VIBRATION CHARACTERISTICS OF A BEAM BEARING HIGH ACCELERATIONS AND JERKS IN A TRIAXIAL CENTRIFUGE

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The load of the rapid maneuver aircraft is characterized by high accelerations, high jerks and multiple directions in space. The ground load simulation test of the aircraft is usually carried out on a triaxial centrifuge. In this paper, considering the three-axis centrifugal motion and elastic vibration of the beam, velocity and acceleration model of the beam are established by the method of motion synthesis and matrix rotation. The acceleration model of the beam is the basis of the load analysis and calculation of the dynamic responses. Dynamic equations of the beam in the three-axis centrifugal environment is established. Based on the dynamic equations, responses of the beam is calculated at the cases of different motion parameters. The influence of the accelerations and jerks on the responses is analyzed. The results show that both the accelerations and jerks can result deformation on the beam. Jerks can cause vibration of the beam and a sudden change of acceleration will induce a significant vibration of the beam. For the beam, a sudden change of acceleration just likes an impact force loading on the beam. The mechanical model and calculation method obtained in his paper can be used to simulate the flight responses of the aircraft with high accelerations and jerks.

Keywords: triaxial centrifuge, high accelerations and jerks, dynamic characteristic

1. Introduction

In the flight with high maneuverability, the aircraft bears a three-dimensional and high acceleration with high change rate (jerk). Some flight accelerations reach 20 g (g is gravity acceleration), and jerks can also reach 15 g/s [1]. High accelerations and jerks in three dimensional space have a great impact on the performance of the aircraft structure, airborne electronic equipment, et al., and even cause fatal damage. In order to ensure the safety in this environment, three axis centrifuge is used as a motion base to simulate the flight environment[2-4]. Our research group carried out studies[5-7] on kinematics model of a three-axis centrifuge and load design, and solved the inversion problem of motion parameters.

In this paper, based on the previous studies, the kinetic equations of beam in the three-axis centrifugal environment are established by Kane method[8,9], and the rigid-flexible coupling dynamic characteristics of the beam are studied. The mechanical model and simulation procedure can be used to calculate not only the responses of the beam in the centrifugal test, but also the results in the flight environment.
2. Kinematic model of the beam under three-axis centrifuge

The schematic diagram of the triaxial centrifuge model studied in this paper is shown in Figure 1\[12\]. The centrifuge consists of three stage rotors. The No. 1 axis is parallel to the No. 2 axis, and the axis of No. 2 and No. 3 are vertical. The rotational angles of the No. 1, No. 2 and No. 3 axis are represented by \( \varphi \), \( \theta \) and \( \alpha \), respectively.

For the beam mounted on the third stage rotor along the length direction of the main arm, the flexibility vibration and rigid coupling motion are considered, the kinematics model of the beam under the three-axis centrifugal environment is established. The unit vectors \( e_1 \), \( e_2 \) and \( e_3 \) are respectively along the directions of the three axes of the body coordinate \((xyz)\) fixed on No.3 axis. The motion of a point \( P' \) in the coordinate fixed on No.3 axis is analyzed. After deformation, \( P' \) arrives at point \( P \).

\[
1u, 2u, 3u \text{ represent the deformation of } P' \text{ along the directions of } x, y \text{ and } z, \text{ respectively.}
\]

The relative position vector of the point \( P' \) before deformation can be expressed as \( r = xe_1 \). The variation of the displacement before and after deformation is expressed as:

\[
u = u_1 e_1 + u_2 e_2 + u_3 e_3,
\]

The velocity and acceleration of the base point \( O_3 \) are respectively expressed as:

\[
\begin{align*}
\dot{v} &= \dot{v}_1 e_1 + \dot{v}_2 e_2 + \dot{v}_3 e_3 \\
\ddot{a} &= \ddot{a}_1 e_1 + \ddot{a}_2 e_2 + \ddot{a}_3 e_3
\end{align*}
\]

The angular velocity of the beam, which is the absolute angular velocity of the third axis, is expressed as:

\[
\omega_3 = \omega_3 e_1 + \omega_3 e_2 + \omega_3 e_3
\]

Thus, based on the method of motion synthesis, the absolute velocity and absolute acceleration of point \( P \) can be expressed as:

\[
\begin{align*}
v &= \dot{v}' + \omega_3 (r + u) + \omega \times \dot{v}' \\
\ddot{a} &= \ddot{a}' + \omega_3 (r + u) + \omega \times (\omega (r + u)) + \omega \times \ddot{v}' + 2\omega \times \dot{v}'
\end{align*}
\]

In Eq. (6), \( \omega \times (\omega (r + u)) \) is the normal acceleration of the point \( P \), \( \dot{v}' \) is the vibration acceleration of point \( P \) caused by the elastic vibration, and \( 2\omega \times \dot{v}' \) represents the Coriolis acceleration caused by the coupling of the elastic vibration and the rigid body rotation.
3. Dynamic equations of the beam

For a beam installed in the triaxial centrifuge, as shown in Figure 2, the dynamic equations are established by Kane method. The equations are expressed as:

\[ F_q + F'_q = 0 \quad (i=1,2,3) \tag{7} \]

expresses the generalized active force on the beam, is the corresponding generalized inertia force, subscript 1, 2, 3 correspond to the axis direction of x, y, z, respectively.

The modal coordinates of the beam is used as the generalized coordinates, and is the generalized velocity. Thus, the generalized inertial force and generalized active force of the beam can be expressed as:

\[ F'_q = -\int_0^t \rho \frac{\partial \mathbf{v}}{\partial Q_i} dx \tag{8} \]

\[ F_q = -\frac{\partial U}{\partial \mathbf{Q}_q} \tag{9} \]

The deformation energy U in Eq.(9) is obtained as following:

\[ U = \frac{1}{2} \int_0^t \left[ \sum_{i=1}^n A_i \left( \frac{\partial u_i}{\partial x} \right)^2 dx + \frac{1}{2} \sum_{i=1}^n E_i \left( \frac{\partial^2 u_i}{\partial x^2} \right)^2 dx + \frac{1}{2} \sum_{i=1}^n \frac{\partial^2 \mathbf{v}_i}{\partial x^2} \right] dx \tag{10} \]

In Eq.(10), the geometry relationship of the deformation is approximated as:

\[ \frac{\partial s}{\partial x} = u_{1,x} + \frac{1}{2} \left( u_{2,x} + u_{3,x} \right) \tag{11} \]

Substituting the velocity, acceleration and deformation energy into Eq.(8) and (9), then into Eq. (7), the Kane equations of the beam are obtained.

Here, to simplify the equations in Eq.(7), the Rayleigh-Ritz method is used to describe the displacement due to elastic deformation, so s, u2, u3 are expressed as:

\[ s(x,t) = \sum_{i=1}^n \Phi_{i,s}(x)Q_i(t) \tag{12a} \]

\[ u_2(x,t) = \sum_{i=1}^n \Phi_{i,2}(x)Q_i(t) \tag{12b} \]

\[ u_3(x,t) = \sum_{i=1}^n \Phi_{i,3}(x)Q_i(t) \tag{12c} \]

Where \( \Phi_{i,s}, \Phi_{i,2}, \Phi_{i,3} \) are the basis functions that are chosen according to the modes of the beam in the inertial system. \( Q_i, Q_2, Q_3 \) are the modal coordinates, they are used as the generalized coordinates. Subscript \( i \) is the order of modal, and \( n_1, n_2, n_3 \) represent the modal truncation numbers of the three directions, respectively.

Substituting the displacement functions in Eq.(12a)-Eq.(12c) into the deformation energy and the generalized active forces are calculated. After linear processing, the generalized active forces corresponding to \( Q_u, Q_2, \) and \( Q_3 \) are as follows:

\[ F_u = -\frac{\partial U}{\partial Q_u} = \sum_{i=1}^n E_i \left( \int_0^t \Phi_{i,u}(x) dx \right) Q_i \tag{13a} \]
The generalized inertial forces of the three directions are following:

\[ F_{i} = \frac{\partial U}{\partial \ddot{Q}_{i}} = \sum_{j=1}^{n} EI \left( \int_{0}^{l} \Phi_{j,i} \Phi_{j,i} dx \right) \ddot{Q}_{j} \]  \hspace{1cm} (13b)

\[ F_{i} = \frac{\partial U}{\partial \dot{Q}_{i}} = \sum_{j=1}^{n} EI \left( \int_{0}^{l} \Phi_{j,i} \Phi_{j,\dot{v}} dx \right) \dot{Q}_{j} \]  \hspace{1cm} (13c)

The generalized partial velocity.

According to Eqs.(14)-(15) and Eq.(7), the dynamic equations of the system are obtained as follows:

\[ M \ddot{Q} + GQ + KQ = F \]  \hspace{1cm} (15)

The dimension of the matrix \( M, G, K \) in Eq. (15) is \( n \times n \), respectively. \( F \) is an \( n \times 1 \) column vector, where \( n = n_{1} + n_{2} + n_{3} \), \( n_{1}, n_{2}, n_{3} \) express the modal truncation number along three axis of the beam, respectively. The damping matrix \( G \) is caused by the coupling of the rigid motions, the rigid-flexible coupling motions, simultaneously.

4. Dynamic responses of the beam in the triaxial centrifuge

For a clamped beam installed in the triaxial centrifuge in Fig.1, Newmark method is adopted to calculate the deformation and responses of the beam. The geometrical parameters and material parameters of the beam are listed in Table 1.

| Young’s modulus of the beam, \( E \)(Nm\(^2\)) | 70×10\(^9\) |
| Poisson’s ratio of the beam, \( v_{b} \) | 0.3 |
| Density of the beam, \( \rho \) (kgm\(^{-3}\)) | 2700 |
| Length of the beam, \( l \)(m) | 0.3 |
| Width of the beam, \( b \)(m) | 0.02 |
| Thickness of the beam, \( h \)(m) | 0.004 |
| Location of the key point | (0.5l,0,0) |

Case 1: Responses of the beam under unidirectional acceleration

In this case, a trapezoidal overload \( (a/g) \) in the z-direction is imposed on the beam. The acceleration of the key point on the beam in the centrifugal environment is set to be consistent with the acceleration in the flight environment, the displacement response of the beam is simulated. The time history of the overload is as shown in Fig.3, namely, the ratio of acceleration in z-direction to gravity acceleration \( g \) varies as shown in Fig.3.

The displacement responses of the free end of the beam is as shown in Fig.4. The results expressed
by full line are obtained by the model established in Eq.(15), those expressed by the dash line are obtained by the software Recurdyn. Both of the results agree well with each other, which means the model established is effective. According to the results, the amplitude of the vibration magnifies significantly at the inflection points of the overload curve, it means that the change of the acceleration leads to elastic vibration.

![Graph](image1)

Fig 3: Variation of the overload  

![Graph](image2)

Fig 4: Displacement responses of the beam  

**Case 2: Responses of the beam under three-axis’s accelerations**

The overloads (accelerations/g) on the key point of the beam is as shown in Fig.5. The displacements of the free end obtained by the mechanical model in Eq.(15) are expressed in Fig.6.

![Graph](image3)

Fig 5: Variation of the three-axis’s overload  

![Graph](image4)

Fig 6: Displacement responses  

![Graph](image5)

Fig 7: Elastic vibration responses
Fig. 7 shows the elastic vibration of the free end of the beam when the three-axis’ accelerations are imposed on the beam. It can be seen that the deformation tendency of the beam is the same with the acceleration curves. Due to the influence of change rate of the acceleration, the displacement response curves are not smooth and the elastic vibration occurs subsequently. In other words, jerks can cause vibration of the beam and a sudden change of acceleration will induce a significant vibration of the beam.

5. Conclusions

Dynamic mechanical model of a beam mounted in a three-axis centrifuge is established. Displacement responses of the beam are simulated when the beam suffered different loads. The results show that acceleration and jerk will lead to deformation on the beam, the deformation tendency of the beam is the same with the acceleration curves. Jerk will motivate elastic vibration of the beam and the influence of the jerk is just like that a sudden impact acts on the beam.

Reference