AN ADAPTIVE FEED-FORWARD VIBRATION CONTROL ALGORITHM FOR MULTIPLE PERIODIC DISTURBANCES OF UNKNOWN FREQUENCIES

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A new adaptive feed-forward cancellation (AFC) control algorithm is proposed in this paper which achieves periodic disturbance cancellation. The control algorithm can be used to eliminate multiple periodic disturbances and is independent of where the disturbances enter the system by employing the concept of equivalent input disturbance (EID). The disturbance signal is tracked by using a modified steepest descent method and it is not necessary to know the exact frequencies of the disturbances. To accelerate the convergence, the metric conversion technique is also introduced in this study thus a uniform rate of convergence can be achieved. Some numerical simulations are also carried out in this paper so that the validity of the proposed control algorithm can be verified.

Keywords: active vibration control, periodic disturbance, adaptive feed-forward control

1. Introduction

Attuning noise and vibration is of great importance in many engineering applications, such as precision machine tools, jet engines, satellites, and marine vessels. The rejection of periodic signals is a fundamental research area in vibration control and has been widely studied [1–3] as it is one of the most common sources in practice. Thus, it is highly desirable to propose a method to control the periodic disturbance.

There are several ways to eliminate periodic signals. One of the most common approaches is based on the internal model principle (IMP), which prescribes that a model of the disturbance should be included in the controller. Another method is the adaptive feedforward cancellation (AFC), where the control signal is the same but with a negative value with the disturbance, and is added to the system to perfectly cancel the disturbance. Bodson et al. [4] has proved that these two methods are equivalent. However, they also pointed out that AFC has advantage over IMP when the frequency of disturbance varies in an unknown way. The AFC approach to deal with known frequency has long been investigated [4–6]. By employing a frequency estimator, Bodson and his co-workers extended the AFC approach to control disturbances with unknown frequency [7–10]. Zhu et al. [11] used a time-frequency-analysis (TFA) based AFC algorithm to reject the vibration with multiple unknown frequencies.

In this paper, a new adaptive method based on the AFC approach is proposed. The disturbance with unknown time-varying frequencies is taken into consideration by using the steepest decent algorithm to minimize the performance function. Based on the steepest decent algorithm, metric conversion is introduced to accelerate the convergence. Several numerical simulations are also performed in this paper to
valid the proposed method, and the performance of the proposed method is compared with other algorithms as well.

2. Problem formulation

The mathematical model of a dynamic system in the Laplace domain can be written as

\[ Y(s) = G_d(s)D(s) + G(s)U(s) \]  

where \( G_d(s) \) is the transfer function between disturbance \( D(s) \) and output \( Y(s) \), and \( G(s) \) is the transfer function between control input \( U(s) \) and output \( Y(s) \). It is usually not different to obtain the transfer function of the control channel \( G(s) \). However, as the disturbance may be various and arbitrary, it is usually difficult to identify \( G_d(s) \). Thus, to overcome the limitations of Eq. (1), the concept of equivalent input disturbance (EID) is employed here. Assuming that an equivalent disturbing signal \( D_e(s) \) is acting on the controller’s input channel to simulate the effect of disturbance \( D(s) \) on the system output \( Y(s) \), Eq. (1) can be rewritten as

\[ Y(s) = G(s)(D_e(s) + U(s)) \]  

Assuming that \( G(s) \) is always controllable and observable, and the plant is stable (which means all the poles of \( G(s) \) lies in the left-half plane), thus it can be proved that EID \( D_e(s) \) always exists according to She et al. [12].

3. Algorithm design

3.1 The steepest decent algorithm

In this section, the control algorithm is proposed based on the adaptive feed-forward control scheme as shown in Figure 1. In the figure, \( d_e(t) \), \( u(t) \) and \( y(t) \) are the counterparts of \( D(s) \), \( U(s) \) and \( Y(s) \) in the time domain.

![Figure 1: Adaptive feed-forward control scheme.](image)

The equivalent disturbance signal \( d_e(t) \) with \( n \) frequency components can be written as

\[ d_e(t) = -\sum_{i=1}^{n} \theta_{c,i} \cos \omega_i t - \theta_{s,i} \sin \omega_i t \]  

where \( \theta_{c,i} \), \( \theta_{s,i} \) and \( \omega_i \) are all unknown. By letting the control signal be

\[ u(t) = \sum_{i=1}^{n} \hat{\theta}_{c,i} \cos \hat{\omega}_i t - \hat{\theta}_{s,i} \sin \hat{\omega}_i t \]  

in which \( \hat{\theta}_{c,i} \), \( \hat{\theta}_{s,i} \) and \( \hat{\omega}_i \) are the estimates of \( \theta_{c,i} \), \( \theta_{s,i} \) and \( \omega_i \) respectively. By using Eq. (2), the output of the plant can then be obtained

\[ y(t) = \sum_{i=1}^{n} \hat{\theta}_{c,i} G_c(\hat{\omega}_i; t) + \hat{\theta}_{s,i} G_s(\hat{\omega}_i; t) - \hat{\theta}_{c,i} G_c(\omega_i; t) - \hat{\theta}_{s,i} G_s(\omega_i; t) \]  

where
\[
G_c(\omega; t) = G_R(j\omega) \cos \omega t - G_I(j\omega) \sin \omega t \\
G_s(\omega; t) = G_R(j\omega) \sin \omega t + G_I(j\omega) \cos \omega t
\]
and \(G_R(j\omega)\) and \(G_I(j\omega)\) are the real and imaginary parts of \(G(j\omega)\), respectively.

The performance function can be defined as
\[
f(\hat{x}) = \frac{1}{2} y^2
\]
where \(\hat{x} = (\hat{\theta}_{c,1}, \hat{\theta}_{s,1}, \hat{\omega}_1, \hat{\theta}_{c,2}, \hat{\theta}_{s,2}, \hat{\omega}_2, \ldots, \hat{\theta}_{c,n}, \hat{\theta}_{s,n}, \hat{\omega}_n)^T\).

Adopting the steepest decent method on Eq. (7), the parameter update law can then be obtained
\[
\dot{\hat{x}} = -\mu \nabla f(\hat{x})
\]
or
\[
\dot{\hat{\theta}}_{c,i} = -\mu y \frac{\partial f}{\partial \hat{\theta}_{c,i}} = -\mu y G_c(\hat{\theta}_i; t) \\
\dot{\hat{\theta}}_{s,i} = -\mu y \frac{\partial f}{\partial \hat{\theta}_{s,i}} = -\mu y G_s(\hat{\theta}_i; t) \\
\dot{\hat{\omega}}_i = -\mu y \frac{\partial f}{\partial \hat{\omega}_i} = -\mu y t (-G_s(\hat{\theta}_i; t) \hat{\theta}_{c,i} + G_c(\hat{\theta}_i; t) \hat{\theta}_{s,i})
\]
where \(\mu\) is the adaptive gain.

Discretize Eq. (8), we have the recursion using the steepest decent algorithm
\[
x[k + 1] = x[k] - \mu \nabla f(x[k])
\]

### 3.2 Metric conversion of the algorithm

The expression of the performance function can be written in the quadratic form
\[
f(\hat{x}) = \frac{1}{2} \hat{x}^T H \hat{x} + b^T \hat{x} + c
\]
in which \(H\) is a \(3n \times 3n\) matrix
\[
H = \nabla^2 f = \text{diag}(H_1 \quad H_2 \quad \ldots \quad H_n)
\]
where
\[
H_i = \begin{pmatrix}
G_c^2 & G_c G_s & t G_c (-G_s \hat{\theta}_{c,i} + G_c \hat{\theta}_{s,i}) - y t G_s \\
G_s^2 & t G_s (-G_s \hat{\theta}_{c,i} + G_c \hat{\theta}_{s,i}) + y t G_c & \\
\text{sym} & t^2 (-G_s \hat{\theta}_{c,i} + G_c \hat{\theta}_{s,i})^2 - y t^2 (G_c \hat{\theta}_{c,i} + G_s \hat{\theta}_{s,i}) & \\
\end{pmatrix}_{\omega = \hat{\omega}_i}
\]
Assuming the estimates are close to the true values of the system, the system output \(y\) should be close to zero, which leads to the following equation
\[
\lim_{y \to 0} H_i = \begin{pmatrix}
G_c^2 & G_c G_s & t G_c (-G_s \hat{\theta}_{c,i} + G_c \hat{\theta}_{s,i}) \\
G_s^2 & t G_s (-G_s \hat{\theta}_{c,i} + G_c \hat{\theta}_{s,i}) & \\
\text{sym} & t^2 (-G_s \hat{\theta}_{c,i} + G_c \hat{\theta}_{s,i})^2 & \\
\end{pmatrix}_{\omega = \hat{\omega}_i}
\]
It is not difficult to obtain that the eigenvalues of Eq. (14) are: \(\lambda_{1,i} = \left|\left|G\right|\right|^2 + t^2 (-G_s \hat{\theta}_{c,i} + G_c \hat{\theta}_{s,i})^2\right|_{\omega = \hat{\omega}_i}, \lambda_{2,i} = \lambda_{3,i} = 0\). However, as the convergence of the algorithm is dependent on the eigenvalues, which are functions of \(t\), the convergence may also be time-dependent. Moreover, the eigenvalues are also dependent on \(G\), the performance surface of the algorithm may also be very eccentric, which results in a low speed of convergence.

To achieve a uniform rate of convergence, \(H\) should have a constant eigenvalue. That is, all the non-zero eigenvalues of \(H\) should be the same. It is possible by performing a metric conversion on Eq. (11) by letting
\( \mathbf{x} = \mathbf{Q}\dot{\mathbf{y}} \) \hspace{1cm} (15)

where

\( \mathbf{Q} = \text{diag}(\mathbf{Q}_1, \mathbf{Q}_2, \ldots, \mathbf{Q}_n) \) \hspace{1cm} (16)

and

\[
\mathbf{Q}_i = \text{diag}\left( \frac{1}{|G|}, \frac{1}{|G|}, \frac{1}{1 - G\hat{e}_{c,i} + G\hat{e}_{s,i}}t \right)_{\omega = \hat{\omega}_i}
\] \hspace{1cm} (17)

Thus, Eq. (11) can be rewritten as

\[
f(\dot{\mathbf{y}}) = \frac{1}{2}\dot{\mathbf{y}}^T\mathbf{Q}_i^T\mathbf{H}\mathbf{Q}_i\dot{\mathbf{y}} + \mathbf{b}^T\mathbf{Q}_i\dot{\mathbf{y}} + c
\] \hspace{1cm} (18)

It is easy to get the expression of \( \mathbf{Q}_i^T\mathbf{H}\mathbf{Q}_i \), which is also a block diagonal matrix, and the eigenvalues of each blocks are 2,0 and 0. Thus, a uniform rate of convergence for each frequency component can be obtained by using the metric conversion given in Eq. (15).

The adaptive iteration procedure can then be obtained by applying the steepest decent method on Eq. (18). Using Eq. (15) and discretising the result leads to the new recursive expression

\[
\mathbf{x}[k+1] = \mathbf{x}[k] - \mu\mathbf{Q}_i^T\nabla f(\mathbf{x}[k])
\] \hspace{1cm} (19)

where a better performance can be obtained to eliminate multiple periodic disturbances with unknown frequency components.

4. Numerical example

In order to verify the efficiency of the proposed method, numerical simulations are carried out to control the force transmissibility of a cantilever beam given by

The transfer function of the plant in this study is

\[
G(s) = \frac{5.078 \times 10^{-7}s^2 + 6.602 \times 10^{-6}s + 0.05012}{1.055 \times 10^{-9}s^4 + 2.109 \times 10^{-8}s^3 + 1.82 \times 10^{-4}s^2 + 9.7 \times 10^{-4}s + 0.9}
\] \hspace{1cm} (20)

and the corresponding frequency response is shown in Figure 2. It could be observed from the figure that the system has two resonant frequencies below 100Hz, which are 12Hz and 65Hz, respectively.

![Frequency response of the plant.](image_url)
4.1 Control of disturbance with known frequencies

In this example, the frequencies of the equivalent disturbance are 8Hz, 12Hz and 65Hz, and their magnitudes are all selected to be one. The proposed method is used here to control the vibration of the plant. As the metric conversion technique is adopted in the control algorithm, simply choosing the adaptive gain $\mu = 1$ would be appropriate for the iteration. The controller is switched on at $t = 5s$, and the results are shown in Figure 3, which illustrates that the control algorithm converges in 3 seconds and successfully suppressed the vibration.

![Figure 3: Control of disturbance with known frequencies using the proposed method.](image)

To show the efficiency of the proposed method, the control algorithm is compared with the classical algorithm given by Eq. (10) as shown in Figure 4. When using the classical algorithm to attenuate harmonic disturbance, a proper selection of the adaptive gain is of vital importance as shown in the figure. The classic algorithm converges very slowly when the adaptive gain is small and becomes unstable easily when the adaptive gain is large. Moreover, no matter what the adaptive gain is, the rate of convergence of the classical algorithm is not as good as the proposed method, as the optimal adaptive gain for each frequency component is different. However, with the introduction of metric conversion, a uniform rate of convergence can be determined and the performance of the algorithm can be much better than the classical algorithm when there are multiple frequency components.

![Figure 4: System output for different algorithms when the frequencies of the disturbance is known.](image)
4.2 Control of disturbance with unknown frequencies

Instead of knowing the exact frequencies of the disturbance, only an estimated frequency could be obtained in practice. In this example, the exact frequencies of the disturbance are respectively 8.125Hz, 12.375Hz and 64.855Hz and the initial estimated frequencies remain 8Hz, 12Hz and 65Hz. Figure 5(a) shows the simulation result of the control algorithm without frequency tuning, where the system output is amplified by the controller due to inaccurate frequency estimations. Thus, it is of vital importance to show the ability of the proposed method to deal with inaccurate initial frequency estimations.

The simulation results of controlling the plant using the proposed method is given in Figure 5(b). The figure shows that the proposed method can successfully eliminate the disturbance with inaccurate frequency estimations. The iteration of the estimated frequencies is given in Figure 6, which shows that all the estimated frequencies converge to the true value in a short time.

![Figure 5: Control of disturbance with unknown frequencies.](image)

(a) Without frequency tuning.  
(b) The proposed method.

![Figure 6: Iteration procedure of the estimated frequency.](image)

(a)  
(b)  
(c)

4.3 Control of disturbance with time-varying frequencies

In this section, we’ll show the ability of the proposed method to deal with sinusoidal disturbance with time-varying frequencies. The frequencies of the equivalent disturbance changes from 8.125Hz, 12.375Hz and 64.855Hz to 7.875Hz, 12.195Hz and 65.08Hz during the time interval $t \in [20s, 40s]$. The simulation result is shown in Figure 7 and the values of estimated frequencies with respect to time are given in Figure 8. The figures show that the algorithm has a very good performance even when the frequencies of the disturbance varies with time. Moreover, it should be mentioned that in this simulation, the Fast Fourier Transform (FFT) analysis is performed on the system output during the first 5 seconds before control starts, which provides initial value for the frequency estimation. It can be found that using the FFT analysis to estimate the frequency can accelerate the convergence of the algorithm.
Figure 7: Control of disturbance with time-varying frequencies.

Figure 8: Iteration procedure of the estimated frequency.

5. Conclusion

This paper proposes a new AFC algorithm which can be used to eliminate periodic disturbances with unknown and time-varying frequencies. The concept of EID is employed to avoid identification of the transfer path from disturbance to the system output. The performance function of the AFC algorithm is derived where the frequencies of the disturbance is assumed to be unknown. Based on the performance function, the steepest decent algorithm is used to track the EID and the metric conversion technique is introduced to accelerate the convergence. Some numerical simulations are performed to validate the algorithm, and it shows that the proposed algorithm has a good performance for sinusoidal disturbance.

REFERENCES


