NONLINEAR VIBRATION ANALYSIS OF A CONTROL FIN WITH FREEPLAY NONLINEARITY

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Abstract. In order to establish a nonlinear dynamic model of the aircraft-control fin, the fixed interface component mode synthesis method (C-B method) was used to divide the overall structure of the aircraft into two linear substructures, fuselage and control fin, with the DOFs corresponding to the nonlinear control stiffness element in the interface. The modal analysis of each substructure is carried out to obtain its natural modal characteristics. By using the force equilibrium and displacement coordination between the substructures, and considering the nonlinear load-displacement relation between the substructures, the low-order modal characteristics of the substructures are synthesized to obtain the nonlinear dynamic characteristics of the whole structure. The FEM software for component mode substitution of all-moved rudder is developed, which is used to simulate the structural dynamic response of the reduced-order control fin model in time domain.
Keywords: nonlinear dynamic model, C-B method, vibration, freeplay

1. Introduction

In the all-moveable rudder, freeplay nonlinearity is unavoidable. For the all-moveable rudder with freeplay nonlinearity, the dynamic characteristics are very complicated, which not only adversely affects the aerelastic performance of the aircraft, but also affects the handling performance and even the fatigue damage problem. Theoretically, the freeplay nonlinearity will cause difficulties in structural dynamics especially for the modal testing, so it is necessary to carry out nonlinear vibration analysis before any modal test of nonlinear control fins.

Several investigators have performed vibration analysis of control fins with freeplay nonlinearity being considered. To establish the nonlinear vibration equation, a fictitious mass method was introduced by Karpel et al.[1-2]. Lee and Kim[3] analyzed an all-movable nonlinear control fin, and the results showed that the nonlinear parameters and initial conditions had strong influences on the nonlinear responses. In this work, we present a nonlinear vibration analysis method for the control fin with freeplay nonlinearity with the help of the C-B method.

2. Vibration analysis of the model with freeplay nonlinearity

For a system with freeplay nonlinearity, the node associated with the nonlinear connected region can be used as an interface, the system is divided into several subsystems, and then the modal synthesis can
be performed by the C-B method [4]. For a system consisting of two substructures, the displacement coordination condition between the substructures is

\[ u_\alpha^p = u_\beta^p + x \]  

(1)

where \( x \) is the relative displacement of the two substructure interfaces in the nonlinear connected region,

\[ q = \begin{bmatrix} p_\alpha^p \\ u_\alpha^p \\ p_k^\beta \\ x \end{bmatrix} \]

(2)

and \( q \) is independent generalized coordinates.

The second coordinate transformation (independent coordinate transformation) of the system is:

\[ p = Sq \]

(3)

\[ S = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & I & 0 & I \end{bmatrix} \]

(4)

The mass matrix and stiffness matrix of the system under independent generalized coordinates are:

\[ \ddot{M} = S^T \ddot{M} S \quad \ddot{K} = S^T \ddot{K} S \]

(5)

The interface force vector of the system under independent generalized coordinates is:

\[ \ddot{F} = S^T \ddot{F} = \begin{bmatrix} 0 \\ f_\alpha^\beta + f_\beta^\beta \\ 0 \\ f_\beta^\beta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ f_\beta^\beta \end{bmatrix} \]

(6)

The integrated system free vibration equation is:

\[ \dddot{M}q + \dddot{K}q = \dddot{F} \]

(7)

This completes the modal synthesis of the two substructures.

For the internal force \( f_\beta^\beta \), it is a function of the relative displacement \( x \) of the interface, that is \( f_\beta^\beta = -f(x) \). when the stiffness between substructures is a freeplay nonlinear stiffness, \( f(x) \) is a nonlinear function of \( x \). Take freeplay nonlinear stiffness as an example:

\[ f(x) = \begin{cases} k(x + \delta) & x \leq -\delta \\ 0 & -\delta < x < \delta \\ k(x - \delta) & x \geq \delta \end{cases} \]

(8)

Express it as a form of linear stiffness and nonlinear stiffness superposition, as follows

\[ f(x) = f_L(x) + f_N(x) \]

(9)

where

\[ f_L(x) = kx \]

(10)
Bring \( f_f^{\beta} = -f(x) = -f_L(x) - f_N(x) \) into the vibration equation and move the linear restoring force \(-f_L(x)\) to the left hand side of the equation. The free vibration equation of the system is:

\[
\ddot{M}q + \ddot{K}q = -\ddot{F}_N
\]  

(12)

where: \( \ddot{K} = \ddot{K} + \ddot{K}_L \), \( \ddot{K}_L = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & k \end{bmatrix} \), \( \ddot{F}_N = \begin{bmatrix} f_N(x) \end{bmatrix} \)

If a dynamic load \( Q \) is applied to the freeplay nonlinear system and a damping matrix \( \tilde{C} \) is introduced, the forced vibration equation of the system is obtained as

\[
\ddot{M}q + \dot{\tilde{C}}q + \ddot{K}q = Q - \ddot{F}_N
\]  

(13)

For this equation, it can be transformed into a state space equation and nonlinear vibration analysis is performed by the Runge-Kutta method.

Define \( y = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \) and introduce the equation \( \dot{q} = \ddot{q} \), then the above equation can be transformed into a state space equation:

\[
\dot{y} = \begin{bmatrix} 0 & I \\ -\tilde{M}^{-1}\ddot{K} & -\tilde{M}^{-1}\ddot{C} \end{bmatrix} y + \begin{bmatrix} 0 \\ \tilde{M}^{-1}Q \end{bmatrix} + \begin{bmatrix} 0 \\ -\tilde{M}^{-1}\ddot{F}_N(x) \end{bmatrix}
\]  

(14)

The time domain response is solved by the Runge-Kutta method later. In each time step, according to the value of \( \ddot{F}_N(x) \), the solution of the next time step is obtained, and the time history of the forced vibration response of the system is obtained.

It should be noted that the forced vibration Eq. (12) is not a decoupled equation. The eigenvalue analysis can be performed using the mass matrix \( \tilde{M} \) and the linear stiffness matrix \( \ddot{K} \) to calculate the modal shape, and perform another modal coordinate transformation on Eq. (12) to obtain the decoupling equation of motion in modal space.

\[
\ddot{M}_m\ddot{r} + \ddot{C}_m\ddot{r} + \ddot{K}_m\ddot{r} = \Phi_m^TQ - \Phi_m^T\ddot{F}_N(\Phi_m\ddot{r})
\]  

(15)

where \( \ddot{M}_m \), \( \ddot{C}_m \), and \( \ddot{K}_m \) are the modal mass matrix, the modal damping matrix, and the modal stiffness matrix, respectively, \( \Phi_m \) is the modal shape matrix, and \( r = \Phi_m^Tq \) is the modal coordinate.

Define \( w = \begin{bmatrix} r \\ \dot{r} \end{bmatrix} \) and introduce equation \( \dot{w} = \ddot{w} \), we can transform the above equation into a state space equation:

\[
\dot{w} = \begin{bmatrix} 0 & I \\ -\ddot{M}_m^{-1}\ddot{K}_m & -\ddot{M}_m^{-1}\ddot{C}_m \end{bmatrix} w + \begin{bmatrix} 0 \\ \ddot{M}_m^{-1}\Phi_m^TQ \end{bmatrix} + \begin{bmatrix} 0 \\ -\ddot{M}_m^{-1}\Phi_m^T\ddot{F}_N(\Phi_m\ddot{r}) \end{bmatrix}
\]  

(16)
If the vibration mode is normalized by mass, the above formula can be simplified to

\[ \dot{w} = \left[ \begin{array}{cc} 0 & I \\ \text{diag}(\omega^2) & -\text{diag}(2\xi \omega) \end{array} \right] \dot{w} + \left[ \begin{array}{c} 0 \\ \Phi^T \mathcal{Q} \end{array} \right] + \left[ \begin{array}{c} 0 \\ -\Phi^T \mathcal{F}_N (\Phi \mathcal{R}) \end{array} \right] \]

where \( \omega \) is the natural frequency of the \( i \)th mode and \( \xi \) is the damping ratio of the \( i \)th mode.

Then the time domain simulation can be carried out by the Runge-Kutta method to obtain the nonlinear vibration response of the system.

### 2.1 Modal analysis of the aft fuselage-control fin model

In this paper, a partial fuselage-control fin model is studied as shown in Fig. 1. The partial fuselage is fixed at both ends. At the joint between the fuselage and the control fin, the translational freedom and the rotational freedom about the z-axis is rigidly connected, the z-axis about the x-axis is connected by a linear torsional spring, and the rotational degree of freedom about the y-axis is coupled through a nonlinear torsional spring connection. The excitation point is at the leading edge of the root chord, and the excitation direction is vertical to the control fin.

![Figure 1: Partial fuselage-rudder FEM model](image)

Firstly, the freeplay nonlinearity is replaced by the linear stiffness and the model is analyzed. The natural frequencies of the 6 lowest order modes are shown in Table 1, and the modal shape are shown in Fig. 2~Fig. 7.

<table>
<thead>
<tr>
<th>Modal order</th>
<th>Modal name</th>
<th>natural frequencies(Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1st bending</td>
<td>2.55</td>
</tr>
<tr>
<td>2</td>
<td>1st torsion</td>
<td>8.67</td>
</tr>
<tr>
<td>3</td>
<td>1st in-plane bending</td>
<td>18.97</td>
</tr>
<tr>
<td>4</td>
<td>2nd bending</td>
<td>122.06</td>
</tr>
<tr>
<td>5</td>
<td>1st in-plane side expansion</td>
<td>206.76</td>
</tr>
<tr>
<td>6</td>
<td>control fin bending</td>
<td>227.44</td>
</tr>
</tbody>
</table>
In order to perform vibration analysis by the C-B method, it is necessary to divide the model into several substructures. From the connection between the fuselage and the control fin, the model is divided into the following two substructures. As show in Fig. 8 and Fig. 9, there is only 1 node at the interface of the two substructures.
The nonlinear stiffness of the freeplay model is replaced by the linear counter part. The modal analysis is carried out by the C-B method. The natural frequencies of the 6 lowest order modes and the results of the original model are shown in Table 2. It can be seen that the natural frequency error obtained by the C-B method is rather small.

Table 2: Natural frequencies of the 6 lowest order modes

<table>
<thead>
<tr>
<th>Modal order</th>
<th>Natural frequencies(Hz)</th>
<th>Original model</th>
<th>C-B method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.55</td>
<td>2.55</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8.67</td>
<td>8.67</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>18.97</td>
<td>18.97</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>122.06</td>
<td>122.09</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>206.76</td>
<td>208.37</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>227.44</td>
<td>227.45</td>
<td></td>
</tr>
</tbody>
</table>

2.2 Time domain vibration response of the model

A simple harmonic excitation is applied at the excitation point, and the nonlinear time domain vibration analysis is performed by the C-B method. In this example, the main focus is on the rotational motion of the control fin. Therefore, the excitation frequency is near the 2nd natural frequency. The excitation force amplitude is 5N, the freeplay angles is 1°, and the simulation time step is 0.005 s. The total simulation duration is 5s, the initial displacement and the initial velocity are both 0, and the modal damping is 0.03. In order to study the influence of the freeplay nonlinearity on the structural vibration response, the vibration analysis of the linear system without considering the freeplay nonlinearity is also carried out by the C-B method, and compared with the vibration analysis result considering the freeplay nonlinearity.

Firstly, the excitation frequency is 7 Hz. Fig. 10 is the time history of the rotation angle, and Fig. 11 is the time history of the z-direction displacement response. It can be seen from Fig. 10 to Fig. 11 that the response amplitude of the structure is significantly increased after considering the freeplay nonlinearity.

Figure 10: Time history of the rotation angle, 7Hz
Figure 11 Time history of the z-direction displacement response of the leading edge, 7Hz

Then, the excitation frequency is set as 10 Hz. The time history of the rotation angle is shown in Fig.12, and Fig. 13 is the time history of the z-direction displacement response of the leading edge. It can be seen from Fig. 12 and Fig. 13 that the response amplitude of the structure is significantly reduced after considering the freeplay nonlinearity.

Figure 12: Time history of the rotation angle, 10Hz

Figure 13: Time history of the z-direction displacement response of the leading edge, 10Hz
3. Conclusion

The above analysis shows that the natural frequency error obtained by the C-B method is rather small.

The freeplay nonlinearity has a significant influence on the vibration response of the structure. Under different excitation frequencies and excitation force amplitudes, the response amplitude may increase, and the response amplitude may also decrease. The reason is that the existence of the freeplay nonlinearity weakens the stiffness of the original linear structure, so that the resonance frequency of the rotational mode is reduced.

In the original linear system, the resonance frequency of the rotational mode is about its natural frequency, which is 8.67 Hz. After considering the nonlinearity, the resonance frequency of the mode is reduced. When the excitation frequency is 7 Hz, the excitation frequency is closer to the resonance frequency of the nonlinear system, so the response amplitude of the structure considering the freeplay nonlinearity is large. When the excitation frequency is 10 Hz, the excitation frequency is closer to the resonance frequency of the original linear system, so the response amplitude of the linear structure is larger.

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REFERENCES