Metrology as a scientific field focuses specifically on the definition and realisation of the units of measurements specified within a particular reference framework but also their required traceability. Such traceability used to be derived through physical objects called artifacts but significant research has resulted in a traceability shift from artifacts (with the kilogram becoming the final unit) to fundamental quantities and nature constants derived mostly in a quantum manner and natural constants. This paper reviews the research work in the area of acoustical metrology and its associated branches and how such a transition between classical to quantum frameworks can take place.

Keywords: acoustics, metrology, interferometry, photon correlation

1. Introduction

The International System of Units (SI) consists of the seven base units, namely the second (s), metre (m), ampere (A), kelvin (K), candela (cd), mole (mol) and the kilogram (kg). In addition, derived units are formed by combinations of their base counterparts: for example the ohm (Ω) derives its traceability through the kilogram, metre, second and ampere as follows:

\[ \Omega : \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3} \cdot \text{A}^{-2} \]  (1)

With the exception of the first six base units, the kilogram remained the very last unit to be traced to an artifact and, indeed, it was announced and agreed that within 2019 it will be replaced by a method that relies on fundamental constants rather than a physical object, the Kibble balance [1].

Ultimately all base units and thus all derived units within the SI will be traceable to fundamental constants derived in a quantum manner, therefore removing any reliance on artifacts and enabling national measurement institutes (NMIs) to develop their own experimental versions and realisations of such units.

In acoustical metrology, the unit of sound pressure is the pascal (Pa) and in terms of SI base units it is defined as:

\[ \text{Pa} : \text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2} \]  (2)
It may thus become evident from Eq. (2) that the pascal derived its traceability through a combined classical and quantum framework. With the potential redefinition of the kilogram in terms of the Kibble balance, its derived traceability will be through a fully quantum framework.

Obvious as it may seem, in order to realise a particular unit, one should also be able to measure the parameter that it is related to; for example to realise the metre, one should be able to measure length directly (as it is indeed the case for its SI realisation).

Acoustics metrology is broadly, but not entirely, divided into three separate technical areas; airborne, marine / underwater and ultrasonics. These areas have their own well-established international calibration standards, however only in the area of ultrasonics, acoustic pressures are directly measured and thus the acoustic pascal is directly realised: in contrast, standards based on the reciprocity principle have been adopted for airborne [2, 3] and underwater acoustics [4]. Due to the definition of reciprocity originally formulated by Lord Rayleigh [5] and adopted nearly two centuries later for the purposes of device calibration, there is no need to actually measure acoustic pressures at all to provide the sensitivity of a sensor. Moreover and as a consequence, the acoustic pascal is not derived in accordance to Eq. (2) for neither airborne nor underwater acoustics, with the only current exception being, as stated, ultrasonics.

Therefore, the area of acoustical metrology has been faced for quite some time with two major challenges; the first is to be able to realise the unit, the acoustic pascal, by measuring acoustic pressures directly and absolutely, while the second is to transition from a classical framework that depends on artifacts to a quantum framework that relies purely on fundamental nature constants.

2. Homodyne interferometry for ultrasonics

Medical ultrasonics was the first major branch in acoustics to propose and introduce a methodology towards a new absolute measurement framework non-reliant on artifacts [6]. The frequency range of interest varied from 500 kHz up to 15 MHz and subsequently increased to 70 MHz [7], even though there are applications where the frequency range of interest extends well above 100 MHz.

As the medium of interest is water where the speed of sound is approximately 1,500 m/s, typical particle displacements lie in the nanometre and picometre ranges. The measurement requirements for such small displacements, along with the need to minimize environmental vibrational contributions to an absolute minimum, necessitated the use of optical homodyne interferometry.

The choice of interferometer architecture may indeed be based on a Michelson or Mach-Zehnder configuration; however, any given system must be able to probe a thin pellicle (using a water-matched membrane with a reflective coating on one side) that follows the propagation of the sound field using a laser measurement beam that is then combined with a laser reference beam so that the amplitude demodulated signal may yield the particle displacement.

Indeed, there are many factors that are taken into careful consideration from a theoretical and experimental perspective in order to optically measure particle displacements due to ultrasound [8], including the development of additional interferometers to calibrate the required optical standard [9, 10]. However, at the fundamental level, the combination of the voltages resulting from the reference ($V_0$) and measurement ($V_i$) paths of the interferometer, in addition to the optical wavelength of the laser source $\lambda$ and refractive index of medium $n$, yields the required particle displacement $d$:

$$d = \frac{V_i \lambda}{V_0 4 \pi n}$$  \hspace{1cm} (3)

The particle displacement $d$ combined with the acoustic frequency $f$ yields the particle velocity $u$:

$$u = 2 \pi f d$$  \hspace{1cm} (4)

Subsequently, the free-field pressure $p$ is derived through the particle velocity, density of the medium $\rho$ and the speed of sound $c_s$ in that medium:
\[ p = \rho c_s u \]  

At this point, it is important to state some fundamental equations that are essential to the following discussion, namely the famous Planck-Einstein equation that relates energy \( E \), the Planck constant \( h \) and frequency \( v \):

\[ E = h v \]  

as well as the relationship between frequency \( v \), speed \( c \) and wavelength \( \lambda \):

\[ v = \frac{c}{\lambda} \]  

Equations (6) and (7) may be combined to yield the relationship between wavelength and the Planck constant:

\[ \lambda = \frac{hc}{E} \]  

By combining the parameters from Eqs. (3) and (4), the pressure equation simplifies to:

\[ p = \rho c_s f \frac{V_1}{V_0} \frac{\lambda}{2n} \]  

By substituting Eq. (8) into Eq. (9), a number of dependences may be observed:

\[ p = \left( \frac{hc}{E} \right) \left( \frac{\rho c_s}{n} \right) \left( \frac{f V_1}{2 V_0} \right) \]  

At this point, it is worth reminding the SI units of the quantities in Eq. namely:

\[ h : \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-1} \]  

\[ c : \text{m} \cdot \text{s}^{-1} \]  

\[ E : \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2} \]  

\[ \rho : \text{kg} \cdot \text{m}^{-3} \]  

\[ f : \text{s}^{-1} \]  

The right-hand side of Eq. (10) has been specifically arranged into three parts. The first part relates purely to quantum terms and when the units displayed in Eqs. (11), (12) and (13) are combined, it is found that the traceability is provided through the definition of the metre exclusively. The second part, contains quantities specifically relating to the experimental environmental conditions (density and refractive index of the medium, as well as the speed of sound) and when the units displayed in Eqs. (14) and (12) are combined, the traceability is provided through the definition of the kilogram, metre and the second. The third part contains quantities relating to the parameters that are ultimately controlled by the metrologist through measurements and it is found that the traceability is provided through the definition of the second as shown in Eq. (15).

Indeed, when all units shown in Eqs. (11) to (15) are included in Eq. (10), the acoustic pascal derives its traceability directly from the kilogram, metre and second as shown in Eq. (2).

3. **Heterodyne interferometry for underwater acoustics**

The technical area of underwater acoustics covers frequencies ranging from a few Hz up to (usually) 1 MHz, thus providing an overlap between audible and ultrasonic frequency ranges in water, as the name suggests. However, it should be mentioned at this point that even though from a physical perspective
sound can be audible in water, the term audible is almost explicitly related to the area of airborne acoustical metrology. Hydrophones for medical ultrasound applications (the technical area discussed previously) are characterized down to a few hundred kHz and for this reason water tanks with a capacity of hundred to a couple of thousand litres are adequate. However, hydrophones for marine and large underwater applications require water facilities with a capacity of a few thousand litres up to potentially small lakes for low frequency and/or large area applications such as for sonar.

For large water tank facilities, where the frequency range of interest can be as high a few MHz down to 20 kHz, the particle displacements due to sound propagation lie within the micrometre range. In addition, due to the large scale of the experimental facilities and hence large volumes of water which contribute to potentially small signal-to-noise ratio (SNR) signals, heterodyne rather than homodyne interferometry is most suited in such situations. For both cases of ultrasonics and underwater acoustics optical measurements, a water tank is required where a pellicle mounted on a frame and placed in the path of propagating sound (in burst mode) is required as shown in Fig. (1).

![Figure 1: water tank with a transparent optical window used for underwater acoustics and ultrasonics optical measurements of sound](image)

Very much like the case of homodyne interferometry discussed before, the optical configuration could be either Michelson or Mach-Zehnder and, indeed, the measurement arm of the interferometer probes a thin pellicle placed on the path of propagating sound.

However, in this case the reference arm of the interferometer is shifted in phase (typically 40 or 80 MHz) and therefore the phase demodulated signal yields the particle velocity [8]. Figure 2 shows a typical signal resulting from a homodyne and heterodyne interferometer used for ultrasonics and underwater acoustics respectively.

![Figure 2: typical signals for homodyne (left) and heterodyne (right) interferometers](image)
The aforementioned acoustic particle velocity is calculated as a function of the measured Doppler shift $\Delta f$ of the signal combined with the optical wavelength $\lambda$ of the laser source:

$$u = \frac{\lambda}{2} \Delta f$$

(16)

By combining Eqs. (5) and (16), the pressure equation simplifies to:

$$p = \rho c_s \left( \frac{\lambda}{2} \Delta f \right)$$

(17)

As previously, Eq. (8) is combined with the pressure equation so that, again, a number of dependencies may be observed:

$$p = \left( \frac{\hbar c_L}{E} \right) \left( \rho c_s \right) \left( \frac{\Delta f}{2} \right)$$

(18)

The first group of quantities, relates to quantum terms and the traceability derived through the metre, the second group relates to experimental environmental conditions (without any dependence to the density of the medium this time) and traceability through the kilogram, metre and second, while the last term relates to the accuracy of the measurement itself and is traceable to the second. Therefore, the acoustic pascal in underwater acoustics can derive its traceability directly from the kilogram, metre and second as shown in Eq. (2).

4. Photon correlation and interferometry for airborne acoustics

Airborne (also referred to as sound-in-air) acoustical metrology is arguably the most traditional area when it comes to new measurement methods. Indeed, it has been the last area that eventually is close to incorporating new metrological approaches and practices.

One reason is that the established calibration method based on reciprocity (coupler as well as free-field) has been the cornerstone and the undisputable standard for decades, offering uncertainties at the second decimal place in decibels. Such uncertainties are usually an order of magnitude smaller compared to the typical end-user requirements. However, the two fundamental issues with reciprocity is that it neither measures acoustic pressures (and hence Pa directly) nor it is artifact-free.

There are different reasons that prevailed up until recently the transition away from such a classical framework. One may recall that for medical ultrasound and marine hydrophone calibrations, a target (thin pellicle) is used that is small enough in dimensions compared to the acoustic wavelength as a probing target. This very feature enables the sufficient optical scattering of the measurement beam (as one side of the pellicle has either aluminium or gold coating) which when mixed with the reference beam results in the required demodulated signal. In addition, the excitation signal in the above cases is in the form of bursts which provides a sufficient time window to capture the optical signal before reflections from the water tank contaminated the main signal. Finally, commercial interferometric systems may be utilized as comparison tools that can validate the development stages of any potentially new optical standard in those technical areas.

Sound-in-air audible acoustical metrology is therefore a different case. In this case, the medium is air and consequently the speed of sound is 3 times lower than in water; in addition, the frequencies of interest lie for frequencies below 20 kHz. Also, the sound source is no longer a transducer that operates in burst mode, but a horn loudspeaker that is designed for continuous mode operation. The measurement environment is also a large (several cubic metres) fully anechoic chamber. All the above factors, prohibit the use of a reflective pellicle as target due to sound diffraction effects; this, consequently, means that optical homodyne or heterodyne interferometry is no longer an option.

There are commercial optical heterodyne dual-beam systems that rely exclusively on the use of airborne particle scattering but there are two fundamental disadvantages; the first is that significant amounts
of seeding are required to increase the required optical scattered signal strength and the second is that significant acoustic pressures are required in order to produce a sufficiently high signal-to-noise ratio. Or, in other words, one may re-formulate the disadvantages as being that in the first case one must measure and accurately quantify the seeded air density and in the second case the pressures required are so high that the vast majority of acoustic devices simply do not operate linearly in such ranges.

All the above issues and drawbacks required a significant amount of work in order to develop and assess new methods that can actually measure acoustic pressures (keeping in mind that reciprocity does not physically measure such a parameter) with seeding levels not affecting the air density and can operate on airborne particle scattering rather than requiring probing targets.

After significant research, gated photon correlation eventually became the most suitable candidate to become the next primary artifact-free optical standard [11] for sound-in-air metrology, enabling the move from classical to quantum dependence. The technique relies on the crossing of two identical laser beams (homodyne mode) at a point in space inside a free-field chamber producing optical interference, an example of the configuration shown in Fig. (3). Airborne particles due to propagating sound scatter photons as they oscillate through the optical fringes; these photons are remotely collected and subsequently analysed to yield the acoustic particle velocity as a function of the acoustic frequency \( f \), optical wavelength \( \lambda \), crossing angle of the laser beams \( \theta \) and the measured time quantity \( \tau \) resulting from the analysis of the captured photon sequences:

\[
\mathbf{u} = \frac{3.832 \, f \, \lambda}{4 \, \sin \theta \, \sin(\pi \, f \, \tau)} \quad (19)
\]

Figure 3: cross section of the free-field chamber showing the horn sound source and the two crossing laser beams where the acoustic particle velocity is remotely measured

It should be mentioned that the optical delivery and collection systems are placed outside the acoustical chamber; Fig. (4) shows typical photon sequences captured by the measurement system. By combining Eq. (19) into Eq. (5), one may deduce the acoustic pressure:

\[
p = \rho \, c_s \, \frac{3.832 \, f \, \lambda}{4 \, \sin \theta \, \sin(\pi \, f \, \tau)} \quad (20)
\]

Substituting Eq. (8) into Eq. (20) and re-arranging into groups as done previously yields the following expression:

\[
p = \left( \frac{h \, c}{E} \right) \left( \rho \, c_s \right) \left[ \frac{3.832 \, f}{4 \, \sin \theta \, \sin(\pi \, f \, \tau)} \right] \quad (21)
\]

The three groups, exactly in the same way as for medical ultrasonics and marine acoustics, underline a dependence (first group) with traceability to the metre, an environmental dependence (second group)
with traceability to the kilogram, metre and second, while also highlighting the experimental accuracy dependence (third group) with traceability through the definition of the second. Equation (21) shows that again in this case the acoustic pascal derives its traceability through the kilogram, metre and the second as shown in Eq. (2).

![Figure 4: captured photon sequences for airborne acoustic particle velocity measurements](image)

Finally, another area in acoustics where acoustic pressure and the realisation of the pascal may take place in a non-artifact manner is infrasonics [12]. In this case, a laser pistophone approach based on optical heterodyne interferometry is considered as an alternative standard method for this frequency range. As opposed to all the optical approaches discussed in previous sections where acoustic pressures are measured at a point in space, in this case a small coupler chamber is utilised where a piston provides the sound excitation. The pressure is derived as a function of the specific heat ratio \( k \), static pressure \( P \) in the chamber with a known volume \( V \) and area of the piston itself \( S \):

\[
p = \frac{k P S}{V} \times \text{x}
\]

In this case, the dynamic pressure is required over the volume. In addition, \( k \) is a dimensionless quantity, where \( S, x \) and \( V \) cancel out dimensionally, thus realising the Pa directly with traceability through the kilogram, metre and the second as shown in Eq. (2).

5. Conclusions

Acoustic metrology covers quite a large range of frequencies, from 0.01 Hz to over 100 MHz, in media such as air and water, with applications ranging from environmental vibrations, audiology, noise monitoring, sonar characterization, tomography and medical ultrasound. The range of end-users is equally complex in scope including manufacturers of transducers and sensors, national measurement institutes, hospitals and universities.

The aim of such a vast metrological area is improvement of the quality of life through traceable measurements of the most important parameter, pressure directly realising the required unit, the pascal. It is also equally important to remove the metrological reliance from existing manufacturers providing specific devices that act both as sensors and artifacts at the same time.

Opinions obviously differ, but metrology should neither necessarily nor exclusively rely on methods not measuring the required physical parameter or not realising its unit simply at the cost of unbelievably small uncertainties that are not required by end-users. Metrology, as the art of measurement, is about understanding the fundamentals of what is required to be measured and how, in an absolute and independent manner.

This paper has outlined and summarized the work that has been taking place within acoustics and the required paradigm shift from an existing classical framework to one that relies on purely fundamental
quantities in nature. People with an active interest and involvement in the area are coming to terms that this should take place fairly soon – and with the redefinition of the kilogram, such a necessary and fundamental transition in the entire area will eventually be realised.

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