A NEW METHOD TO GENERATE MODE WAVES IN DUCTS BY RIGID SPLITTERS

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In duct acoustics experiments, a controller is often required to generate the desired modes while suppressing all other cut-on modes. The present paper proposes a new method to actualize such a duct mode controller in the absence of flow by inserting in-duct rigid splitters in the sound source section. The interval between adjacent splitters is determined to ensure only plane wave propagating and each region separated by the splitters contains a loudspeaker flush mounted on the duct wall. The plane waves in each region can be easily controlled through properly setting the amplitude and phase of the corresponding loudspeaker, which finally generating a particular dominant mode in the experimental section. The relation between the settings of loudspeakers and the desired mode number is deterministic and the complex amplitudes of loudspeakers are controlled separately. By using numerical simulations, this method is checked in ducts with circular, annular and rectangular cross-section shapes and the mode wave dominance is found to be quite good.

Keywords: mode, controller, rigid splitters

1. Introduction

One of the first preparation steps in conducting duct acoustics experiments is to design a controller which is capable of generating desired modes while suppressing all other cut-on modes. The existing mode controlling techniques can be mainly divided into two categories. One is adding restrictor plate inside the duct between the loudspeaker and the experimental section. The restrictor plate lets certain modes pass through while the other modes are suppressed [1, 2]. This method can be applied in a broadband source excitation and only one loudspeaker is needed. But the undesired modes may still exist with large sound energy and this method cannot be extended into flow duct. The other is an array of loudspeakers flush mounted on the duct wall in a plane perpendicular to the duct centreline[3-8], which is usually called as spinning mode synthesizer in a circular or annular duct. By adjusting the amplitudes and phases of the loudspeakers [3, 4], individual duct modes or a given combination of modes can be generated. However, it is not easy to accurately set up the loudspeakers’ amplitudes and phases. Therefore, a complicated feedback or feedforward control system[3, 5-7] is required with the pressure signals monitored by an array of wall-mounted microphones in the downstream of the loudspeakers. The performance of such a device may greatly depend on the efficiency of the control system. This paper develops a new device with in-duct rigid splitters to generate desired modes in the absence of flow. The amplitudes and phases of each loudspeaker are determined by the desired mode number and can be controlled separately.
In Section 2, the method of mode generation without flow is illustrated in detail. The performance of the mode controller in ducts with different cross-section shapes are accessed by numerical simulations in Section 3. The conclusions are summarized in Section 4.

2. Mode generation method

2.1 Sound field generated by a wall-mounted loudspeaker in a semi-closed duct

Consider a semi-closed duct as shown in Fig. 1 and assume the duct size is so small or the frequency is so low that only plane wave is cut-on. A loudspeaker and a microphone are mounted on the duct wall at \( z = z_0 \) and \( z = z_1 \), respectively. Assume the angular frequency and complex amplitude of the loudspeaker is \( \omega \) and \( \zeta \). By using Green function method [9], the harmonic sound waves \( p_r \) and \( p_l \) radiated from the loudspeaker can be represented as

\[
p_r = -\frac{i\zeta}{2 S_D k} e^{ik(\ar - k(z - z_0))},
\]

\[
p_l = -\frac{i\zeta}{2 S_D k} e^{i(k(\ar + k(z - z_0))},
\]

which drop off all the cut-off modes. These expressions can be used in the duct region not very near the loudspeaker. Here \( S_D \) is the area of cross section and \( k = \omega / c_0 \). Assume the left end of the duct is rigid and the right end of the duct is a soft plane with reflection coefficient \( R_{ef} \). The sound wave reflected by the left end is \( p_b = -\frac{i\zeta}{2 S_D k} e^{i(k(\ar - k(z - L_R)))} \) where \( L_R \) is the distance from the loudspeaker to the left end, so the forward propagating sound wave in the duct with \( z_0 < z < 0 \) is \( p_r + p_b \). The sound wave reflected by the right end is thus \( R_{ef} (\hat{p}_r + \hat{p}_b) e^{i(\ar + kz)} \) where \( \hat{ } \) means the complex amplitude. Considering multiple reflections by both ends, the sound field in the duct with \( z_0 < z < 0 \) can be represented as

\[
p(z, t) = \left[ 1 + R_{ef} R' + R'^2 + ... \right] \left[ (\hat{p}_r + \hat{p}_b) e^{i(\ar - kz)} + [R_{ef} + R'^2 R' + R'^3 R'^2 + ...] (\hat{p}_l + \hat{p}_b) e^{i(\ar + kz)} \right]
\]

where \( R' = e^{-i(2L_R + k)L} \) and the complex amplitude \( A \) for the superimposed forward propagating sound wave is

\[
A = -\frac{i\zeta}{2 S_D k} \left[ e^{-ikL_R} + e^{-i(2L_R + k)L} \right] \left[ 1 + R_{ef} R' + R'^2 R'^2 + ... \right].
\]

From Eqs. (3) and (4), it can be seen that the pressure at a fixed location is proportional to the complex amplitude \( \zeta \) of the loudspeaker. So, in experiments, the measured pressure by the microphone shown in Fig. 1 can be used an input into a control algorithm to adjust \( \zeta \) to be the desired value since it directly reflects \( \zeta \).

Figure 1: Schematic diagram of a semi-closed duct.
2.2 Circular duct

Figure 2 shows axial section and cross section of mode controller in a circular duct. The left side of the mode controller is ended by rigid wall and the right side is open to the experimental duct. The main duct in the sound source section is separated into \( N_s \) equal secondary fan-shaped ducts by inserting rigid splitters. The interval between adjacent splitters is determined to ensure only plane wave cut-on, which means \( N_s \) is large enough to meet the requirement

\[
f_c > f_{\text{max}}
\]

where \( f_c \) is the cut-off frequency of the secondary duct and \( f_{\text{max}} \) is the upper limit of frequency in experiments. Here, the number \( N_s \) of the secondary ducts is limited to even numbers. In this case, \( f_c \) is the minimum value of the cut-off frequencies of \((0, 1)\) and \((N_s/2, 0)\) modes in the main circular duct, thus only the mode waves with \( m = -(N_s/2 - 1) \) are cut-on in the main duct.

In each secondary duct, there is a loudspeaker to generate sound wave and a microphone to monitor the sound field, both flush mounted on the duct wall. The measured pressure of the microphone is proportional to the complex amplitude of the loudspeaker in the same secondary duct, as described in Section 2.1. Therefore, the amplitudes and phases of the loudspeakers in different secondary duct can be controlled separately, making the control algorithm simpler than that used in spinning mode synthesizer [3, 5-7]. EE and FF planes, which are drawn as red and blue dotted lines in Fig. 2, denote the axial positions of loudspeakers and microphones, respectively. Note in order to ensure only plane waves need to be considered, the loudspeakers should be located far enough from the interface DD plane \((z = 0)\).

The sound pressure in the inlet of the experimental duct can be written as

\[
p_{\text{inlet}}(r, \varphi, z, t) = \sum_m \sum_n \left[ C_{mn} J_m(k_{mn} r) e^{i(ax-k_{mn} z)} + D_{mn} J_m(k_{mn} r) e^{i(ax+k_{mn} z)} \right] e^{i\omega t}.
\]

Here, \( m \) and \( n \) are the spinning and radial mode numbers, respectively. \( k_{mn} \) and \( k_{mn} \) are the radial- and axial-wave numbers in the inlet duct, respectively.

In a secondary duct numbered \( i \), the sound pressure near DD plane can be represented as

\[
p_{MC,i}(z, t) = A_i \left( e^{-ix} + R_{ef,i} e^{ix} \right) e^{i\omega t},
\]

according to Eq. (3). Considering the axial asymmetry of the mode controller, the reflection coefficients \( R_{ef,i} \) in all secondary ducts should be the same, so their subscript \( i \) will be dropped in the next illustration.

Based on the continuity of sound pressure on DD plane, we obtain

\[
\int_{S_i} p_{MC,i}(z=0) dS = \int_{S_i} p_{\text{inlet}}(z=0) dS, \quad i = 1, 2, \ldots, N_s
\]

where \( \int_{S_i} \cdot dS \) is the integral on the cross section of a secondary duct numbered \( i \). After substituting Eqs. (6) and (7) into Eq. (8) and with some manipulations, the sound pressure continuity equations become

\[
A_i \left( 1 + R_{ef} \right) S_D = \sum_{m=-(N_s/2-1)}^{N_s/2-1} \chi_m S_{\varphi,m}, \quad i = 1, 2, \ldots, N_s.
\]

Here, \( S_D \) is the area of the secondary duct, \( \chi_m = \sum_n (C_{mn} + D_{mn}) \int_0^R J_m(k_{mn} r) dr \) and \( S_{\varphi,m} = \int_{\varphi_{m-1}}^{\varphi_m} e^{i\omega t} d\varphi \) (the range of \( \varphi \) for a secondary duct numbered \( i \) being \( \varphi_{m-1} \sim \varphi_m \)).

There are \( N_s \) equations in Eq. (9) while the number of coefficients \( \chi_m \)'s for cut-on modes is \( N_s - 1 \). So, \( \chi_m \)'s can be determined by specifying the ratios of \( A_i \) to each other, i.e., \( A_1 : A_2 : \cdots : A_{N_s} \). If a single...
mode with \( m = m_0 \) is required, which means the coefficients \( \chi_m \) s with \( m \neq m_0 \) should be zero, \( A_1 : A_2 : \cdots : A_{N_s} \) need to be controlled to be equal to \( S_{\varphi,m_0,1} : S_{\varphi,m_0,2} : \cdots : S_{\varphi,m_0,N_s} \). According to Eq. (4), \( A_i \propto \zeta_i \) where \( \zeta_i \) is the complex amplitude of the loudspeaker in a secondary duct numbered \( i \). Therefore, the ratios of \( \zeta_i \) to each other should be controlled as

\[
\zeta_1 : \zeta_2 : \cdots : \zeta_{N_s} = S_{\varphi,m_0,1} : S_{\varphi,m_0,2} : \cdots : S_{\varphi,m_0,N_s}.
\] (10)

![Diagram](image)

Figure 2: (a) Axial section and (b) cross section of mode controller in a circular duct.

(Here the number of secondary ducts in mode controller is \( N_s = 4 \).

### 2.3 Annular duct

Figure 3 shows the axial section and cross section of mode controller in an annular duct. The main duct in the mode controller is separated into \( N_s \) equal secondary annular sector ducts by inserting rigid splitters. The method to generate the desired spinning modes in an annular duct is almost the same with that in a circular duct, except that the calculation of cut-off frequency \( f_c \) should be based on the eigenvalue in the annular duct. The ratios of \( \zeta_i \) should be controlled as Eq.(10) to generate a single mode with \( m = m_0 \).

![Diagram](image)

Figure 3: (a) Axial section and (b) cross section of mode controller in an annular duct.

(Here the number of secondary ducts in mode controller is \( N_s = 4 \).)
2.4 Rectangular duct

Figure 4 shows the axial section and cross section of mode controller in a rectangular duct. The main duct in the mode controller is separated into $N_x \times N_y$ equal secondary rectangular ducts by inserting rigid splitters. Assume the duct is reflection symmetry about both $x = H/2$ and $y = W/2$ planes. So, the reflection coefficients $R_{i,j}$s in all secondary ducts are nearly equal to each other. The mode generation method in a rectangular duct is also similar to that in a circular duct. If $(m_0, n_0)$ mode is expected to be dominant in the inlet duct, two requirements should be met:

(a) $N_x$ and $N_y$ should satisfy

$$N_x \geq \frac{2H f_{\text{max}}}{c_0}, \quad N_y \geq \frac{2W f_{\text{max}}}{c_0};$$

(b) The ratios of complex amplitudes $\zeta_{i,j}$ of loudspeakers in different secondary ducts should be controlled as

$$\zeta_{1,1} : \zeta_{1,2} : \cdots : \zeta_{i,j} : \cdots : \zeta_{N_x,N_y} = S_{1,1,m_0,n_0} : S_{1,2,m_0,n_0} : \cdots : S_{i,j,m_0,n_0} : \cdots : S_{N_x,N_y,m_0,n_0},$$

where

$$S_{i,j,m_0,n_0} = \int_{y=-N_y/2}^{N_y/2} \int_{x=-N_x/2}^{N_x/2} \frac{m_0 \pi x}{H} \cos \left( \frac{n_0 \pi y}{W} \right) dx dy.$$

3. Numerical simulations

In this section, the mode generation method is checked in three kinds of ducts by using the module “acpr” in COMSOL. The mode amplitude ratio is used to access the performance of the mode controller. It is defined as the ratio of the desired mode’s amplitude to the next highest amplitude mode expressed in dB [7]. The loudspeakers are simplified to monopole point sources and the rigid splitters are modelled as interior sound hard boundaries. The complex amplitudes of point sources are assumed to be well controlled to be the desired values, in other words, the ratios of $\zeta_j$ to each other is given in the numerical simulations. The maximum mesh size is set to $c_0/10 f_{\text{max}}$ and the main duct is terminated with a PML (perfectly matched layer) domain.

3.1 Circular duct

As shown in Fig. 2, the number of secondary ducts is $N_x = 4$. The radius of the main duct is set as...
$R = 5 \text{ cm}$, so, $f_{\text{max}} < 3335 \text{Hz}$ according to Eq. (5). The test section duct is lined with a locally reacting liner whose impedance is $2 - i$. Five point probes are arranged in the inlet duct to decompose the sound field into different spinning modes.

If the mode with $m = 1$ is required to be dominant in the inlet duct, the complex amplitudes of the point sources should be controlled as $\zeta_1 : \zeta_2 : \zeta_3 : \zeta_4 = i : -1 : -i : 1$ according to Eq. (10). Set the frequency as $f = 2700 \text{Hz}$ which is larger than the cut-off frequency ($2010 \text{Hz}$) for the desired mode. Mode waves with $m = -1 \sim 1$ are cut-on in the main duct. The simulated sound field and the modal decomposition are shown in Figs. 5(a) and 5(b), respectively. It can be seen from Fig. 5(b) that the mode amplitude ratio is greater than 30 dB.

3.2 Annular duct

The mode controller with $N_s = 4$ is used and the outer and inner radii are set as $R_o = 2 \text{ cm}$ and $R_d = 6 \text{ cm}$, respectively. So, $f_{\text{max}} < 2668.1 \text{Hz}$. The impedance of the outer wall in the test section is set as $2 - i$ and there are five point probes in the inlet duct for modal decomposition.

Like in circular duct, the complex amplitudes of the point sources should be controlled as $\zeta_1 : \zeta_2 : \zeta_3 : \zeta_4 = i : -1 : -i : 1$ to generate a single mode with $m = 1$. Set the frequency as $f = 2000 \text{Hz}$ so that the range of $m$ for cut-on mode waves is $-1 \sim 1$ in the main duct. The simulated sound field and the modal decomposition are shown in Figs. 6(a) and 6(b), respectively. The mode amplitude ratio in this case is greater than 30 dB.
### 3.3 Rectangular duct

As can be seen in Fig. 4, the main duct is separated by rigid splitters into \( N_x \times N_y = 2 \times 4 = 8 \) equal secondary ducts. The width \( W \) and the height \( H \) of the duct are set as 16 cm and 8 cm, respectively. So, \( f_{\text{max}} < 4287.5 \text{Hz} \) according to Eq. (11). The impedances on the upper \((x = 0)\) and lower \((x = H)\) duct walls are both set as \(2 - i\). Sixteen point probes are placed in the inlet duct wall for modal decomposition.

According to Eq. (12), the complex amplitudes of the point sources should be controlled as \(\xi_{1,1} : \xi_{1,2} : \xi_{1,3} : \xi_{1,4} : \xi_{2,1} : \xi_{2,2} : \xi_{2,3} : \xi_{2,4} = 1 : \sqrt{2} : -1 : -1 : -\sqrt{2} : 1 : 1 : 1\) to generate a single \((1,1)\) mode. Set the frequency as \(f = 3000 \text{Hz}\) at which the mode waves with \(m = 0\)~1 and \(n = 0\)~3 are cut-on. The simulated sound field and the modal decomposition are shown in Figs. 7(a) and 7(b), respectively. From Fig. 7, it can be seen that amplitude of incident \((1,1)\) mode is at least 30 dB larger than that of other incident modes, so the desired incident mode is indeed dominant in the inlet duct.

![Modal decomposition](image-url)

**Figure 7:** (a) The simulated sound field in a rectangular duct and (b) modal decomposition in the inlet duct.

### 4. Conclusions

A new mode generation method is developed by inserting in-duct rigid splitters in the sound source section and controlling the complex amplitudes of wall-mounted loudspeakers in equal separated secondary ducts. In all secondary duct, only plane wave is cut-on so that the complex amplitudes of loudspeakers are easily determined by the desired modes through sound pressure continuity condition at the interface between the mode controller and the main duct. The complex amplitudes of loudspeakers can be controlled separately and microphones can be flush mounted in each secondary duct to give inputs into the control system.

This method is checked in ducts with circular, annular and rectangular cross-section shapes by using numerical simulations and modal decomposition is made with simulated data. From the results, it can be seen that the performance of the mode controller can be very good provided the ratios of complex amplitudes of loudspeakers to each other satisfy the requirement associated with the desired mode number.

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REFERENCES


