THE 50TH ANNIVERSARY OF THE TRANSPORT THEORY IN CONDENSED MATTER PHYSICS

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In 1966 Woon Siong Gan coined and invented the name transport theory during his PhD works at the physics department of Imperial College London. This is the first introduction of transport theory into condensed matter physics. Transport theory was previously used in neutron transport theory for nuclear reactor design. In 1967 Philip Anderson and Volker Heine changed the name of the solid state theory group to condensed matter theory group of the Cavendish Laboratory, Cambridge University to unify solid state physics and liquid state physics and to reflect the significance of phase transition in condensed matter physics. Gan’s PhD thesis(1969) is entitled Transport Theory in Magnetoacoustics. It pioneered statistical mechanics approach to ultrasound propagation in semiconductor in the presence of high magnetic fields and low temperatures. Thus his PhD thesis also played a role in the founding of the field of condensed matter physics. In 1971 the international journal Transport Theory and Statistical Physics was launched by Taylor & Francis. Today transport theory is the key foundation of theoretical materials design. In this paper transport phenomena and transport theory are explained. Today quantum transport theory has a more important role than classical transport theory. Quantum transport theory has given rise to three Nobel physics awards: quantum Hall effect, Anderson localization, and Onsager reciprocal relations.

Keywords: transport theory, condensed matter physics

1. Introduction

This year will be celebrating the 50th Anniversary of the transport theory in condensed matter physics. Woon Siong Gan coined and invented the name transport theory during his PhD works at the physics department of the Imperial College London in 1966. Gan’s PhD thesis[1] Transport Theory in Magnetoacoustics is dated 1969. Prior to this transport theory has been used only in neutron transport theory for the design of nuclear reactors. Gan’s PhD thesis first introduced transport theory into condensed matter physics. Transport theory is a manifestation of the Boltzmann transport equation which is used in classical statistical mechanics. This is an integral equation in terms of the Boltzmann probability distribution function. Transport theory also introduced the role of phase transition into condensed matter physics. It is the theory of transport phenomena. Electrons and phonons are transport phenomena. There is a singularity behaviour of the transport properties during the phase transition. It is of interest to note...
that in the year 1967, Philip Anderson and Volker Heine changed the name of the solid state theory group at the Cavendish Laboratory, Cambridge University to the condensed matter theory group to reflect the important role of phase transition and this is also the unification of solid state physics and liquid state physics to form condensed matter physics. Hence his PhD thesis also played a role in the founding of the field of condensed matter physics. Today transport theory is the key foundation of the theoretical design of new materials. Its status in condensed matter physics is equivalent to that of the Yang Mills theory in particle physics.

At the start transport theory was a classical theory. However, as time evolved, it has developed into the quantum transport theory since the 1980s. Quantum transport theory has given rise to the award of three Nobel physics prizes: the Quantum Hall Effect, the Anderson Localization, and the Onsager Reciprocal Relations. Today quantum transport theory plays a more important role than classical transport theory in condensed matter physics.

2. What is transport theory?

Transport theory is the theory of transport phenomena. Examples of transport phenomena are electrons and phonons (J. M. Ziman, Electrons and Phonons, The Theory of Transport Phenomena in Solids, Oxford University Press, 2001) and ultrasound attenuation in solids. Transport phenomena have wide applications in condensed matter physics, the motion and interaction of electrons, holes, and phonons are studied under transport phenomena. In biomedical engineering, some transport phenomena of interest include microfluidics, thermoregulation, and perfusion. In mechanical engineering, transport phenomena are studied in reactor design, analysis of molecular or diffusive transport mechanisms and metallurgy.

In materials, atoms are arranged in a particular way. Any stimulus state takes the material state away from thermal equilibrium. Then material responds by transferring energy, charge, spin, momentum etc from one spatial part to another. Transport theory is an attempt to construct a theory that relates the material response to the stimulus. All materials are used for their response to stimulus, e.g. wool(sweater), silicon(computer chip), copper(wire), carbon (writing) etc. Key material question: what atoms and how should I arrange them to get a desired response to a particular type of stimulus. Hence transport theory lays key foundation of theoretical materials design.

3. What is transport phenomena

Transport phenomena are ubiquitous throughout the engineering discipline. Transport phenomena encompass all agents of physical change in the universe. Moreover, they are considered to be fundamental building blocks which developed the universe and which is responsible for the success of life on earth. However, the scope is limited to the relationship of transport phenomena to artificial engineered systems.

In physics, transport phenomena are all irreversible processes of statistical nature stemming from the random continuous motion of molecules, mostly observed in fluids. Every aspect of transport phenomena is grounded in two primary concepts: the conservation laws and the constitutive equations. The conservation law, which in the context of transport phenomena are formulated as continuity equation, describes how the quantity being studied must be conserved. The constitutive equations describe how the quantity in question responds to various stimuli via transport. Prominent examples include Fourier’s
Law of Heat Conduction, and the Navier Stokes equation which describe respectively the response of heat flux to temperature gradients and the relationship between fluid flux and the forces applied to the fluid.

These equations also demonstrate the deep connection between transport phenomena and thermodynamics, a connection that explains why transport phenomena are irreversible. Almost all of these physical phenomena ultimately involve systems seeking their lowest energy state in keeping with the principle of minimum energy. As they approach this state, they tend to achieve true thermodynamic equilibrium, at which point there are no longer any driving forces in the system and transport ceases. The various aspects of such equilibria are directly connected to a specific transport: heat transfer is the system's attempt to achieve thermal equilibrium with its environment, just as mass and momentum transport move the system towards chemical and mechanical equilibrium.

Examples of transport processes include heat conduction (energy transfer), fluid flow (momentum transfer), molecular diffusion (mass transfer), radiation and electric charge transfer in semiconductors.

Transport phenomena have wide application. For example, in solid state physics, the motion and interaction of electrons, holes and phonons are studied under transport phenomena.

Quantum transport in mesoscopic systems-introduction to electronic transport in mesoscopic systems

It is well known that the conductance $G$ of a macroscopic conductor is given as $G = \sigma A / L$, where $\sigma = $ conductivity, the intrinsic property of the conducting material, $A =$ area of cross-section of the conductor and $L$ its length. So the conductance decreases as the cross-sectional area is reduced and it increases as the length of the conductor is reduced.

One may suspect that the conductance goes to infinitely large values as the length of the conductor is made extremely small. But this is not true. The above mentioned simple scaling law or the so called Ohmic behaviour breaks down at mesoscopic length scales(sub-micrometre length scales) to be defined precisely later. It does not become infinite, but it reaches a limiting value $G_c$.

To understand the cause of the breakdown of this simple scaling law, one has to take into account the quantum nature of the electrons, according to which the electron is not a classically tiny charged particle quantum mechanical wave-particle. This wave characteristic of the electron is responsible for many analogies in the field of Anderson localization, propagation of light through a random medium, and mesoscopic condition through a disordered sample with static disorder. The mesoscopic length scales (usually submicrons) are defined as length scales at which the wave character of electron has definite effect on the measurable physical property such as conductance. The conductance no longer monotonically varies but it shows jumps or steps in units of $G_c = 2e^2 / h$. This is a universal character independent of the material of the sample.

The physics of electron transport is called mesoscopic transport physics when (i) the de-Broglie wavelength associated with the electrons, (ii) the mean free path which is the distance travelled by the electron before its initial momentum is destroyed, and (iii) the phase relaxation time which is the distance travelled by the electron before it initial phase is destroyed because of the order of the sample size. In this regime, all our classical intuition about electron transport breaks down. New interacting effects appear, such as the above mentioned conductance quantization, universal conductance fluctuations, interface effects in mesoscopic fields(Aharanov-Bohm oscillation),interesting phenomena in driven quantum dots such as Coulomb blockades, Kondon effect, non-equilibrium Kondo effect, quantum dynamics of the Kondo effect.
4. Onsager reciprocal relations

Onsager reciprocal relations is an example of transport phenomena. Onsager reciprocal relations express the equality of certain ratios between flows and forces in thermodynamic systems out of equilibrium, but where a notion of local equilibrium exists. Reciprocal relations occur between different pairs of forces and flows in a variety of physical systems. For example, consider fluid systems described in terms of temperature, matter density, and pressure. In this class of systems, it is known that temperature differences lead to heat flows from the warmer to the colder parts of the system. Similarly, pressure difference will lead to matter flow from high pressure to low pressure regions. What is remarkable is the observation that when both pressure and temperature vary, temperature differences at constant pressure can cause matter flow (as in convection) and pressure differences at constant temperature can cause heat flow. Perhaps surprisingly, the heat flow per unit of pressure difference and density (matter) flow per unit of temperature difference are equal. This equality was shown to be necessary by Lars Onsager using statistical mechanics as a consequence of the time reversibility of microscopic dynamics (microscopic reversibility). The theory developed by Onsager is much more general than this example and capable of treating more than two thermodynamic forces at once with the limitation that the principle of dynamical reversibility does not apply when (external) magnetic fields or Coriolis forces are present, in which case the reciprocal relations break down.

5. The quantum Hall effect

The quantum Hall effect is a quantum mechanical version of the Hall effect, observed in two-dimensional electron systems subjected to low temperatures and strong magnetic fields, in which the Hall conductance undergoes quantum Hall transitions to take on the quantized values

$$\sigma = I_{\text{channel}} = \nu e^2 V_{\text{Hall}} h$$

where $I_{\text{channel}}$ = channel current, $V_{\text{Hall}}$ = Hall voltage, $e$ = elementary charge, and $h$ = Planck’s constant. The prefactor $\nu$ is known as the filling factor and can take on either integer or fractional values. The quantum Hall effect is referred to as the integer or fractional quantum Hall effect depending on whether $\nu$ is an integer or fraction respectively. The striking feature of the integer quantum Hall effect is the persistence of the quantization (i.e. the Hall plateau) as the electron density is varied. Since the electron density remains constant when the Fermi level is in a clean spectral gap, this situation corresponds to one where the Fermi level is an energy with a finite density of states, though these states are localized. The fractional quantum Hall effect is more complicated as its existence relies fundamentally on electron-electron interactions. The fractional quantum Hall effect is also understood as an integer Hall effect although not of electrons but of charge flux composites known as composite fermions. There is also a new concept of the quantum spin Hall effect which is an analogue of the quantum Hall effect where spin currents flow instead of charge currents.

6. Anderson localization

In condensed matter physics, Anderson localization is the absence of diffusion of waves in a disordered medium. This phenomenon is named after the American physicist P W Anderson who was the first to suggest that electron localization is possible in a lattice potential, provided that the degree of randomness (disorder) in the lattice is sufficiently large, as can be realized for example in a semiconductor with impurities or defects. Anderson localization is a general wave phenomenon that applied to the transport of electromagnetic waves, acoustic waves, quantum waves, spin waves, etc. This phenomenon is to be distinguished from weak localization, which is the precursor effect of Anderson localization., and
from Mott localization, named after Sir Nevill Mott where the transition from metallic to insulating behaviour is not due to disorder but to a strong mutual Coulomb repulsion of electrons. It is shown that strong disorder can be employed to obtain high quality wavefront due to the Anderson localization phenomenon in a transverse Anderson localizing optical fiber.

7. **International Journal on Transport Theory**

   In 1971, the International Journal of Transport Theory and Statistical Physics was published by Taylor & Francis. It is still in existence today although the name has been changed to Journal of Computational & Theoretical Transport in 2014.

8. **Books on transport theory**

   4. Quantum Transport by Supriyo Datta, Cambridge University Press, 2005. This is on quantum transport theory.
   5. Quantum Transport: Introduction to Nanoscience by Yuli Y Nazarov & Yaroslav M Blanter. This is on quantum transport theory.
   6. Quantum Transport in Nanoscale Devices by D Vasileska, D Mamaluy, I Knezevic, H R Khan, and S M Goodnick. This is on quantum transport theory.

9. **Transport theory used in phase transition and in metamaterials design**

   The following is an example on how the transport theory can play a role in the theoretical design of new materials. It is of interest to note that in a metamaterial there is a singularity behaviour of the transport properties at the point of phase transition or at the resonance frequency. The permittivity will rise to infinity with a sudden drop to negative infinity and then with a gradual rise in the negative region. This is similar to the hyperbolic shape. Some examples of transport properties are conductivity, permeability, permittivity, viscosity, thermal conductivity, diffusivity etc. Hence by studying the singularity behaviour of different transport properties at the point of phase transition, one can discover new materials theoretically.

**REFERENCES**