EBR SCHEMES ON UNSTRUCTURED MESHES FOR APPLIED AEROACOUSTICS PROBLEMS

Tatiana Kozubskaya
Keldysh Institute of Applied Mathematics, Moscow, Russia
email: kozubskaya@imamod.ru

The paper presents an overview of the Edge-Based Reconstruction Schemes for solving Euler equations on unstructured meshes and gives some recent examples of using these schemes for aviation-industry-oriented problems.

Keywords: computational aeroacoustics, computational fluid dynamics, high accuracy method, finite-volume scheme, unstructured mesh

1. Introduction

The paper presents an overview of the Edge-Based Reconstruction (EBR) Schemes for solving Euler equations on unstructured meshes and gives some recent examples of using these schemes for aviation-industry-oriented problems.

The EBR schemes possess higher accuracy as compared with the traditional finite-volume second-order methods at lower costs as compared with the very high order algorithms. The higher accuracy is provided thanks to the quasi-1D reconstruction of variables on extended edge-oriented stencils on unstructured meshes so that in case of uniform grid-like meshes, the EBR schemes reduce to a high-order finite difference method. The lower costs result from the quasi-1D nature of these schemes. The EBR schemes have been extended to hybrid unstructured meshes and equipped with WENO-based shock-capturing techniques (so called WENO-EBR schemes). They have been implemented for interface regions of sliding meshes and are currently being adapted to prismatic boundary layers within the strand-mesh technology. The schemes are used for scale-resolving simulations of complex turbulent flows and associated acoustic fields.

In the paper we show some recent numerical results. In particular, we simulate the aeroacoustics of turbulent subsonic (M=0.9) and hot under-expanded jets, the noise generated by the turbulent flow over the tandem of square cylinders (FP7 VALIANT case) and over the three-component airfoil (30P30N model).

2. Edge-Based Reconstruction Schemes Unstructured Meshes

2.1 Basic Ideas of EBR Scheme

Consider the EBR schemes in more details and describe its main properties briefly for the Euler equations
\[
\frac{\partial \mathbf{Q}}{\partial t} + \nabla \cdot \mathbf{F}(\mathbf{Q}) = 0, \quad \mathbf{F}(\mathbf{Q}) = (\rho \mathbf{u}, \rho \mathbf{uu} + \rho \mathbf{I}, (E + p) \mathbf{u})^T
\]

(1)

written with respect to conservative variables \( \mathbf{Q} = (\rho, \rho \mathbf{u}, E)^T \). Here \( \rho \) – density, \( \mathbf{u} \) – velocity vector, \( p \) – pressure, \( \mathbf{I} \) – identity matrix, \( \mathbf{F} \) – the flux vector. The general formulation of edge-based vertex-centered schemes can be represented as

\[
\frac{d \mathbf{Q}}{dt}(i) = -\frac{1}{v_i} \sum_{j \in \mathcal{N}(i)} h_{ij} \left[ \mathbf{n}_i \right]_{ij}
\]

(2)

where \( v_i \) – the volume of the dual cell built around vertex \( i \), \( \mathcal{N}(i) \) – a set of first-level neighbours of node \( i \), \( h_{ij} \) – the numerical flux which is calculated as \( h_{ij} = \mathbf{F}(\mathbf{Q}) \cdot \mathbf{n}_i \), \( \mathbf{n}_i = \int_{\partial C_i} \mathbf{n} ds \), \( \partial C_i \) – the boundary of cell interface between nodes \( i \) and \( j \), \( \mathbf{n}_i \) – the oriented square of cell surface \( C_i \). A key point in the formulation (2) of EBR schemes is that both the numerical flux \( h_{ij} \) and the oriented square \( \mathbf{n}_i \) are evaluated at the edge \( ij \) midpoint only. A specific scheme of the EBR family is defined by the method of calculating flux \( h_{ij} \) on an extended edge-based quasi-1D stencil. We calculate \( h_{ij} \) as a solution of approximate Godunov-type Riemann solver or flux-splitting methods basing on the left and right states \( \mathbf{F}_{LR} \), \( \mathbf{F} = \mathbf{F}_{LR} n_i + \mathcal{F}_{LR} n_j \) or/and \( \mathbf{Q}_{LR} \) which, in their turn, are determined with the use of quasi-1D reconstructions. The reconstructions are built in a way that they transform to the corresponding high-order finite-difference approximations when applied to uniform grid-like (or TI) meshes. We denote an EBR scheme as EBRn scheme if its highest theoretical order (which is reachable on TI meshes) is equal to \( n \).

**Figure 1:** Quasi-1D stencils of EBR schemes: on TI-mesh (left), on arbitrary triangular mesh (right).

Figure 1 schematically shows the extended quasi-1D edge-based stencil we use for the numerical flux evaluation on a triangular unstructured mesh. The reconstructions for calculating the left and right states involve the values in the points with local numeration \( \{ \mathbf{r}_i \} \), \( i = 1, 6 \). The values in the points that do not coincide with the mesh vertices are determined with the help of linear interpolation on the correspondent edges which are intersected by the edge \( ij \) direction. In 3D, the reconstructions of left and right states are built in a similar way with replacing the intersected edges with the intersected faces. In particular, in the EBR5 scheme the reconstruction operators \( \mathbf{R}_{LR} \) acting on mesh function \( \psi \) can be written in terms of divided differences \( \Delta \psi_{s,t/2} = (\psi_{s+1} - \psi_s)/\Delta \psi_{s,t/2} \), \( \Delta \psi_{s,t/2} = |r_{s+1} - r_s| \) as

\[
\begin{align*}
\mathbf{R}_{LR}^\uparrow (\psi_s) &= \psi_s + \frac{\Delta \psi_{s,t/2}}{2} \left( \frac{1}{15} \Delta \psi_{s,t/2} + \frac{11}{30} \Delta \psi_{s/2,t/2} + \frac{4}{5} \Delta \psi_{s/2,3/2} - \frac{1}{10} \Delta \psi_{s,3/2} \right) \\
\mathbf{R}_{LR}^\downarrow (\psi_s) &= \psi_s - \frac{\Delta \psi_{s,t/2}}{2} \left( \frac{1}{15} \Delta \psi_{s+1,t/2} + \frac{11}{30} \Delta \psi_{s,t/2} + \frac{4}{5} \Delta \psi_{s/2,3/2} - \frac{1}{10} \Delta \psi_{s,3/2} \right) \\
\end{align*}
\]

(3)
All the stencils and metrical coefficients which provide the algorithm are determined at the preprocessing stage and the total required costs of EBR schemes appear not significantly higher than in the case of second-order TVD-type finite-volume methods.

The thorough description of EBR schemes is given in [1-4].

2.2 WENO-EBR Scheme for Shock Capturing

To treat possible discontinuities and solutions with high gradients, we equip the EBR schemes with the quasi-1D WENO techniques [5]. To implement shock capturing, we use the Riemann solver of Roe written in characteristic variables to which we apply the WENO-reconstruction in concordance with the classical finite-difference WENO scheme. In doing so, in our quasi-1D unstructured case, we represent the involved lower-order reconstructions and corresponding smoothness monitors in terms of divided differences along the 1D-stencils (Fig.1). The resulting WENO-reconstruction operator $\mathcal{R}_{\psi}^{\text{WENO},L}$ as applied to the left state of mesh function $\psi$ is given by the following formulas:

$$
\mathcal{R}_{\psi}^{\text{WENO},L}(\psi) = \psi_l + \sum_{k=1}^{3} \omega^{L,k}_{\psi} \mathcal{R}_{\psi}^{L,k}(\psi),
$$

with the weights $\omega^{L,k}_{\psi}$: 

$$
\omega^{L,k}_{\psi} = \frac{\sigma^{L,k}_{\psi}}{\sum_{k=1}^{3} \sigma^{L,k}_{\psi}}, \quad \sigma^{L,k}_{\psi} = \frac{\Omega^k}{(10^{-10} + IS^{L,k})}, \quad \Omega^1 = \frac{1}{10}, \quad \Omega^2 = \frac{6}{10}, \quad \Omega^3 = \frac{3}{10}
$$

which are defined in dependence on smoothness monitors $IS^{L,k}$:

$$
IS^{L,3} = \frac{13}{12} \left( \Delta\psi_{5/2} - \Delta\psi_{3/2} \right)^2 + \frac{1}{4} \left( 3\Delta\psi_{5/2} - \Delta\psi_{3/2} \right)^2,
$$

$$
IS^{L,2} = \frac{13}{12} \left( \Delta\psi_{3/2} - \Delta\psi_{5/2} \right)^2 + \frac{1}{4} \left( \Delta\psi_{7/2} + \Delta\psi_{5/2} \right)^2,
$$

$$
IS^{L,1} = \frac{13}{12} \left( \Delta\psi_{3/2} - \Delta\psi_{5/2} \right)^2 + \frac{1}{4} \left( \Delta\psi_{9/2} - 3\Delta\psi_{7/2} \right)^2.
$$

2.3 Hybrid EBR Scheme for LES-based Simulations

As is known, to provide correct LES-based turbulent-flow predictions, the scale resolving approaches require a carefully-calibrated balance between the numerical dissipation and instability. The method should be both accurate and stable enough, to allow for a correct representation of arising small physical instabilities and damping “parasitic” numerical oscillations. To meet the above requirements, in most cases we use the hybrid scheme of [6] that combine upwind and central-difference parts for the numerical-flux evaluation. In contrast to the original scheme our implementation is based on the high-order edge-based reconstructions

A presence of shocks in the regions of developed turbulence strongly complicates the process of building an appropriate hybrid scheme. We first design the following three-term hybrid formulation blending the central-difference, upwind and shock-capturing reconstructions:

$$
\mathcal{R}_{\psi}^{\text{HYBRID},L}(\psi) = \sigma^3 \mathcal{R}_{\psi}^{\text{WENO},L}(\psi) + (\sigma - \sigma^3) \mathcal{R}_{\psi}^{L}(\psi) + (1-\sigma) \mathcal{R}_{\psi}^{R}(\psi)
$$

where reconstructions $\mathcal{R}_{\psi}^{\text{WENO},L}(\psi)$ and $\mathcal{R}_{\psi}^{L}(\psi)$, $\mathcal{R}_{\psi}^{R}(\psi)$ are defined in (4) and (3) correspondingly, $\sigma \in [0,1]$ is the weight coefficient of upwind component of the hybrid scheme proposed in [6]. According to (6), the central-difference reconstruction and the corresponding numerical flux dominate when $\sigma$ varies.
near zero, the upwind (with no limiters) reconstruction is working when $\sigma$ is of moderate values and the WENO reconstruction gets involved when $\sigma$ approaches to 1. The development of sensor for switching to the complete shock-capturing scheme still present a problem. In practice, we use different problem-adjusted techniques. In most case, we control the value of $|p_i - p_j| / \min\{p_i, p_{i+1}\}$, where $p_i, p_{i+1}$ are the pressure values in two neighboring points.

2.4 EBR-Based Implementations

The current implementations of the EBR schemes mainly concerns the extension to strand and sliding meshes, immersed boundary method for moving objects, and dynamic adaptation of mesh-deformation type.

3. Numerical Results

3.1 Noise Generated by Turbulent Flow around Two Square Cylinders

The turbulent flow around a tandem of square cylinders at different angles of attack is simulated at $M=0.2$ and $Re=182000$ [7]. The numerical results are compared with the experimental data of NLR, the Netherlands. The case is one of the four generic cases on airframe noise studied in European FP7 VALIANT Project.

For turbulence treatment the hybrid Delayed Detached-Eddy Simulation (DDES) method with the standard Spalart-Allmaras turbulence model is used. The tetrahedral mesh consists of 13M nodes and 72M tetrahedrons. Along the span direction it contains 90 uniform steps, to support a close of uniform mesh in the LES region.

![Figure 2: Instantaneous flow fields in the central cross-section: density (left) and vorticity magnitude (right). Angle of attack 0°](image)

The turbulent content well captured by the numerical simulations can be seen in Fig. 2. Large scale quasi-2D coherent vortex shedding is visible. The quasi-2D structures are dispersed by the small-scale chaotic 3D turbulence. The vortices travel with the mean flow then break on the 2nd strut and slowly decay further downstream.

The far field acoustic spectra are experimentally measured in several control points on the pressure suction sides positioned with 15° resolution at 2-meter radius around the center of the upstream strut in the mid-span plane. The observers form an angle of 105° (obs. 1), 90° (obs. 2), 75° (obs. 3), and 60° (obs. 4) with respect to the downstream direction.
In Fig. 3 we see the dominant peaks in the surface-pressure spectra that are well captured for both struts with the maximum discrepancies of about 5 dB in level and about 20 Hz in frequency that seems quite accurate. The divergence for the far-field results with respect to the experimental data is in whole not larger than the ones observed for the surface-pressure spectra.

### 3.2 Jet Noise

The jet noise predictions using the EBR schemes are presented in [8] where the cases of subsonic jet (M=0.9, Re=) and underexpanded hot jet (Re=1269200) are studied. For both cases the ILES approach was used with the preliminary RANS-computed nozzle exit profiles.

Three meshes of 1.52 M, 4.13 M and 8.87 M vertices are used for the predictions. Fig. 4 (left) presents a snapshot of pressure time-derivative distribution that shows the acoustic field obtained on last one. Fig. 5 show the 1/3-octave spectra in the far field points at different observer angles in comparison with
the experiment data. It is seen that the mesh refinement provides the convergence and a better agreement with the reference data.

![Figure 5: Subsonic jet farfield acoustics: 1/3-octave spectrums versus the experimental data.](image)

The sound wave patterns of different nature and directions of propagation which are generated by the underexpanded hot jet are distinctively seen in Fig. 4 (right). The strongest ones are high-frequency Mach waves generating in the initial region of the jet shear layer and propagating predominantly at the angles of high emission. The presence of concentric wave patterns corresponds to the broadband shock cell noise generating by the interaction of shock cells with the turbulence. Fig. 6 shows the noise directivity at distance $100D$ and 1/3-octave spectra at different observer angles. The acoustics results in far field are rather well correlated with the reference data.

![Figure 6: Underexpanded hot jet far field noise: OASPL at distance 100D (top left) and 1/3-octave spectrums at different observer angles compared with the experiment and reference computation.](image)

### 3.3 Noise of High-Lift Devices (HLD)

In order to validate the EBR schemes for predicting the airframe noise from high-lift devices we study numerically the model configuration NASA MD-30P30N [10] that represents an unswept wing with deployed high-lift devices. This configuration allows to obtain a reasonable-quality numerical solution at rather low costs by apply periodic boundary conditions.

The 30P30N configuration was computed using a mesh of 36 million nodes made by extrusion of a 2D base mesh in the spanwise direction. The IDDES method [11] was used for simulating the turbulent flow. The view of the instantaneous flow field and the FW/H surface are shown in Fig. 7. Figure 8 presents the far field spectra at the maximum point of slat noise radiation ($\theta=290^\circ$, $0^\circ$ corresponds flow streamwise direction) compared with the reference data (taken from the paper [12]) obtained using different scale-resolving numerical algorithm on various meshes.
Figure 7: Instantaneous field of pressure time derivative and FW/H surface contour.

Figure 8: 30P30N HLD far field noise: reference predictions (left), our predictions with solid slat (blue) and permeable (black) FWH surfaces (right).

4. Conclusion

The paper presents the EBR schemes and their recent modifications in the light of simulating far field noise in aviation-oriented applications. We show the results of using these schemes for different representative cases. Thus, we predict the HLD airframe noise where a proper consideration of the near-wall turbulent flow is important, and the jet noise where the correct resolution of free turbulence is vitally needed. Basing on the performed predictions and other our practice, we pay attention to the EBR schemes as an effective compromise between costs and accuracy for solving applied aeroacoustics problems.

give the different examples of the aeroacoustics problems numerically studied using these schemes.

All the computations have been carried out using the in-house code NOISEtte.

The work is supported by the Russian Foundation for Basic Research (Project 18-01-00445).

REFERENCES


