the frequency response of a system by splitting it into smaller subdomains characterized by their surface impedances.

In the next section, we propose a way to experimentally identify matrix $Z^B$. The only requirement is that the subdomain $B$ has to be passive when coupled to other subdomains.

### 3. Identification of acoustic impedance of a passive subdomain

As explained in previous section, the following equation can be written for a passive subdomain

$$\mathbf{p}^{\text{coup}} = Z^B \mathbf{v}^{\text{coup}},$$

where $\mathbf{p}^{\text{coup}}$ and $\mathbf{v}^{\text{coup}}$ are the coupling pressures and velocities measured directly on the whole system. These two vectors are linked by the surface impedance matrix $Z^B$ which is intrinsic to subdomain $B$ and doesn’t depend on the system the subdomain $B$ is coupled to. If $N$ independent experiments ($N$ being the number of patches on the coupling surface) are performed - moving $N$ times the point source in subdomain $A$ for example - , the following system of equations can be written

$$\mathbf{P}^C = Z^B \mathbf{V}^C,$$

where each column of matrices $\mathbf{P}^C$ and $\mathbf{V}^C$ represents one experiment as expressed by Eq. (8). Thus, matrix $Z^B$ can be identified by inverting the system of equations (9)

$$Z^B = \mathbf{P}^C(\mathbf{V}^C)^{-1}.$$
However, as any inverse method, the proposed approach is ill-posed and the matrix $V^C$ is ill-conditioned. Rather than using a classical Tikhonov regularization, we propose here an original way to regularize the problem.

To identify the matrix $Z^B$ one thus need a set of $N$ independent experiments. The idea is here to collect, for $Q$ sets of $N$ experiments, $Q$ identified matrices $Z^B_{q,\text{id}}$ ($q = 1 ... Q$). The median values of each term of matrices $Z^B_{q,\text{id}}$ is then computed to construct the identified matrix $Z^B_{\text{id}}$. Taking the median (and not the mean) has a regularization effect on the results as will be demonstrated in the next section.

It is possible to create $Q$ sets of experiments by only using $M$ source positions ($M > N$). The number of possible different combinations of sources is given by

$$Q = \frac{M!}{N!(M-N)!}$$  \hspace{1cm} (11)

where $M!$ represents the factorial of number $M$.

4. **Experimental validation**

The experimental setup consists in a rectangular box with five walls considered as rigid as can be seen in Figure 2(a). This box is passive as no source is acting directly inside. The box is placed on absorbing mounts in a quiet room (not anechoic). The objective of this experiment is to identify the matrix $Z^B$ of surface acoustic impedance of the rectangular box. To achieve this goal, the coupling surface (the sixth surface of the box) is virtually divided into 20 rectangular patches ($5 \times 4$) and a pU probe is placed at the centre of each to measure the coupling pressures and velocities, see Figure 2(b). As a consequence, the matrix to be identified is of dimensions $20 \times 20$ and consists then in 400 complex values to determine at each frequency. In order to identify this matrix, at least 20 sources positions (sequentially moved) are needed. By considering more positions of the source, one can obtained several versions of the same matrix (as previously presented). In the present example, 25 positions of the source have been used. This can lead to 53130 different combinations of 20 source positions leading to 53130 versions of the same matrix $Z^B$ (Eq. (11)). Taking the median values for each term of the matrix at each frequency has a regularization effect as will be seen in the following results.
Figure 2: (a) Picture of the experimental setup and (b) positions of the point sources and positions of the microflown sensors used to identify the surface impedance of the cavity.

The identification using the described process is plotted in Figure 3 for two terms of the matrix $\mathbf{Z}^B$. To validate this approach, the results are compared to reference values of the impedance computed using analytical expressions for a rectangular box with rigid walls. We assume here a sound speed of $c_0 = 340\text{m/s}$, a density of air of $\rho_0 = 1.29\text{kg/m}^3$ and a damping of $\eta_0 = 0.01$. The comparison is very good even if the picks are slightly underestimated (surely due to uncertainty in the estimation of the damping in the reference computation). However, the overall level as well as the shapes of the curves are well identified by the proposed approach.

Figure 3: comparison between the identified surface impedance of the cavity and a reference solution (analytical formula of a cavity with rigid walls). (a) $\mathbf{Z}_{1,1}^B$; (b) $\mathbf{Z}_{3,16}^B$. Blue solid line: reference; red dashed line: identification.
This identified matrix could be then coupled to other subdomains to compute the vibro-acoustic response of the whole system, even if the whole system is not the one used during the characterization process.

5. Conclusion

In this work, a methodology to in-situ characterize a vibro-acoustic subdomain was proposed. In the framework of the Patch Transfer Function method, this methodology allows identifying the surface acoustic impedance matrix of a passive subdomain. The measurements can be performed directly on the whole system without physically uncoupling the subdomain under investigation.

An original way to regularize the ill-posed problem was proposed and rely on the use of median metric on sets of experiments.

Finally, the methodology was successfully applied on a real test case consisting in a rectangular box.

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