TUNABLE DIRAC CONES IN TWO-DIMENSIONAL ACOUSTIC METAMATERIALS WITH MATRYOSHKA STRUCTURE

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Dirac cones of an acoustic system are the foundation of most topological phase transitions and topological states, and have recently become a research hotspot. Although the Dirac cones, Dirac-like cones, double Dirac cones and semi-Dirac points are all skillfully designed, it is still indispensable to realize a tunable Dirac cone in a novel acoustic structure. We propose two-dimensional acoustic metamaterials with matryoshka structure to achieve tunable Dirac cones and topological spin states. Dirac points can be obtained on the dispersion curves owing to the high symmetry. The concentric circular scattering units of the matryoshka structure are arranged in honeycomb lattices. By a rotating-scatterer mechanism to break the symmetry, the Dirac cone at K (K') is split and the topological spin states appear at the band valley. The existence of a topological transition with opposite Chern numbers as the rotating angle varies is also verified, and chair edge states are obtained along the interfaces separating the topologically opposite spin states insulators. Moreover, the frequency of the Dirac cone is tuned by rotating the inner structure in a double-layer matryoshka structure.

Keywords: Dirac cones, acoustic metamaterials, matryoshka structure, chair edge states

1. Introduction

In recent years, the topological states and the physical phenomena of acoustic systems have gradually become research hot spots [1]. Because the acoustic topology protection of boundary state transmission can effectively suppress backscattering and has a strong robustness to defects [2-4], it has great application prospects in acoustic communication, noise control and so on. Analogous to electronic systems, the topological phase transition and robust boundary states in band gap structures
can be excited by breaking the time-reversal symmetry, using temporal (or spatial) modulation [Fleury et al., 2016] and coupling resonance [2] or reducing the space symmetry [4] in artificial periodic structures, such as sonic crystals. The key to realize an acoustic topological phase transition is to degrade the dispersion relation of Dirac cones by introducing additional degrees of freedom, so as to obtain the spin states and then construct the acoustic topological insulator [5]. Therefore, the formation of Dirac cones in a band structure is the foundation of most topological phase transitions and topological states.

In this paper, we proposed two-dimensional acoustic metamaterials with matryoshka structure to achieve tunable Dirac cones and topological spin states. The matryoshka structure is usually composed of concentric circular scattering units, which can obtain multiple band gaps owing to the local resonances [6]. When we arrange these concentric circular scattering units in honeycomb lattices, Dirac points can be obtained on dispersion curves owing to the high symmetry. By a rotating-scatterer mechanism to break the symmetry, the Dirac cone is split and the topological spin states appear at the band valley. We also demonstrate the existence of a topological transition with opposite Chern numbers as the rotating angle varies, and obtain chair edge states which are along the interfaces separating the topologically opposite spin states insulators. Moreover, the frequency of the Dirac cone is tuned by rotating the inner structure in a double-layer matryoshka structure.

2. RESULTS AND DISCUSSION

Figure 1 (a) Layout of single-layer matryoshka structure. (b) Calculated band structures, where the solid line refers to the dispersion relation of the structure with $\alpha = \pi/9$, and the dashed line refers to the dispersion relation of the structure with $\alpha = \pi/6$. 

Figure 1(a) shows the schematic diagram of a unit cell of the single-layer matryoshka structure, where the white part is metal steel, which is considered as a rigid body in the calculation process, and the other parts correspond to air. The whole structure consists of a hexagonal array of slotted cylinders.
with three splits, and can be treated as a Helmholtz resonator with three necks. The structural geometry can be described by the lattice constant \( a (=12 \text{ mm}) \), the radius of the slotted cylinders \( r (=6 \text{ mm}) \) and \( R (=8 \text{ mm}) \), and the split width \( W (=0.4 \text{ mm}) \). Additionally, the orientation of the slotted cylinder is characterized by the angle \( \alpha (0 \leq \alpha \leq \pi/3) \) with respect to the vertical direction. In this paper, all calculations were carried out using COMSOL Multiphysics finite-element method (FEM) software. During the calculations, both the air and steel were considered, and their parameters were density \( \rho = 1.25 \text{ kg/m}^3 \), sound velocity \( C = 343 \text{ m/s} \) for the air background; and density \( \rho = 7800 \text{ kg/m}^3 \), sound velocity \( C = 5100 \text{ m/s} \) for the steel structure. Owing to the large difference between the acoustic impedance, the shear deformation of steel was neglected.

As shown in Fig. 1(a), it is noted that when \( \alpha = \pi/6 \), the whole structure has \( C_{3v} \) symmetry consistent with a hexagonal lattice, which has both a threefold rotational symmetry and three mirrors. When the internal steel structure rotates by a certain angle, \( \alpha \) is not equal to \( \pi/6 \), and the symmetry of the whole structure reduces to \( C_3 \) and only has threefold rotational symmetry. The band structures of the single-layer matryoshka structure with \( \alpha = \pi/6 \) (dash line) and \( \alpha = \pi/9 \) (solid line) are shown in Fig. 1(b). A Dirac cone (at 16.924 kHz) appeared at the inequivalent hexagonal corners \( K (K') \) of the first BZ for the single-layer matryoshka structure with \( \alpha = \pi/6 \), owing to the mirror symmetry protection. For \( \alpha = \pi/9 \), the Dirac cone at point \( K (K') \) splits and forms a pair of frequency extrema in the band structure owing to a symmetry mismatch between the steel structure and hexagonal lattice, as shown in Fig. 2(b). These two frequency extrema are respectively marked with valley states \( K_1 \) (at 16.924 kHz) and \( K_2 \) (at 17.563 kHz), and form a complete band gap.

Figure 2 shows the curves of two frequency extreme points varying with angle \( \alpha \), where the blue line represents valley states \( K_1 \) and red line represents valley states \( K_2 \). The band gap structure around the Dirac cone went from open to closed to open again when the rotating angle \( \alpha \) was changed from 0 to \( \pi/3 \) degrees. For \( \alpha = \pi/6 \), the band gap was closed, and the band gap was widest for \( \alpha = 0 \) or \( \pi/3 \). The color illustrations in Fig. 2 give the distribution of the absolute sound pressure at the valley states \( K_1 \) and \( K_2 \) for \( \alpha = \pi/9 \) and \( \alpha = 2\pi/9 \). When the rotating angle \( \alpha = \pi/9 \), the Dirac cone split and formed two valley states \( K_1 \) and \( K_2 \) owing to the loss of mirror symmetry. The pressure distributions suggested that the two valley states had an opposite vortex in the corner point of hexagonal unit cell: the valley state \( K_1 \) at the upper edge of the band gap had a clockwise chiral vortex, and the valley state \( K_2 \) at the lower edge of the band gap had an anti-clockwise chiral vortex. When the rotating angle \( \alpha = 2\pi/9 \), the two topological valley states exchanged their positions at the edge of the band gap. For the upper edge of the band gap, the valley state was occupied by \( K_2 \) with an anti-clockwise chiral vortex, and the valley...
state $K_1$ with a clockwise chiral vortex appeared at the lower edge. The whole band structure was reversed when the rotation angle $\alpha$ passed through $\pi/6$, and this inversion process excited the topological spin states and topological phase transitions.

![Diagram of band-edge frequencies](image)

Figure 2 Band-edge frequencies depicted for the acoustic system with different rotation angle $\alpha$, where the color illustrations show the distribution of absolute sound pressure at the valley states $K_1$ and $K_2$ for $\alpha = \pi/9$ and $\alpha = 2\pi/9$.

![Diagram of double-layer matryoshka structure](image)

Figure 3 (a) Layout of double-layer matryoshka structure. (b) Calculated band structures. The solid line refers to dispersion relation of the structure with $\alpha = \pi/9$, $\beta = \pi/9$, and the dashed line refers to dispersion relation of the structure with $\alpha = \pi/6$, $\beta = \pi/9$. 
Based on a single-layer matryoshka structure, the topological spin states and topological insulators were designed by rotating the scattering units. Subsequently, a double-layer matryoshka structure was designed to obtain a tunable Dirac cone. As shown in Fig. 3, the double-layer matryoshka structure had two layers of slotted cylinders. The whole geometry could be described by the lattice constant $a$ (=12 mm), the radius of the outer cylinders $r_1$ (=3 mm) and $R_1$ (= 4 mm), and the radius of the inner cylinders $r_2$ (=1.5 mm) and $R_2$ (= 2.5 mm), and the split width $W$ (=0.8 mm). Orientation of the outer and inner cylinder were characterized respectively by the angle $\alpha \leq \pi/3$ and $\beta \leq \pi/3$ with respect to the vertical direction. The band structures of the double-layer matryoshka structure with $\alpha = \pi/6, \beta = \pi/9$ (dashed line) and $\alpha = \pi/9, \beta = \pi/9$ (solid line) are shown in Fig.3 (b). Similar to single-layer structure with $\alpha = \pi/6$, the double-layer matryoshka structure also exhibited a Dirac cone at 15.306 kHz, and when the outer layer was rotated, the Dirac cone at point K (K') split and formed a pair of topological spin states. It is also noted that the Dirac cone still existed although the symmetry of the inner structure was mismatched with the hexagonal lattice.

Figure 4 (a) Frequencies of two topological spin states varying with angle $\alpha$. (b) Frequencies at Dirac cone with different $\beta$ of the double-layer matryoshka structure.

Figure 4(a) shows the curves of the frequencies of two topological spin states varying with angle $\alpha$. Compared with a single layer, the band gap structures also changed from open to closed to open again when angle $\alpha$ was varied from 0 to $\pi/3$ degrees, and the only difference was that the curves of the double layer was asymmetrical owing to the mirror symmetry mismatch between the inner and outer structures. The frequencies at the Dirac cone with different $\beta$ are given in Fig. 4(b). A tunable Dirac cone was obtained when rotating the inner layer. All results suggested that the Dirac cone only depended on the mirror symmetry of the outer structure. Hence, no matter how the inner structure was rotated, the band gap will close and form the Dirac cone when the symmetry of the outer structure matches with the hexagonal lattice, and the band gap opens when the outer symmetry is mismatched.
Moreover, the frequency position of the Dirac cone is affected by $\beta$, and it can be tuned by rotating the inner structure.

3. CONCLUSIONS

In this paper, matryoshka structures are proposed to achieve a Dirac cone and topological phase transitions. The results suggest that a Dirac cone appears at the inequivalent hexagonal corners $K$ ($K'$) for the single-layer matryoshka structure with $\alpha = \pi/6$, owing to the mirror symmetry protection. Additionally, the topological spin states and topological phase transitions with opposite Chern numbers are achieved by a rotating-scatterer mechanism to break the mirror symmetry. Movement of the acoustic edge states are further observed along the interfaces separating the topologically different valley insulators, and their transmission properties are determined in a zigzag bending channel. Moreover, a double-layer matryoshka structure is designed to obtain a tunable Dirac cone by rotating the inner layer. The results suggest these novel topological structures with a tunable Dirac cone offer the potential design of novel tunable acoustic topological materials and devices for practical applications in sound control fields.

REFERENCES


