This paper investigates nonlinear dynamic behaviors of a rotating blade subjected to the blast pulse. The rotating blade is simplified as a presetting, pretwist, rotating functionally graded material cylindrical panel. According to the power-law distribution, the effective material properties are assumed to be graded along the thickness direction. The centrifugal and Coriolis effects are considered in the structural model. Nonlinear strain-displacement relationships of the functionally graded material cylindrical panel are obtained by the Green strain tensor. The aerodynamic pressure is derived by the first-order piston theory. Based on the first-order shear deformation theory, nonlinear partial differential governing equations are obtained by using the Hamilton principle. Galerkin procedure is used to acquire the dynamic discrete system. Numerical simulations about nonlinear transient dynamic responses of the system subjected to the blast pulse are performed by considering the effects of different parameters of the functionally graded material cylindrical panel.

Keywords: transient analysis, functionally graded material, cylindrical panel, blast pulse

1. Introduction

Due to the smooth and continuous variation of the volume fraction for the constituent materials, the material properties such as Young’s modulus, density and thermal expansion coefficient, also change continuously. Therefore, functionally graded material (FGM) structures can relieve the problems of interfacial debonding and stress concentration [1]. In recent years, they have been regarded as one of the most promising candidates for future in many engineering fields, such as aerospace, rocketry, and many others. Such FGM blades are composed mostly of ceramic outside and metal inside, thereby having mechanical toughness and low thermal conductivity [2]. Much research has been carried out on the FGMs due to their excellent thermomechanical properties [3]. Many investigations have been paid more attention to FGM beams [4,5]. With the requirement of engineering application, more complex structures, such as FGM plates and shells [6-8], are being concerned.

The research about the modeling and behaviors of the rotating blade has been studied. Yao et al. [9,10] investigated nonlinear vibrations of the pretwist, presetting, thin-walled rotating beam. Sun et al. [11] presented a dynamic model to explain forced vibration behaviors of the rotating plate subjected to
a distribution force and a point force, respectively. Hu and Tsuiji [12] studied free vibration of the rotating twisted cylindrical panel by means of the Rayleigh-Ritz method.

In order to analyze vibration behaviors of the rotating pretwisted cantilever blade made of FGM, one needs to present a blade model that takes into account the continuously varying properties of constituent materials along the thickness direction and the rotational motion at the same time. Because of the complexity of the model, vibration behaviors of rotating FGM blades [13,14] have been rarely analyzed so far. It should be noted that the above mentioned scholars about rotating FGM blades attached more importance to the free vibration of the beams. However, almost no one has been focused on the nonlinear transient responses of the rotating pretwisted FGM cylindrical panels, which can realize the optimization design of the parameters for the blades. In this paper, a new dynamic model is developed to investigate the rotating pretwisted cantilever blade of the aero-engine compressor, which is made of functionally graded material (FGM). The rotating FGM blade is simplified as a presetting, pretwist, rotating FGM cylindrical panel. According to the power-law distribution, the effective material properties are assumed to be graded along the thickness direction. The centrifugal and the Coriolis effects are considered in the structural model. Based on the first-order shear deformation theory [15], nonlinear strain-displacement relationships of the FGM cylindrical panel are obtained by the Green strain tensor. The aerodynamic pressure is derived by the first-order piston theory [16].

Nonlinear partial differential governing equations are obtained by using the Hamilton principle. Galerkin procedure is used to acquire the dynamic discrete system. Numerical simulations about transient dynamic responses of the system subjected to the blast pulse are performed considering the effects of parameters of the FGM cylindrical panel and the blast pulse.

2. Formulation

The rotating pretwisted blade is modeled as a rotating FGM cylindrical panel with length \( L \), thickness \( h \), radius \( R \) and subtended angle \( \Phi \), as shown in Fig. 1. The FGM cylindrical panel is fixed on a rigid hub with the radius \( R_0 \) at a presetting angle \( \phi \), which rotates with a varying rotating speed \( \Omega(t) \). The rotating speed is \( \Omega(t) = \Omega_0 + f \cos \Omega t \), which includes the steady-state rotating speed \( \Omega_0 \) and the periodic perturbation rotating speed \( f \cos \Omega t \).

An inertial reference Cartesian coordinate system \( (X, Y, Z) \) is located at the center of the hub \( O_0 \). The origin of the rotating coordinate system \( (x_0, y_0, z_0) \) with the unit vector \( (i_1, i_2, i_3) \) is located at the blade root \( O \). The distance between the \( O \) and the center of the cylinder \( O_1 \) is \( e \). A curvilinear coordinate system \( (x, \theta, z) \) is located at the mid-surface of the FGM cylindrical panel.

The pretwist angle at an arbitrary cross section of the FGM cylindrical panel is \( K_0(x) \). The pretwist angle changes linearly along the spanwise direction, which is given by
where \( k = K / L \), and \( K \) is the pretwist angle at the free end of the FGM cylindrical panel.

### 2.1 Material properties of FGM

The material properties of the FGM cylindrical panel are assumed to be varied continuously and smoothly along the thickness direction according to the power-law distribution. The volume fractions of the ceramic and the metal can be written as follows [2]

\[
V_c (z) = \frac{2z^n}{h}, \quad V_m (z) = 1 - V_c (z),
\]

where \( V \) is the volume fraction, the subscripts \( c \) and \( m \) represent the ceramic and metal, respectively. Superscript \( n \) is the volume fraction index.

The effective material properties, such as Young’s modulus \( E \), mass density \( \rho \) and thermal expansion coefficient \( \alpha \), are determined according to a proper homogenization scheme by the linear rule of mixture

\[
P = P_c V_c + P_m V_m,
\]

where the typical temperature dependent material properties can be expressed as [17]

\[
P_i = P_0 \left( P_2 T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3 \right), \quad i = c, m,
\]

where \( P_0, P_1, P_2, P_3 \) are the coefficients of the temperature, and \( T \) is the temperature in Kelvin.

### 2.2 Strain-displacement relationship

The position vector of an arbitrary point \( Q_0 \) on the mid-surface for the undeformed FGM cylindrical panel is given by

\[
r_0 (0) = x i_1 + R \sin \theta i_2 + (R \cos \theta - e) i_3.
\]

The FGM cylindrical panel has a uniform rate of pretwist along the \( x \) axis since \( k \) is a constant. Therefore, \( i_2 \) and \( i_3 \) are the rotating vectors around the \( x \) axis. The covariant base vectors \( a_i \) and \( a_2 \) on the mid-surface associated with the point \( Q_0 \) are defined as the partial derivatives of \( r_0 (0) \) with respect to the \( x \) and \( \theta \), respectively. The unit vector \( a_3 \) perpendicular to \( a_i \) and \( a_2 \). Based on the Frenet-Serret formula, the above vectors are derived as follows

\[
a_i = \frac{\partial r_0 (0)}{\partial x} = i_1 + k(e - R \cos \theta) i_2 + k R \sin \theta i_3, \quad a_2 = \frac{\partial r_0 (0)}{\partial \theta} = R \cos \theta i_2 - R \sin \theta i_3, \quad a_3 = \frac{a_i \times a_2}{|a_i \times a_2|} = \frac{1}{B} (-k \sin \theta i_1 + \sin \theta i_2 + \cos \theta i_3),
\]

where \( B = \sqrt{1 + (k \sin \theta)^2} \).

An arbitrary point \( Q \) outside the mid-surface of the undeformed FGM cylindrical panel is obtained by

\[
r (0) = r_0 (0) + z a_3 = \left( x - \frac{z}{B} k \sin \theta \right) i_1 + \left( R \sin \theta + \frac{z}{B} \sin \theta \right) i_2 + \left( R \cos \theta - e + \frac{z}{B} \cos \theta \right) i_3.
\]

The displacement vector with the displacement components \( u(x, \theta, z, t) \), \( v(x, \theta, z, t) \) and \( w(x, \theta, z, t) \) is defined as follows

\[
U = u(x, \theta, z, t) a_i + v(x, \theta, z, t) a_2 + w(x, \theta, z, t) a_3.
\]
The position vector of an arbitrary point $Q$ outside the mid-surface of the deformed FGM cylindrical panel is given by

$$r = R_i + r^{(0)} + U.$$  \hfill (9)

A local orthogonal coordinate system $(\xi, \eta, \zeta)$ issuing from the point $Q$ before deformation is introduced. Based on the Green strain tensor, we derived the accurate strain-displacement relationship [18].

### 2.3 Displacement and constitutive equations

According to the first-order shear deformation theory, the displacement field of the FGM cylindrical panel is assumed to be

$$u(x, \theta, z, t) = u_0(x, \theta, t) + \varphi_z(x, \theta, t), \quad v(x, \theta, z, t) = v_0(x, \theta, t) + \varphi_y(x, \theta, t), \quad w(x, \theta, z, t) = w_0(x, \theta, t),$$  \hfill (10)

where $u_0$, $v_0$ and $w_0$ symbolize the displacements of an arbitrary point on the mid-surface of the FGM cylindrical panel along the $x$, $\theta$ and $z$ directions. $\varphi_x$ and $\varphi_0$ are the rotation of the transverse normal about the $\theta$ axis and the $x$ axis, respectively.

The thermo-elastic constitutive equation of the FGM cylindrical panel is given by

$$
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{xy} \\
\sigma_{xz} \\
\sigma_{yx} \\
\sigma_{yy} \\
\sigma_{yz} \\
\sigma_{zz}
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & 0 & 0 & 0 \\
Q_{12} & Q_{22} & 0 & 0 & 0 \\
0 & 0 & Q_{66} & 0 & 0 \\
0 & 0 & 0 & k_0Q_{55} & 0 \\
0 & 0 & 0 & 0 & k_0Q_{44}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{xy} \\
\varepsilon_{xz} \\
\varepsilon_{yx} \\
\varepsilon_{yy} \\
\varepsilon_{yz} \\
\varepsilon_{zz}
\end{bmatrix} -
\begin{bmatrix}
\alpha \\
\alpha \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\Delta T
\end{bmatrix},
$$  \hfill (11)

where $k_0$ is the shear correction factor and chosen as $5/6$.

In addition,

$$Q_{11} = Q_{22} = \frac{E}{1 - \nu^2}, \quad Q_{12} = \frac{\nu E}{1 - \nu^2}, \quad Q_{44} = Q_{55} = Q_{66} = \frac{E}{2(1 + \nu)},$$  \hfill (12)

where $\nu$ denotes Poisson’s ratio.

### 2.4 Equations of motion

The centrifugal force components of the FGM cylindrical panel along the $x_0$, $y_0$ and $z_0$ axes are given as follows

$$F_{x_0} = \rho \Omega^2 (R_0 + x - \frac{z}{B} k \sin \theta), \quad F_{y_0} = \rho \Omega^2 [(R + \frac{z}{B}) \cos (\phi - \theta) - e \cos \phi] \sin \phi,$$

$$F_{z_0} = \rho \Omega^2 [(R + \frac{z}{B}) \cos (\phi - \theta) - e \cos \phi] \cos \phi.$$  \hfill (13)

The centrifugal force is expressed by

$$\mathbf{F} = F_{x_0} \mathbf{i}_1 + F_{y_0} \mathbf{i}_2 + F_{z_0} \mathbf{i}_3.$$  \hfill (14)

The centrifugal force components of the FGM cylindrical panel in the curvilinear coordinate system $(x, \theta, z)$ are derived by

$$F_x = \mathbf{F} \cdot \mathbf{a}_1, \quad F_\theta = \mathbf{F} \cdot \mathbf{a}_2, \quad F_z = \mathbf{F} \cdot \mathbf{a}_3.$$  \hfill (15)

Based on the quasi-steady first-order piston theory [16], considering the curvature effect, the aerodynamic pressure in supersonic flow can be given by
\[
P_a = -\frac{\rho_w U_a^2}{\sqrt{M_a^2 - 1}} \left[ \frac{\partial w}{\partial x} + \left( \frac{M_a^2 - 2}{M_a^2 - 1} \right) \frac{1}{U_a} \frac{\partial w}{\partial t} - \frac{w}{2R\sqrt{M_a^2 - 1}} \right],
\]

where \( U_a \) = \( U_c \cos(\phi + kx) \), \( M_a \), \( \rho_w \), and \( U_a \) denote Mach number, the free stream air density and the velocity of incoming flow, respectively.

In this paper, we consider the single point excitation. Due to the blades with the high rotating speed, the excitation is instantaneous. The pulse load can be expressed by

\[
P_0 = P_{at} \delta(x - x_0) \delta(\theta - \theta_0)/R,
\]

where \( x_0 \) and \( \theta_0 \) are the location of the pulse load along \( x \) and \( \theta \) axes, respectively.

The blast load is given by

\[
P_{air} = P_m(1 - t/t_p)e^{-\alpha wt}, \quad 0 < t < t_p,
\]

where \( \alpha \) is the decay parameter and \( t_p \) is the positive phase duration.

Hamilton’s principle is utilized to derive the equation of motion for the FGM cylindrical panel

\[
\int_0^t (\delta U - \delta K + \delta W) dt = 0,
\]

where \( t \) is the time, \( \delta K \) is the virtual kinetic energy, \( \delta W \) is the virtual work done by the external forces and \( \delta U \) is the virtual strain energy, which induced by the deformation and centrifugal force [18].

The edges of the FGM cylindrical panel are clamped at \( x = 0 \) and free at \( x = L \), \( \theta = -\frac{\Phi}{2} \), \( \phi = \frac{\Phi}{2} \).

The form of the displacements and rotations are given in Ref [18].

The effects of the Coriolis force and periodic perturbation rotating speed are neglected. The first two modes of transverse nonlinear vibration are mainly considered. According to the studies given by Nosir [20], the inertia terms of \( u_0 \), \( v_0 \), \( \phi_0 \), and \( \phi_0 \) can be neglected. Therefore, the displacements \( u_0 \), \( v_0 \), \( \phi_0 \), and \( \phi_0 \) can be expressed by the transverse displacement \( w_0 \). Then the Galerkin procedure is adopted, the nonlinear ordinary differential equations of the FGM cylindrical panel are obtained by

\[
\ddot{w}_1 + (\mu_1 + \mu_2)w_1 + m_1w_1 + m_2w_2 + m_3w_3 + m_4w_4^2w_2 + m_5w_5^2 + m_6w_6w_2 + m_7w_7^2
\]

\[+ m_{18}w_1w_2^2 + m_{19}w_2^3 + m_{110} = m_{11}P_1(t),
\]

\[
\ddot{w}_2 + (\mu_21 + \mu_22)w_2 + m_{11}w_1 + m_{21}w_2 + m_{22}w_3^3 + m_{23}w_2^5w_2 + m_{25}w_1^5 + m_{13}w_6w_2 + m_{27}w_2^2
\]

\[+ m_{28}w_1w_2^2 + m_{29}w_2^3 + m_{210} = m_{211}P_2(t),
\]

where \( w_1 \) and \( w_2 \) are the amplitudes of radial vibration for the two modes. \( \mu_i \) and \( \mu_{1i} \) \((i = 1, 2)\) are the structural damping coefficients and the aerodynamic damping coefficients, respectively.

### 3. Numerical results

Nonlinear equations of motion for the FGM cylindrical panel in Eq. (24) will be solved by the Backward Differentiation Formula (BDF) in FORTRAN software. In this paper, the outer surface of the FGM cylindrical panel is assumed to be ceramic (Si₃N₄) rich and the core is assumed to be metal (SUS304) rich. The material properties of the FGM cylindrical panel are given in Ref [8]. Poisson’s ratio is selected as \( \nu = 0.3 \). Unless otherwise noted, the geometric parameters of the FGM cylindrical panel are given as follows, which the length is \( L = 0.3 \text{m} \), the thickness is \( h = 0.002 \text{m} \), the radius is
$R = 0.1m$, the subtended angle is $\Phi = 60^\circ$, the presetting angle is $\phi = 30^\circ$, the pretwist angle is $K = 30^\circ$, the hub radius is $R_0 = 0.5m$, the decay parameter is $\alpha = 0.5$, the positive phase duration is $t_p = 0.15ms$ and the air blast load is $P_m = 1 \times 10^6 N/m^2$. We perform nonlinear transient responses of the FGM cylindrical panel. The free end represents the location of the FGM cylindrical panel at $x = L$. We assume that the blast load acts on the tip end of the FGM cylindrical panel.

In order to validate our method, we perform transient responses of a simply supported FGM plate, which is composed of aluminum and alumina, subjected to a step load. The parameters are given, which the length is $a = 0.2m$, the width is $b = 0.2m$, the thickness is $h = 0.01m$, the step load ($P_{\text{step}} = P_m$, $0 < t < t_p$) is $P_m = 1 \times 10^6 N/m^2$ and the pulse duration is $t_p = 0.5ms$. The dimensionless center displacement acquired by the BDF in FORTRAN software is compared with the results of ANSYS software and the results given by Reddy [21], as shown in Fig. 2. The results of the BDF are in good agreement with ANSYS software and the literature.

Then, we investigate the effect of time step on nonlinear transient responses of the FGM cylindrical panel. Fig. 3 shows the effect of time step on the dimensionless free end radial displacement time history of the structure under the blast load. It is obvious that the transient dynamic responses of $dt = 10^{-5} ms$ and $dt = 10^{-6} ms$ are roughly the same. But, the result of $dt = 10^{-4} ms$ has obvious difference with $dt = 10^{-5} ms$ and $dt = 10^{-6} ms$. According to the above analysis, in order to ensure the accuracy and save the computing time, we use $dt = 10^{-5} ms$ in the following calculations.

![Figure 2: Comparison of nonlinear transient responses of the FGM plate under the step load](image)

![Figure 3: The effect of time step on the dimensionless free end radial displacement time history](image)

![Figure 4: The effect of positive phase duration on the dimensionless free end radial displacement time history](image)

![Figure 5: The effect of rotating speed on the dimensionless free end radial displacement time history](image)

Parametric studies for nonlinear transient dynamic responses of the FGM cylindrical panel under the blast load are discussed, as shown in Figs. 4-7. In Fig. 4, the effect of positive phase duration on the dimensionless free end radial displacement time history of the FGM cylindrical panel is performed. It is...
obvious that with the increase of the positive phase duration, the amplitude of the blast load increases, and the radial displacement of the system increases gradually. The effect of the rotating speed on the dimensionless free end radial displacement time history of the FGM cylindrical panel is depicted in Fig. 5. With the increase of the rotating speed, the radial displacement of the system increases.

In Fig. 6, the effect of the volume fraction index on the dimensionless free end radial displacement time history of the FGM cylindrical panel is investigated. It is observed that with the increase of the volume fraction index, the radial displacement of the system increases and the frequency of the system decreases. This is due to the fact that with the increase of the volume fraction index, the stiffness of the system will decrease, and the system is more easily to deform. The effect of temperature on the dimensionless free end radial displacement of the FGM cylindrical panel is discussed in Fig. 7. It is obvious that with the increase of the temperature, the radial displacement of the system increases and the frequency of the system decreases. This is the reason that with the increase of the temperature, the stiffness of the system decreases, which makes the system easy to deform.

![Figure 6: The effect of volume fraction index on the dimensionless free end radial displacement time history](image1)

![Figure 7: The effect of temperature on the dimensionless free end radial displacement time history](image2)

## 4. Conclusions

A presetting, pretwist, rotating FGM cylindrical panel model is developed to investigate the rotating pretwisted cantilever blade. According to the power-law distribution, the effective material properties are assumed to be graded along the thickness direction. The centrifugal and the Coriolis effects are considered in the structural model. Based on the general thin panel theory and the first-order shear deformation theory, nonlinear strain-displacement relationships of the FGM cylindrical panel are obtained by the Green strain tensor. The aerodynamic pressure is derived by the first-order piston theory. Nonlinear partial differential governing equations are obtained by using the Hamilton principle. Galerkin procedure is used to acquire the dynamic discrete system. Numerical simulations about transient dynamic responses of the system subjected to the blast load are performed. The amplitude of the blast load increases and the radial displacement of the system increases gradually when the positive phase duration increases. With the decrease of the rotating speed, the radial displacement of the system decreases. With the increase of the volume fraction index, the radial displacement of the system increases and the frequency of the system decreases. The increasing of the temperature can increase the radial displacement and decrease the frequency of the system.

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